New Analytical Model of Distributed Coordination Function

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Summary
Wireless access networks of various types are the most frequently used technologies in the last mile solution i.e. bringing connectivity to the customer or end user. Wireless connections are also increasingly used inside buildings instead of cables, mainly in cases where the support of mobile users is needed. They are mostly using notebooks, organizers and PDAs for an access to company intranet by Wi-Fi technology. More often than beforewireless IP phonesare also used with Wi-Fi. The systems for measuring and controlling the quality of service are needed more and more. The 802.11b and 802.11g standards defineas one of wireless media the function called Distributed Coordination Function. It is based on the random principle and ensures rightful access to media. 802.11e contains the Enhanced Distributed Coordination Function, which can provide prioritized access for time critical traffic. The objective of the paper is to mathematically represent the Distributed Coordination Function such that it can be used as the basis of EDCF analysis.

Key words:
DCF, WI-FI, QoS, Collision Probability

1. Introduction

Wireless networks IEEE 802.11 are mostly known as Wi-Fi (Wireless Fidelity) networks. The standard describes physical and link layers. The basic idea of Wi-Fi networks was to create a technology that will replace the clumsy and not very flexible cables in office and home buildings. In the course of time it has been shown that although highly promising, the Wi-Fi technology does not have the potential to absolutely replace cable technologies. Wireless networks reach lower transfer speeds, higher latencies and are less secure against potential eavesdroppers. However, wireless networks have become an excellent connectivity alternative for mobile users, who, thanks to Wi-Fi, are not limited by cable reach.
In many cases, Wi-Fi networks and technologies are nowadays used for almost absolutely different purposes than they are designed for. They are often applied as an extremely cheap solution of the last-mile connectivity problem, and are used by both home users and companies. This means that Wi-Fi technology is used for creating outdoor connections, but it has never been designed for the task. The main reason is the price. Thanks to the very high spread of the technology and extreme production series, current prices of hardware are extremely low, many times lower than the prices of very sophisticated solutions.
In 802.11b and 802.11g, the Distributed Coordination Function (DCF) [1] is used to ensure access to the media for wireless stations. The following sections include a mathematical analysis of this function.

2. Distributed coordination function and its mathematical model

2.1 Basic description of Distributed Coordination Function, media access competition

Theoretically, the Distributed Coordination Function uses the Carrier Sense Multiple Access with Collision Avoidance (CSMA/CA) [1] media access system. Practically, this model cannot be ensured all the time, and the whole process is mostly analogous to the Carrier Sense Multiple Access with Collision Detection (CSMA/CD) [2] systems.
It follows in part from the above that the biggest problem of DCF is collisions. The algorithm aims to prohibit all collisions, but it is not possible. Every frame is therefore confirmed by the second side of communication. The whole process is described in. To enhance the access model, the frames can also include information about the time for which the media will be occupied [1][2][3].

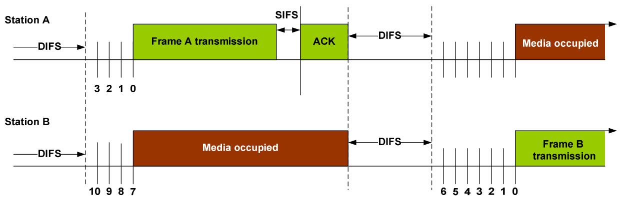

Fig. 1 Media access model of Distributed Coordination Function [1]

For media access, the Distributed Coordination Function uses the so called Contention Window (CW). Its size is for every station and traffic category limited by the interval between CWmin and CWmax. While the Wireless station

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(STA [1]) has data to send, it detects if the media is occupied. If not, STA generates a random value from the interval <0, w – 1>, where w is equal to CWmin. After that, STA starts to count down to zero. During the whole countdown, STA also detects if the media is clear. If not, the countdown is stopped [1].
In the media access competition stage and in the countdown, a media access model is used that, for the sake of synchronization, splitts the time into timeslots. Because of this, the probability of collision is, especially with a higher number of stations, relatively high. For the 802.11b technology the CWmin attribute value is 31 and 1023 for CWmax. For 802.11g the CWmin value is even lower, only 15. The probability that all stations intend to send data at same time, is low but possible. This probability grows with rising number of stations and higher utilization of the media.

2.2 Basic collision probability model for DCF

In the first place, it is necessary to introduce the probability model for one single station. After that, the model should be extendeded to a higher number of stations. Every station can be in one of the following states:

1) Station is in idle state, it has no data to transfer
2) Station has data to transfer and is waiting for media release
3) Station has data to transfer and is transmitting

When considering the hidden node problem [4], collision may occur only when switching between the second and the third state.
Let pv be the non-zero probability of a new data frame arriving on the data input of wireless station. Assume that this probability does not change in time and thus stays constant at every moment.
The next event is when the frame has arrived at the wireless station data input. In that case, the wireless station waits until the wireless media is clear and then it generates a new backoff w from interval <0; W0>, when W0 = CWmin. After that, and when no collision occurs, the data are transferred. In the other case, a new backoff is generated.
Analytical models of the Distributed coordination function [5][6] mostly premise that the probability of collision pk is independent of the number of previous unsuccessful transmission attempts. The probability that collision will not occur is 1 – pk. With these preconditions, the Basic Probability Model of Distributed Coordination Function can be proposed (Figure 2).

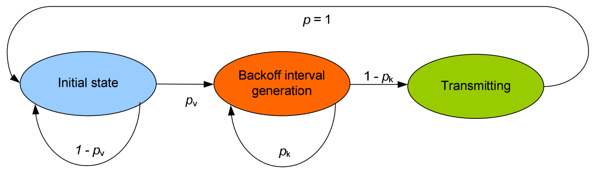

Fig. 2 Basic Probability Model of Distributed Coordination Function

The probability of collision illustrates the fact that two or more independent wireless stations generate the same backoff interval at the same time. Another possibility is that the station which is generating its backoff interval generates the value of backoff equal to the current countdown value of another station.

2.3 Bianchi model

Bianchi proposes in his paper [5]:
The key approximation in our model is that, at each transmission attempt, and regardless of the number of retransmissions suffered, each packet collides with constant and independent probability p.
Bianchi goes on the introduce two new variables: variable W, which determines the actual maximum value of backoff interval and m, which indicates the maximum backoff stage, influencing the maximum value of backoff interval. As stated above, the maximum size of backoff interval is limited by the value of CW_{max}. The following formula results from all these facts:

$$W = 2^m \cdot CW_{min} \tag{1}$$

Based on these facts, Bianchi proposes the following probability model

$$\begin{cases} P\{i,k|i,k+1\} = 1 & k \in (0, W_i - 2) \quad i \in (0,m) \\ P\{0,k|i,0\} = (1-p)/W_0 & k \in (0, W_0 - 1) \quad i \in (0,m) \\ P\{i,k|i-1,0\} = p/W_i & k \in (0, W_i - 1) \quad i \in (1,m) \\ P\{m,k|m,0\} = p/W_m & k \in (0, W_m - 1) \end{cases} \tag{2}$$

The first formula defines the probability that in every time-slot, the backoff interval will be lowered by one. This probability is equal to 1 in the case that the wireless media is not occupied. The second equation illustrates the fact that after successful transmission, the new backoff interval is chosen from <0; W_0 - 1>, while W_0 is equal to CW_{min}. The last two equations describe the collision situations, while the third corresponds to generating new backoff from a higher interval, and the fourth determines the probability that the maximum backoff stage has been reached.

Based on these equations, Bianchi proposes a Markov chain, which is shown in.

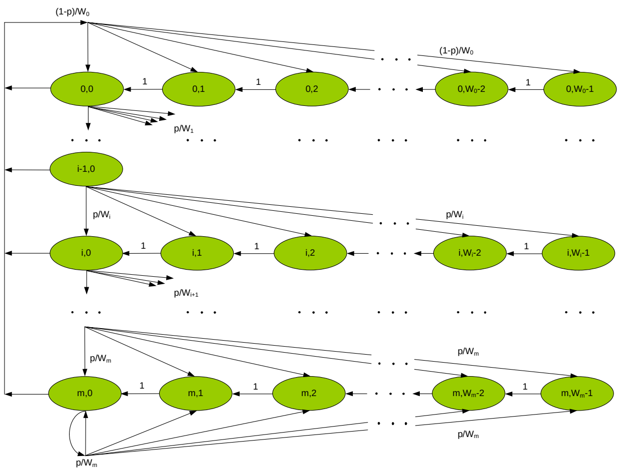

Fig. 3 Markov chain describing backoff stage interval [5]

Bianchi's model simplifies and facilitates the definition of a Distributed Coordination Function system, and allows constructing a relatively very accurate analytical model, which describes the Distributed Coordination Function. With this model, the simulations of system behavior can be carried out and the throughput calculations can be made. As stated above, Bianchi's model is based on the fact that the probability of collision is all the time the same, independent of the number of stations or the number of previous collisions.
This assumption is not completely accurate, mainly when it is necessary [there is need] to analyze the probability of collision at a particular moment. When the first transmission attempt is achieved, the backoff interval is chosen from a relatively small interval. Assume that two wireless stations are trying to get a transmission opportunity. For both stations, it is their first attempt, without any previous collisions. The probability of collision is in this situation increasing rapidly.
The probability that two stations need to send data over wireless media at exactly the same time is very low. But when transmission is requested while the media is occupied by another station, the probability of backoff generation at the same time is growing. The whole fact is demonstrated in Figure 4.

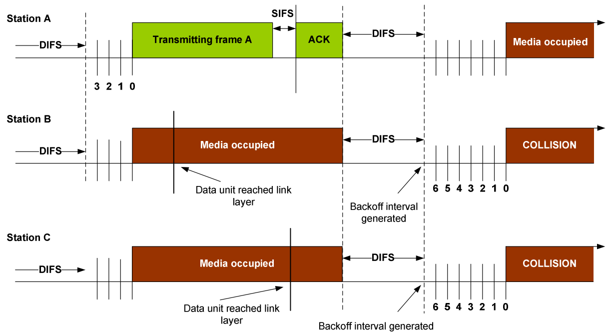

Fig. 4 Generation of backoff interval for two stations in case that data frames reach link layer at different times, but while the media is occupied

In this case, stations attempting to transfer data over wireless media are waiting until it is clear and then they generate the backoff interval and start the countdown.

3. Collision probability analysis

Assume that wireless stations have already started the process leading to data frame transmission over wireless media. The momentthe media is released, both of them generatea backoff from w from the interval <0, $CW_{\min}$>. The probability of generating a particular number W_n is

$$p_{Wn} = \frac{1}{w}, \qquad (3)$$

The probability that the second station generates a random number W_m from the same interval as W_n and that collision will occur is expressed by the formula:

$$p_c(m \mid n) = \frac{p_{Wn} \cdot p_{Wm}}{p_n}. \qquad (4)$$

When these two events are independent and W_m and W_n are generated from the same interval, we can claim that the probability of p_{Wn} and p_{Wm} is the same. The whole formula can now be reduced to.

$$p_c = \frac{1}{w}, \qquad (5)$$

In real service operation there are often more than two stations. A standard wireless network can support tens of stations. The probability of collision in the case that all stations connected to one network will request media access at the same time will be derived from the following:

Because the speed of countdown is at all times the same for all stations, collision will occur when two or more stations choose the same W. First, the set of all possible events has to be determined. If the number of stations is equal to n, the set of all possible events will be equal to

$$M = V'(n,w) = w^n \tag{6}$$

The next logical step is to determine the set of events that will lead to collision. A simpler way is to first define the set of events that will not lead to collision:

$$N = V(n,w) = \prod_{i=0}^{n-1}(w-i) \tag{7}$$

The probability that collision does not occur can be expressed as

$$p_{nc}(n,w) = \frac{N}{M} = \frac{\prod_{i=0}^{n-1}(w-i)}{w^n} = \prod_{i=0}^{n-1}\frac{w-i}{w} \tag{8}$$

The probability of collision is inverse to p_{nc}

$$p_c(n,w) = 1 - p_{nc} = 1 - \prod_{i=0}^{n-1}\frac{w-i}{w} \tag{9}$$

Previously published models [5][6] count all the time with the same and independent collision probability. From (9) it follows that the probability of collision depends on the number of stations generating the backoff and on the size of the interval from which backoff is chosen. In Table 1 the probabilities of collision for various numbers of stations and various backoff stages are given. The values are also shown in Figure 5.

Table 1: Probability of collision for various numbers of stations and various backoff stage

Backoff stage[-]	1	2	3	4	5	6	7
Window size w [-] / Number of stations	15	30	60	120	240	480	1024
1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0667	0.0333	0.0167	0.0083	0.0042	0.0021	0.0010
4	0.3529	0.1880	0.0970	0.0492	0.0248	0.0125	0.0062
8	0.8988	0.6403	0.3858	0.2121	0.1112	0.0570	0.0288
16	1.0000	0.9929	0.8890	0.6487	0.4002	0.2233	0.1181
32	1.0000	1.0000	1.0000	0.9894	0.8851	0.6524	0.4069

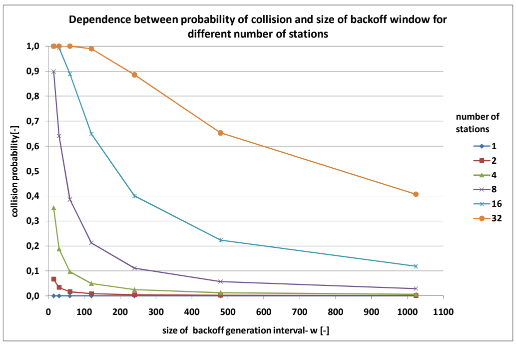

Fig. 5 Dependence of probability of collision on size of backoff window for different numbers of stations

4. Conclusion

With the 802.11e standard, wireless media access methods have mostly been changed. For their mapping, the basic access methods need to be analyzed. Previous models are very strong in throughput simulations but not very realistic in the mathematical analysis of the particular state of system. This paper analyzed the Distributed Coordination Function by means of the probability of collision, and showed that with a higher number of stations, the collision probability rises very fast. Subsequent work was targeted at analyzing the probability of collision when the stations are in different backoff stages, and also at modeling incoming traffic. Conclusions from the present work will be published very soon.

Acknowledgments

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