

Evolutionary Optimized Networks and Their Properties

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Summary

Networks in the real world have a variety of structures and they are different in many respects. Among them, in both natural and artificial networks, they often show scale-free as the result of optimization of growth. An important feature of many complex networks is the structure and performance. Such networks with desirable properties become important in a variety of applications such as in supply chain networks, computer and transportation networks etc.

In this paper we present a methodology of evolutionary design of optimized networks in which the structure of a network is designed to optimize various performance measurements. We propose a methodology in which a complex system optimizes its network structure in order to optimize its overall object function. Especially these in turn depend on two critical measures of the network performances, congestion and economy in terms of design cost. In this paper, we use the genetic algorithm (GA) as a tool of optimization.

We also propose some methodologies to investigate the properties of evolved networks. The objective functions of GA are the combination of the congestion function which is defined by node betweenness and the density of links. We show that an evolutionary optimization process can account for the observed regularities displayed by most networks. Using a graph theoretical case study, we show that when design cost is paramount the Star network emerges and when congestion is important the dense network is found. When congestion and design cost requirements are both important to varying degrees, other classes of networks such as the network with multiples hubs including scale-free emerge. Four major types of networks are encountered: (a) sparse exponential-like networks, (b) sparse scale-free networks, (c) star networks and (d) highly dense networks. The evolutionary consequences of these results are outlined.

Key words:

Traffic network, Congestion, Optimal network, Genetic algorithm

1. Introduction

One of the outstanding problems in complex adaptive systems found in engineering, biology, ecology, economics, sociology, and so on, is explaining and predicting the emergence of self-organized network structures with very interesting properties [1]. Recently, there have been attempts to propose mechanisms for the emergence of the scale-free topologies for such networks. Barábasi and Albert have suggested preferential attachment as a mechanism and these results provide

valuable insights into the structure of the scale-free networks [2]. An important feature of many complex systems, both natural and artificial, is the structure and organization of their interaction networks with interesting properties. Such networks are found in a variety of applications such as in supply chain networks, computer and communication networks etc. Networks in the real world a variety of structures and they are different in many respects. However, the questions of why and how the different network configurations emerge, what is the significance of these different topologies, why do we find similar topologies in diverse applications, and what, if any, is the common underlying governing principle remain to be investigated further.

We propose a general conceptual framework for self-organization of a network by evolutionary adaptation, modeled after Darwin, in which the system's, i.e. the network's (We use these terms interchangeably in this paper), objective is to maximize its chances of overall survival by adapting its configuration according to the environmental pressure. The basic premise is that networks found in nature today exhibit certain characteristic configurations and properties because the same helped them survive the test of time and natural selection. A network typically serves to transport material, energy, and/or information; thus the idea of survival, in all the discussion to follow, is a general one to mean performance towards achieving the design objectives of the network. Therefore, the novel hypothesis is that although human-engineered networks such as supply chains or communication networks have not necessarily 'emerged' by evolutionary adaptation, the underlying design principles that led to their creation could be very similar to those that caused natural networks to evolve to their present forms. The universality of scale-free and other features found in a variety of networks, natural or otherwise, lends support to this view. The proposed framework seeks to shed light on these principles and their guiding influence on network evolution. We will illustrate how the external environment, which imposes or demands certain survival objectives, critically determines the optimal configuration. These insights can be valuable for the study and analysis of all networks under various service environments. In this spirit, the framework applies equally to natural as well as human-engineered networks.

We use two objectives, network congestion and design cost and they are often conflicting, requiring a trade-off in the design. We use the term congestion as a measure of the effectiveness of the information flow over the network to accomplish its functions. By design cost we mean the link density and we want to minimize it. As noted, these are often conflicting objectives. For example, in the design of engineered systems such as chemical plants, automobiles etc., as one tries to minimize the congestion of the system while keeping the costs down, robustness suffers and vice versa. Design and control engineers are instinctively aware of this compromise. Just as in real-life cost or economic constraints are an unavoidable reality, in nature, too, there are such cost constraints. We summarize all the above ideas in the following postulates of our theory for self-organization of complex systems by evolutionary adaptation:

- (i) For complex systems such as complex networks, nature adapts the system configuration (i.e. topology) via the processes of evolution and natural selection so as to maximize an overall survival fitness function under a given environmental selection pressure.
- (ii) The overall survival fitness function consists of both short and long-term survival components. These components are dependent on two important measures determined by the configuration - the efficiency and robustness of the system towards performing its functions or objectives. Efficiency governs the short-term survival, whereas robustness impacts the long-term survival.
- (ii) Depending on the system's functional goal and its survival environment, nature chooses to optimize for efficiency, robustness or both under cost constraints, for 'average case' or 'worst-case' survival.

2. A Graph Theoretic Formulation

Consider a system, i.e. a network, consisting of several individual members or nodes. Let us assume that the survival of this hypothetical system depends on the ability of each node to communicate or interact with all other nodes in an efficient and robust manner. The interactions can be through the exchange of material, energy, and/or information. As long as a node is connected to another node, it can communicate with it and pass on messages to others who are part of the overall connectivity. Therefore, the communication between a pair of nodes need not be direct but could occur via one or more intermediate nodes. The set of nodes in direct communication can change, thus the system is adaptive. An example of such a system would be a supply chain, where the interaction of the different nodes via direct links facilitates the transfer of goods for some economic purpose. This adaptive system

can be modeled as a graph G of n vertices and e edges. The vertices represent the nodes and an edge indicates direct communication between the nodes it connects. Before we proceed any further we would like to lay down the essential concepts and definitions of our graph theoretic framework. In this paper, we use the terms network and graph, nodes and vertices, interchangeably.

We know that Internet topology is scale-free, as was confirmed by Barábasi and his team [1]. To state this simply, the nodes of a scale-free network are not connected randomly (i.e., evenly). Scale-free networks include many 'very connected' nodes, and hubs of connectivity shape the way the network operates. The ratio of very connected nodes to the number of nodes in the rest of the network remains constant as the network changes in size.

Unlike scale-free networks, random connectivity distributions are another large classification of networks characterized by evenly connected nodes. Before Barábasi and his team made their discovery about Internet connectivity, researchers presumed the Internet was a random network without well-connected nodes, or that the number of well-connected nodes was statistically insignificant. Although not all nodes in a random connectivity type of network are connected to the same degree, most have a small average number of connections. Also, as a randomly distributed network grows, the relative number of very connected nodes decreases.

Although the differences between these two classifications of networks is significant, it is worth mentioning that both scale-free and randomly distributed networks can be found in 'small world' networks. A small-world network does not require many hops to get from one node to another. The science behind this idea is there are only six degrees of separation between any two people, or nodes in this case, in the world; so, in both scale-free and randomly distributed networks, with or without very connected nodes, it may not take many hops for a node to make a connection with another node. However, in a scale-free network, many transactions would be channeled through a well-connected hub.

These two types of networks also handle traffic congestion differently. Because the Internet is scale-free, a malfunctioning router automatically prompts Internet protocols to bypass the missing nodes by sending packets to other routers. If the malfunctioning router carries a large amount of traffic, its absence places a significant burden on its neighbors. Although routers do not break down because of too much traffic, if they do, they simply create a queue and process as many packets as they can and drop the rest. Therefore, too much traffic sent to a router results in a denial-of-service attack, and. Only a small percentage of the packets can be processed. Because the sender of lost packets does not get a confirmation that its message

arrived, the sender re-sends the packets again, which only increases the congestion. Therefore, the removal of several large nodes could easily create a disastrous disruption on the Internet [3]. It should be noted that random networks do not handle traffic congestion in the same way. Thus, it has been concluded that the handling of traffic congestion depends on the network topology.

From a users' point of view, the problem lies in the links of a network. For example, in order to connect the Internet, a user pays an Internet service provider (ISP). If the user has several PCs, connecting to the Internet through different ISPs would be ideal from the viewpoint of traffic. But this solution is unrealistic because of the cost. In this sense, users are limited by the constraint of links.

Our goal in this study is to find a network structure that can prevent congestion with the least number of links. In order to reach this goal, we apply our knowledge of optimal information transmission in organizations [4] to

actual networks such as the Internet. We use the genetic algorithm (GA), which can alleviate traffic congestion under the constraint of the number of links.

In this paper, a network is modeled as a graph with the objective to send traffic without congestion. In this model, a node represents network equipment and a link represents a network cable between two nodes. We assume new traffic is created at randomly chosen nodes and sent to randomly chosen nodes.

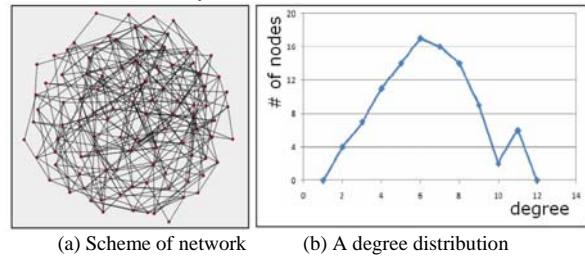


Fig. 2 The initial network and degree distribution.

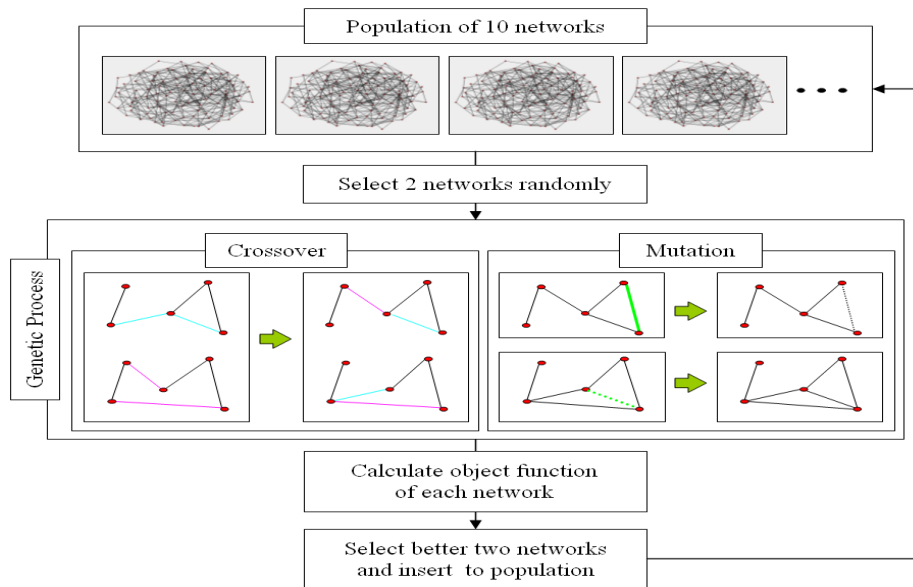


Fig. 1 The basic scheme of the genetic algorithm for optimizing networks.

3. The Evolutionary Optimization Model

We take an undirected graph G having a fixed number of nodes n and links defined by the binary adjacency matrix $A = \{a_{ij}\}$, $1 \leq i, j \leq n$. Given a pair of nodes i and j , $a_{ij} = 1$ if node i and j is linked and $a_{ij} = 0$ otherwise. The adjacency matrix A is $n \times n$ matrix because of award for all nodes and a symmetry matrix because of an undirected graph.

3.1 Initial networks

The first generated network is shown in Fig. 2. The average link number per node is 7, and this obeys a Poisson distribution. In other words, this network is a random network. We generated ten random networks that resemble this and applied the genetic algorithm.

3.2 Genetic algorithm

In this study, the system uses the genetic algorithm to generate an optimized network structure [5]. We used the

MGG model for the change of generations [6]. But, in this study, we chose two better matrices for a next generation.

The first, networks were generated by a specified probability about links. The model uses an encoded network by the binary adjacency matrix for the mutation and crossover. Next, the most suitable matrices among the parents and children matrices are chosen, and the others are eliminated. After repeating each step we obtain the optimized model.

We used the multi-point crossover. After crossover, each element in the matrix switches to a reverse state with a specific probability. In this paper, the network is an undirected graph, and so, if one element is reversed, the symmetry element is reversed at the same time. There is the possibility that an isolated network appears after crossover and mutation. In this paper, when an isolated node appears in a new network that the node has 0 distances to another node, we dump the network. Therefore, we can use non-isolated matrices.

Finally, after long generations have passed, we obtain an optimal network what is reduced traffic congestion and low density. Fig. 1 illustrates our MGG algorithm.

We use crossover rate at 0.7, and mutation rate is set at $2/n C_2$, i.e. reverses of two links per one generation. We create 10 different networks as individuals at the beginning. And, we stop until the object function has almost the same value match on $n C_2$ generations.

4. Definition of objective functions

4.1 Congestion measure

In this paper, we assume that the flow of the packet on the network obeys Poisson distribution. Then, the probability that a packet creates on a certain pass emerges

$$\frac{n\rho}{n(n-1)} = \frac{\rho}{n-1}, \tag{1}$$

where ρ is the probability of packet creation. And, the number of input packet P_i to each node can emerge

$$P_i = \frac{\rho}{n-1} \times \beta_i = \rho \left(\frac{\beta_i}{n-1} \right), \tag{2}$$

where β_i is node betweenness of node i [7].

And, if number of output packet P_o is 1, queue size of a node is

$$\frac{\frac{P_i}{P_o}}{1 - \frac{P_i}{P_o}} = \frac{\rho \left(\frac{\beta_i}{n-1} \right)}{1 - \rho \left(\frac{\beta_i}{n-1} \right)}, \tag{3}$$

according to queuing theory. [8]

The total expected length of the queues for the each betweenness, $\lambda(\rho)$ [4][8][9], is used the function what decrease a traffic congestion. It is defined as

$$\lambda(\rho) = \sum_{i \in n} \frac{\rho \left(\frac{\beta_i}{n-1} \right)}{1 - \rho \left(\frac{\beta_i}{n-1} \right)}. \tag{4}$$

The minimization of $\lambda(\rho)$ involves the simultaneous minimization of the size of the queues and the number of links, which is associated with the cost.

The congestion measure Eq. (4) has a problem. If betweenness or ρ has big value, like Eq. (5), then we cannot use congestion measure. Because $\lambda(\rho)$ has minus value.

$$\beta_i > \frac{n-1}{\rho} \quad \text{or} \quad \rho > \frac{n-1}{\beta_i} \tag{5}$$

So, in recent paper, high packet creation rate cannot be used in optimization.

In this paper, we use a normalization of $\lambda(\rho)$. We assume that the probability ρ is the probability of one packet creation on network in a unit time. Therefore, Eq. (1) is changed to Eq. (6).

$$\frac{\rho}{n(n-1)} \tag{6}$$

It is the rate of input packet for each node in a unit time. And, apply Eq. (6) to Eq. (3).

$$\frac{\rho \left(\frac{\beta_i}{n(n-1)} \right)}{1 - \rho \left(\frac{\beta_i}{n(n-1)} \right)} \tag{7}$$

Finally, we obtain new congestion measure Eq. 8.

$$\lambda(\rho) = \sum_{i \in n} \frac{\rho \left(\frac{\beta_i}{n(n-1)} \right)}{1 - \rho \left(\frac{\beta_i}{n(n-1)} \right)} \quad (8)$$

We can use all of the probability of packet creation. ($0 \leq \rho \leq 1$)

4.2 Link density

Many essential features of links are displayed by complex systems: for example, memory, stability and homeostasis emerge from the underlying network structure [10][11]. Different networks exhibit different features at different levels, but most complex networks are extremely sparse and exhibit the so-called small-world phenomenon [12].

The inverse measure of sparseness is called the link density, α , is defined in terms of a_{ij} as

$$\alpha = \frac{1}{{}_n C_2} \sum_{i < j} a_{ij} \quad (9)$$

where ${}_n C_2$ is combination that from n nodes any two nodes can form a linkable combination and it is the maximum number of links. So, we obtain relative ratio what number of links divided by maximum number of links, define link density α ($0 < \alpha \leq 1$). Therefore, the Eq. (9) is the normalized number of links.

4.3 The object function to be minimized

The evaluation function of our optimization algorithm is optimization of both Eq. (8) and Eq. (9) at the same time. However, the range of the two functions are gain as

- $0.02 \leq \alpha \leq 1$
- $0 < \lambda(\rho) \leq 5.3$

There is big difference occurs. Especially, the maximum $\lambda(\rho)$ is the average when $\omega = 0$, $\rho = 1$. Therefore, the overall object function is normalized as follows

$$E(\omega, \rho) = \omega \lambda(\rho) + 5(1 - \omega) \alpha, \quad (10)$$

where weight variable is $0 \leq \omega \leq 1$, and ω is a parameter controlling the linear combination of both.

We know that when $\omega = 0$ it is the minimization problem for link density only and when $\omega = 1$ it is only the minimization congestion measure.

5. Analysis functions

5.1 Polarization measurement

The simulations search for a network topology with different a value of ω and ρ . In particular, for any such network (with betweenness β), we focus on polarization θ , which is defined as follows:

$$\theta = \frac{\max_{i \in n} \beta_i - \langle \beta \rangle}{\langle \beta \rangle} \quad (11)$$

We use the polarization θ for analyzing the results [4][9]. The polarization Eq. (11) shows that how long the maximum betweenness ($\max_{i \in n} \beta_i$) is separated from average betweenness ($\langle \beta \rangle$) of the network. This type of polarization is used in our characterization of the different phases. If this value is large, the variance of betweenness is large. That is traffic bottlenecks are easy to detect.

For example, a circle network and perfect connected network that complete network (See Fig. 3) have the minimum value of polarization. That is all nodes have same value of betweenness. In case of this, the value of polarization is 0.

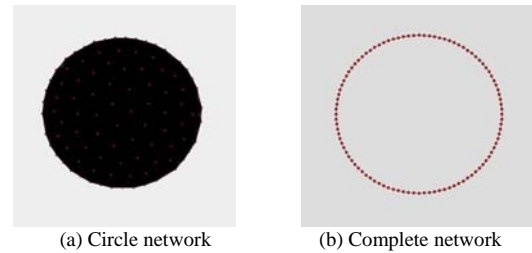


Fig. 3 A network with minimum polarization.

But, star network (See Fig. 4) has maximum value of polarization. Because, the only one node has very big betweenness but the other nodes have very small betweenness. That is the star network has a big difference between the maximum value and average of betweenness.

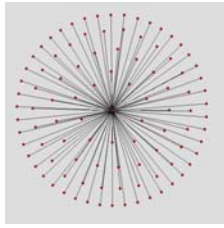


Fig. 4 A network with maximum polarization : Star network

5.2 Entropy measurement

In information theory, entropy is a measure of the uncertainty associated with a random variable. The term by itself in this context usually refers to the Shannon entropy [13], which quantifies, in the sense of an expected value, the information contained in a message, usually in units such as bits. Equivalently, the Shannon entropy is a measure of the average information content one is missing when one does not know the value of the random variable. Shannon's entropy represents an absolute limit on the best possible lossless compression of any communication: treating messages to be encoded as a sequence of independent and identically-distributed random variables, Shannon's source coding theorem shows that, in the limit, the average length of the shortest possible representation to encode the messages in a given alphabet is their entropy divided by the logarithm of the number of symbols in the target alphabet.

For example, a fair coin has entropy of one bit. However, if the coin is not fair, then the uncertainty is lower (if asked to bet on the next outcome, we would bet preferentially on the most frequent result), and thus the Shannon entropy is lower. Mathematically, a coin flip is an example of a Bernoulli trial, and its entropy is given by the binary entropy function. A long string of repeating characters has an entropy rate of 0, since every character is predictable. The entropy rate of English text is between 1.0 and 1.5 bits per letter, or as low as 0.6 to 1.3 bits per letter, according to estimates by Shannon based on human experiments.

We define the degree entropy on a certain value of H as

$$H(\{p_k\}) = -\sum_{k=1}^{n-1} p_k \log p_k, \tag{12}$$

where p_k is the frequency of nodes having degree k and

$$\sum_{k=1}^{n-1} p_k = 1. \tag{13}$$

This type of informational entropy will be used in our characterization of the different phases. Entropy measures of this type have been used in characterizing optimal channel networks and other models of complex systems [14] although they are typically averaged over time. Thus, we analyze uncertainty of network through the link entropy.

In the appeared optimized network, a random network (see Fig. 5) has minimum entropy ($H = 2.17$) i.e. it is instability state, low uncertainty and regular link distribution.

On the other hand, a star network (see Fig. 6) has maximum entropy ($H = 0.56$) i.e. it is stability state, high uncertainty and irregular link distribution. And, there is hard to exist in general world, because all things will change to state of high entropy.



Fig. 5 A network with the minimum entropy : Random network

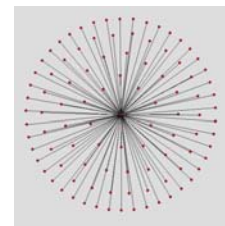


Fig. 6 A network with the maximum entropy : Star network

6. Simulation Results

6.1 Optimized networks (1)

We evaluated the results of the genetic algorithm through optimized networks what is optimized results according to each objective function Eq. (8) and (9).

In the case in which $\omega = 1$, the result optimized is the congestion measure $\lambda(\rho)$. The optimization result is the complete network shown in Fig. 7; the average link number is 98.36 and the link density is 0.99. Approximately 98% of the nodes have 99 links as the link distribution. In other words, traffic congestion does not occur if each node is connected directly to all the other nodes.

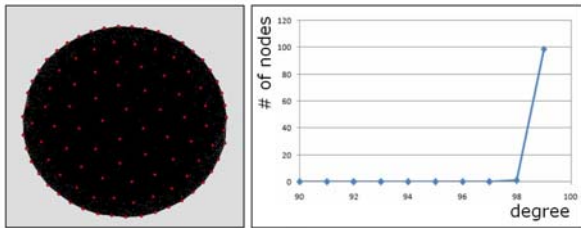


Fig. 7 The optimized network and degree distribution : $\omega=1$

It follows that only the link density α is minimized in the case in which ω is 0. The optimization result is a tree structure (hierarchical) network shown in Fig. 8. This expresses the network with the least number of links.

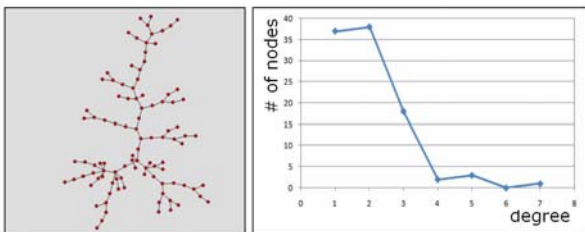


Fig. 8 The optimized network and degree distribution : $\omega=0$

By the above results, we know that the genetic algorithm performs well.

6.2 Optimized networks (2)

Fig. 9 is the result networks that optimized weight variable ($0 \leq \omega \leq 1$) and packet creation rate ($0 \leq \rho \leq 1$) while changing every 0.1. We can obtain many network topology on each ω and ρ . Especially, we can find some rule in the change of each variable. And, we decide about a hub in this simulation. A hub node is defined that when a node has large link, i.e. connect to 10% nodes.

If packet creation rate and weight variable is low, there are line like tree network topologies, i.e. on the conditions that network dose not utilize and link constraint is very high, a network is develop to line like tree network topology. On the other hand, if packet creation rate or weight variable grow up, the hierarchy structure network topology disappears and becomes the hub structure until ω and ρ what obtain star topology. Especially, when only weight variable is high, there is single hub topology i.e. a hub tends to become one hub structure. But, when only packet creation rate is high, there are multi hub topologies i.e. the number of hubs tends to increase.

Star topologies appear when packet creation rate or weight variable grow up. In case of star topologies, if packet creation rate is low, obtain single star topologies,

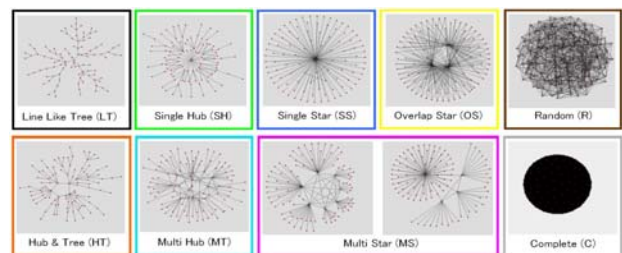
but if packet creation rate is high, obtain multi star topologies. Especially, in multi star topologies, star topology that there 3 or 4 hub is appeared when packet creation rate is low, and star topology what increase number of hubs (For example, 8 ~ 11 hubs) is appeared when packet creation rate is high.

Because small star structure is connected, a route scatters the networks of the multi-star topology and has a property to be able to prevent concentration of the traffic. Besides, we know that change when hub structure changes into star structure pass through multi-star structure and.

When packet creation rate and weight variable is higher than each values what appear star networks, we can obtain dense random networks. The changes from star networks to the dense networks is the foregoing description that if star hub can develop, a network become a star network topology, otherwise became a random network. Thus, there is high total queue length of the network to have a high dense link. But, we know that polarization has shrinking very much than star structure. Even if this has high total queue length, there is effective in scattering routs.

		← Low < Link Density > High →														
		ω														
		0	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.95	1		
↑	ρ	0.05	LT	LT	LT	SH	SH	HT	SH	SH	MS	SS	SS	SS	C	
		0.1	LT	LT	SH	SH	MH	SH	SH	SS	SS	SS	SS	SS	C	
Low	0.2	LT	HT	HT	SH	SH	MS	MS	SS	SS	SS	SS	R	R	C	
	0.3	LT	HT	SH	SH	MS	MS	SS	SS	SS	SS	OS	R	R	C	
< Congestion >	0.4	LT	HT	SH	SH	MS	MS	SS	OS	SS	SS	OS	R	R	C	
	0.5	LT	HT	HT	MS	MS	SS	OS	OS	OS	OS	R	R	R	C	
High	0.6	LT	MH	HT	MS	SS	OS	OS	OS	OS	OS	R	R	C	C	
	0.7	LT	HT	MH	MS	SS	OS	OS	OS	OS	OS	R	R	R	C	C
↓	0.8	LT	MH	MH	MS	OS	OS	OS	OS	OS	R	R	R	R	C	C
	0.9	LT	MH	MS	MS	OS	OS	OS	OS	OS	R	R	R	R	C	C
		0.95	LT	MH	MS	OS	OS	OS	OS	R	R	R	R	R	C	C

(a) Configuration of optimized networks for different values



(b) Some topologies of evolutionary optimized networks.

Fig. 9 The optimized networks for each ω and ρ .

In case of $\omega=0.95$, it becomes near to the complete network, because constraint to minimize link density becomes very weak. And, we know that the change to the complete network from the high-density network change

suddenly. It changes in particular to $\alpha \approx 1$ suddenly from $\alpha \approx 1.2$ when see the link density. As for this, the number of the links becomes largest, but both congestion measure and polarization value become the smallest network structure. Thus, in case of very high ω and ρ , an influence of congestion measure becomes very high, and an influence of the link density becomes trifling. Therefore, the network comes to have process minimizing only congestion measure.

7. Analysis of the optimized networks

We plot of Polarization and Entropy on Fig. 10. Polarization appears a network characteristic by the distribution of the betweenness. And, entropy appears a network characteristic by the distribution of the link. Therefore, Fig. 10 show assortment of networks by network characteristics. In this case, we know that networks do phase transition through the topology.

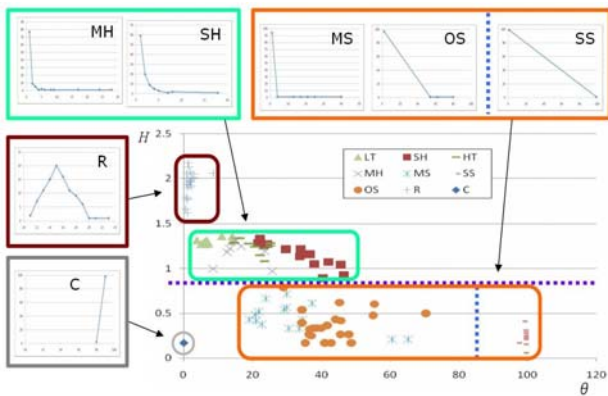


Fig. 10 Comparison in terms of polarization and entropy measurements.

First, on phase transition by entropy, there are 3 type of layer; high-entropy is high-dense layer, middle-entropy is hub layer, and low-entropy is others. And, in the low-entropy, there are complete layer and star layer. Especially, we bind up star networks in one layer; for all that single star networks have very high polarization. Because, they have star hub typical first network topology.

We know that complete, random and single star network is easily assorted by network topology and link distribution. But, assortment of others network is very hard. However, we assorted hub layer and star layer what is multi star and overlap star networks in like Fig.10.

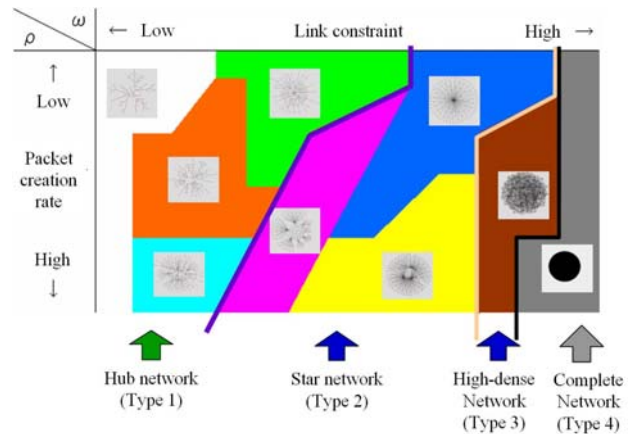


Fig. 11 Classification of optimized networks in terms of link density constraint (ω) and packet creation rate (ρ).

Finally, we can find that optimized networks have 4 different types depending on the values ω and ρ .

- Type 1: A hub network
- Type 2: A star network
- Type 3: A high-dense network
- Type 4: A complete network

In Fig. 11, we classify different types of optimized networks. There are some phase transition any four layers according to ω and ρ . And, in Fig. 11, if ω is high and ρ is low, only one hub is grown up in a network, otherwise some hubs are grown up on same time.

8. Conclusion and Future works

Many network topologies are suggested in recent research. But, we do not know what a optimal network topology is at various purposes. We want to find optimal network topology to answer the each purpose. We simulated a case of optimization network topology based on the flow of traffic according to a genetic algorithm. And, we make an addition to the constraint of a link.

We are designed optimized communication networks what are considered packet creation rate and constraint of link. We can classify according to network characteristic the result networks according to change of importance of link constraint ω and packet creation rate ρ . There are a hub network, a star network, a high-dense network and a complete network. And, according to link density, a hub network and a star network are low density, and a high-dense network and a complete network are high density.

In future work, we intend to increase the number of nodes and analyze the optimized network. In the current

study, we cannot yet declare conclusively what our hub network is because our networks had a small number of nodes. But, the optimization of large network needs very long time. We think about another appropriate method which is module network. It is that the big network can be connected by some small network. First, we optimize some small networks. And, a large network is made from connection of small networks each other. The point is how connect networks to networks do. Finally, we analyze obtain new networks.

References

- [1] R. Albert, H. Jeong, and A.L. Barabási, "Error and attack tolerance of complex networks," *Nature*, vol.406, no.6794, pp.378-382, 2000.
- [2] R. Albert, H. Jeong, and A.L. Barabási, "Internet: Diameter of the world-wide web," *Nature*, vol.401, no.6749, pp.130-131, 1999.
- [3] A.L. Barabási, *Linked: The New Science of Networks*, Perseus Publishing, Cambridge, MA, 2002.
- [4] A. Cabrales, A. Arenas, A. Díaz-Guilera, R. Guimerá, and F. Vega-Redondo, "Optimal information transmission in organizations: Search and congestion," *Working Papers* 2004.77, 2004.
- [5] H. Sato, M. Kubo, and A. Namatame, "GA-based complex network generating system and its characteristics," *IPSI SIG Notes*, vol.2007, no.19, pp.97-100, 2007.
- [6] H. Sato, O. Isao, and K. Shigenobu, "A new generation alternation model of genetic algorithms and its assessment," *Journal of Japanese Society for Artificial Intelligence*, vol.12, no.5, pp.734-744, 1997.
- [7] U. Brandes, "A faster algorithm for betweenness centrality," 2001.
- [8] A.O. Allen, *Probability, statistics, and queuing theory with computer science applications*, Academic Press Professional, Inc., San Diego, CA, USA, 1990.
- [9] V. Cholví, V. Laderas, L. López, and A. Fernández, "Self-adapting network topologies in congested scenarios," *Physical Review E (Statistical, Nonlinear, and Soft Matter Physics)*, vol.71, no.3, 2005.
- [10] S.H. Strogatz, "Exploring complex networks," *Nature*, vol.410, no.6825, pp.268-276, 2001.
- [11] S.A. Kauffman, *The Origins of Order: Self-Organization and Selection in Evolution*, Oxford University Press, 1993.
- [12] D.J. Watts and S.H. Strogatz, "Collective dynamics of 'small-world' networks," *Nature*, vol.393, no.6684, pp.440-442, 1998.
- [13] C.E. Shannon, "A mathematical theory of communication," *Bell system technical journal*, vol.27, 1948.
- [14] R.V. Solé and O. Miramontes, "Information at the edge of chaos in fluid neural networks," *Phys. D*, vol.80, no.1-2, pp.171-180, 1995.



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