Fuzzy Multi-State Allocation of the Diesel Engine Fuel Supply System

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Summary
To reduce the inapplicability of binary state models in system reliability analysis, fuzzy multi-state reliability allocation model is presented in this paper. In the presented model, the fuzzy concept and multi-state reliability theory are integrated based on the state detection technology of diesel engine, and take the technology and economy into account at the same time. Fuzzy multi-state reliability theory was analyzed in detail firstly. Then, the system states allocation and optimization model were established. In the diesel engine fuel supply system which includes the fuzzy components, components' states were represented by the functional characteristics, equivalence classes founded by using lower boundary Points, and the system states were allocated in minimum cost situation at last. Finally, an example was given to verify the effective of the proposed model.
Key words:
Multi-state reliability, Fuzzy concepts, States allocation, Diesel engine fuel supply system, Optimization model

1. Introduction

Since late sixties, researchers have been looking for various techniques to optimize system reliability [1]. Initially, binary state models were used, but its inapplicability were exposed when regarding several of complex large-scale system, and multi-state models were introduced later [2]. In fact, multi-state models have many limitations too, because it requests the user to distinguish various states and the probability it's occur. Until the late nineties, the limitations are broken through when researchers have begun to use fuzzy concepts to optimize system's reliability.

The problem of state allocation and optimization been widely treated by many authors these years, especially along with the advances in computing intelligence, such as fuzzy set theory, neural network, genetic algorithms, rough set theory, and so on, provide a stronger tool for allocation and optimization of system state. Most of the attention to this issue has been given to the redundancy allocation problem. Actually, if resources want to be best utilized one should pay attention to relationship between components states and its cost, and then arrive at some optimal redundancy level [3-5].

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For the fuel supply system of diesel engine, because of the specificity and complexity of its internal structure mechanism, and a mass of mutual transformation of fatigue, wear, deformation and corrosion in among them, we cannot obtain the anticipated effect if we still used binary state models [6]. In this paper, the fuzzy concept and multi-state reliability theory are integrated to the states allocation of diesel engine fuel supply system [7-12]. The structures of this paper are as follows: Fuzzy multi-state reliability theory was analyzed in detail in section 2. The system states allocation and optimization model were established in section 3. In the presented model, the functional characteristics which used to distinguish each component's states were obtained by using the state detection technology of diesel engine in minimum cost situation. An example is given in section 4 to verify the feasibly of the model. Conclusions are given in section 5.

2. The Fuzzy Multi-State Theory

2.1 The Description of the Fuzzy State [13-18]

When states are fuzzy, the system states are characterized through membership function, namely the degree that the system belongs to a certain state. The degree the system belongs to a certain state is ratiocinated by the degree each component belongs to there states, therefore, the membership function of system is the function of every components membership function. So, when the states of the system and the components are fuzzy, the fuzzy state of the system can be represented by the fuzzy component state vector as follows:

$$(\tilde{x}):\{\tilde{x}_1,\tilde{x}_2,\cdots,\tilde{x}_i,\cdots,\tilde{x}_n\} \quad (1)$$

where $\tilde{x}$ is the fuzzy state of the system. $\tilde{x}_i$ is the fuzzy state of the i^{th} component.

The functional characteristic of the system and the components play a direct-viewing role in descries each of the fuzzy states and its membership function. For instance, the time to start a vehicle in an ambient temperature is the functional characteristic of the state of the vehicle that determines the degree that the vehicle is in perfect, or moderate, or failed state. Therefore, to a system which including several components, the components fuzzy states can be defined by its functional characteristic. For instance, the i^{th} component fuzzy state $\tilde{x}_i$ associates to its functional characteristic value of y_i through degree of membership $\mu_{\tilde{x}_i}(y_i)$. Where y_i takes real values, $\mu_{\tilde{x}_i}(y_i)$ is the degree to which y_i belongs to the state $\tilde{x}_i$. Table 1 provides definitions for the functional characteristics and the states of the components and the system.

Table 1: Description of the fuzzy states

Type	Fuzzy state	Functional characteristic
1^{th} component	$\tilde{x}_1$	y_1
2^{th} component	$\tilde{x}_2$	y_2
…	…	…
i^{th} component	$\tilde{x}_i$	y_i
n^{th} component	$\tilde{x}_n$	y_n
system	$\phi(\tilde{x})$	z

2.2 The Equivalence Class Description

Equivalence class is a subset of given set induced by an equivalence relation on that given set. In the multi-state system theory the equivalence class is defined as follows:

$$S_j \subset S \ \ by \ \ S_j = \{x \in S | \phi(x) = j\} \tag{2}$$

where S_j is the j^{th} Equivalence class.

However, when the states are fuzzy the equivalence classes will defined as follows:

$$\tilde{S}_j \subset \tilde{S} \ \ by \ \ \tilde{S}_j = \{\tilde{x} \in \tilde{S} | \phi(\tilde{x}) = j, \ \mu_{\tilde{x}}(z)\} \tag{3}$$

For instance, a system consists of 3 components. A component state vector $[\tilde{1},\tilde{1},\tilde{1}]$, defines a state of the system i.e. $\phi(\tilde{x}) = \phi(\tilde{1},\tilde{1},\tilde{1})$ .Similarly, another components state vector $[\tilde{1},\tilde{1},\tilde{2}]$ defines a state of the system i.e. $\phi(\tilde{x}) = \phi(\tilde{1},\tilde{1},\tilde{2})$. $\phi(\tilde{1},\tilde{1},\tilde{1}) = \phi(\tilde{1},\tilde{1},\tilde{2})$ implying that the increase in the state of the component 3 from $\tilde{1}$ to $\tilde{2}$ is not important when states of the component 1 and 2 are $\tilde{x}_1 = \tilde{x}_2 = \tilde{1}$ $\tilde{1}$ .Further more, $\phi(\tilde{1},\tilde{1},\tilde{1})$ and $\phi(\tilde{1},\tilde{1},\tilde{2})$ belong to the same equivalence class. However, the degree to which $\phi(\tilde{1},\tilde{1},\tilde{1})$ and $\phi(\tilde{1},\tilde{1},\tilde{2})$ belong to the same system state may be different based upon the value of the functional characteristic of the system z. for some values of $\tilde{x}_1$ & $\tilde{x}_2$, the increase of $\tilde{x}_3$ might become important. Under this situation the system states may be different. For instance, the value of the functional characteristic of the system is z_1, when the structure function is $\phi(\tilde{1},\tilde{1},\tilde{1})$, the membership function will be $\mu_{\phi(\tilde{1},\tilde{1},\tilde{1})}(z_1)$. Similarly, the value of the functional characteristic of the system is z_2 when the structure function is $\phi(\tilde{1},\tilde{1},\tilde{2})$, the membership function will be $\mu_{\phi(\tilde{1},\tilde{1},\tilde{2})}(z_2)$. Therefore, even if structure functions such as $\phi(\tilde{1},\tilde{1},\tilde{1})$ and $\phi(\tilde{1},\tilde{1},\tilde{2})$ belong to the same equivalence class, the membership functions may not be the same. Under this condition, membership function is an important element describing the equivalence class.

3. The Allocation Model of System State

3.1 System State Allocation Process

To achieve the desired membership function of the system state $\phi(\tilde{x})$, the membership functions of the components in the component state vector must be equal to or greater than the membership function of the system. The preferred states of the components and their membership functions will depend on several factors such as cost. Therefore, the component state vector that satisfies the condition i.e. $\phi(\tilde{x}) \geq \tilde{j}$ must belong to the $\sum \tilde{S}_j$, $j = \tilde{j}, \cdots, \tilde{M}$. $\sum \tilde{S}_j$ is called the feasible space of the component state vectors and $\sum \tilde{S}_j$ is the j^{th} equivalence class that belongs to the component state vector space set $\tilde{S}$ .The value of the functional characteristic of each component state must remain in a range that satisfies the minimum requirement of the component membership function. These ranges are the feasible space of the functional characteristics. Cost in each component is the function of its functional characteristic. Therefore, the functional characteristic values are identified in the feasible ranges of the component states at minimum costs. These values of the functional characteristics are called minimum cost points. The cost of each of the component state vector in its feasible space is determined at minimum cost points. Cost of each of the component state vectors is compared with the other in order to select the component state vector that incurs the least cost. Figure 1 provides the process of system state allocation.

3.2 The Equivalence Class Establishment Method

In a biggish system, each of the equivalence classes has large number of component state vectors. Therefore, it is cumbersome to identify each of the component state vectors that belong to an appropriate equivalence class by evaluating each of them on an individual basis. Actually, we need not determine the equivalence classes by evaluating all of the component state vectors one at a time; we need only to specify when a decrease in the state of any one of the *n* components forces a decrease in the system state. These special component state vectors are lower boundary points.

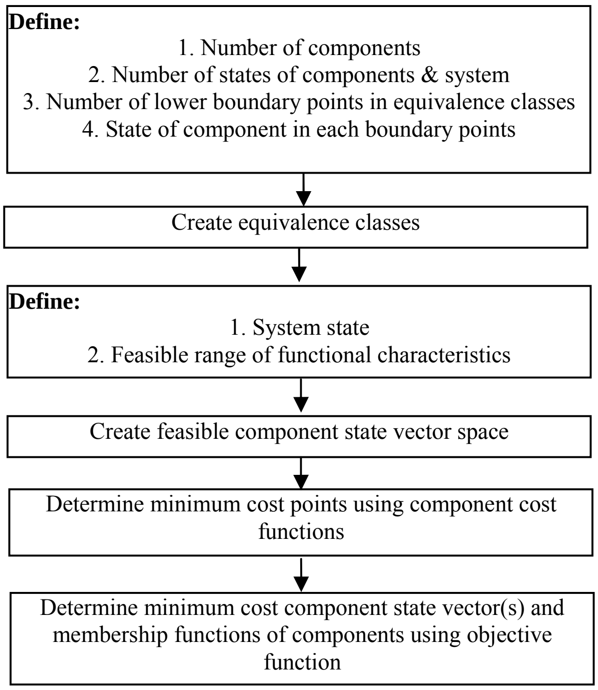

Fig. 1 The allocation process

For BCS structures, the lower boundary points to level 1 are the minimal-path vectors. The lower boundary points to each level $\tilde{k}$ ($\tilde{k} = \tilde{1},\tilde{2},\cdots,\tilde{M}$) is enough to describe the system completely. So we can generate an equivalence structure function by decomposing the MCS into several BCS structures, the Steps in Equivalence class establishment as follows:

(1) All the component state vectors are generated by considering every combination of the components states.

(2) The state of each of the components in the component state vector is converted to binary states as follows:

$$y_{ij} : I(\tilde{x}_i \geq \tilde{j}), \quad i = 1,2,\cdots,n ; \quad \tilde{j} = \tilde{1},\tilde{2},,\tilde{M}_i \tag{4}$$

where $I(\tilde{x}_i \geq \tilde{j}) = \begin{cases} 1, \tilde{x}_i \geq \tilde{j} \\ 0, \tilde{x}_i < \tilde{j} \end{cases}$

For instance, a component state in a component（this component has 4 number states）state vector is $\tilde{2}$. Then the state $\tilde{2}$ can be converted to a binary state representation using expression (2) as [1 1 1 0]. Similarly, states of the rest of the components in the component state vector is converted to binary states.

Using the above method, the MCS structure function $\phi(\tilde{x}) : (\tilde{x}_1, \tilde{x}_2, \cdots, \tilde{x}_n)$ can be converted to the BCS structure function:

$$\phi^k(y) : \left(y_{11}, \cdots, y_{1M_1}, y_{21}, \cdots, y_{2M_1}, \cdots, y_{n1}, \cdots, y_{nM_1}\right) \tag{5}$$

where $\phi^k(y) = f(\phi(\tilde{x}) \geq k)$, $k = \tilde{1},\tilde{2},\cdots,\tilde{M}$.

(3) All the lower boundary points are identified in each of the equivalence classes.

(4) Each of the elements in the lower boundary points is compared against the corresponding element in expression (3), and allocates either 0 or 1 to the result. Then, we multiply all these results, finally the outcome will be either1 or 0.

Above result can be expressed mathematically as follows:

$$\min(y_{ij}), \quad i = \{1,2,\cdots,n\} \tag{6}$$

(5) Step 4 is repeated for each component state vector on each of the lower boundary points in the same equivalence class. We take the maximum of all values. This can be expressed mathematically as follows:

$$\phi^k(y) = \max_{x \in L_K}(\min(y_{ij})) \tag{7}$$

(6) Step 5 is repeated equivalence classes. Then each component state vector in each of the MCS structure function is found by summing the BCS structure function for all levels:

$$\phi(\tilde{x}) = \sum_{k=1}^{M} \phi^k(y) \tag{8}$$

3.3 The Optimization Model of the System State Allocation

In the process of the system allocation, we expect the system perform at an equal to or greater than state $\tilde{j}$ with certain membership function i.e. $\phi(\tilde{x}) \geq \tilde{j}$. Allocation of the system state to component level is done by optimizing the objective function. The objective function is based upon the component cost functions. Component cost functions are the functions of component states and membership functions. Eventually, component

cost functions are based upon the functional characteristics.
Objective Function:

$$\text{Min.cost} = \sum_{i=1}^{n} f\left(y_i, \mu_{\tilde{x}_i}(y_i)\right) \quad (9)$$

where y_i is the functional characteristic value of the i^{th} component; $i = 1,2,\cdots,n$, n is the total number of components in the system.
Constraints:

$$\text{Min}\left(\mu_{\tilde{x}_1}, \mu_{\tilde{x}_2}, \cdots, \mu_{\tilde{x}_n}\right) \geq \mu_{\phi(\tilde{x})} \quad (10)$$

$$\phi(\tilde{x}) \geq \tilde{j} \quad (11)$$

$$\tilde{x} \in \sum_{i=j}^{M} \tilde{S}_i \quad (12)$$

The membership functions of the states of the components must be equal to or greater than the desired membership function of the system state. Therefore, when the desired membership of the system state is $\mu_{\phi(\tilde{x}) \geq \tilde{j}}$, the feasible spaces of the states of the i^{th} component are determined as Figure 2.

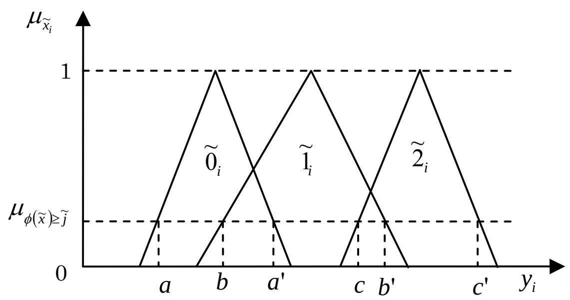

Fig. 2 Feasible spaces of the ith component functional characteristic

4. Case study

The diesel engine fuel supply system mainly consists of the low-pressure oil circuit and the high-pressure oil circuit. The components in low-pressure oil circuit mainly consist of oil tank, low-pressure tubing, low-pressure oil delivery pump and fuel filter. The components in high-pressure oil Circuit mainly consists of high-pressure fuel injection pump, speed governor, plunger matching parts, delivery valve assembly, high-pressure tubing and fuel injector. For study convenience, the diesel engine fuel supply system is simplified as seven parts: oil tank, tubing, oil delivery pump, fuel filter, fuel injection pump, speed governor and fuel injector, the hardware graph and the reliability block diagram of the system can be represented as Figure 3.

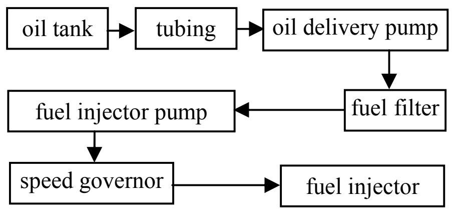

Fig.3 The Reliability block diagram of the diesel engine fuel supply system

Because the failure of some components (the main components) of the fuel supply system would result in the failure of the system i.e. deterioration of any component will affect the function of the system, the relationship of the components in the system can be considered in series. Each component state is determined by its functional characteristic value, and the scale of the functional characteristic value is defined from 0 to 10. There are 5 fuzzy states in the system. For study convenience, the states of the system and components are assumed to be triangular and trapezoidal. With the passage of time, the system deteriorates from its highest state toward its lower states. The state and the measurement range of functional characteristic are represented as Table 2 and 3:

Table 2: State of the system

States	Measurement range
$\tilde{0}$: failure	0-3
$\tilde{1}$: poor performance	2-5
$\tilde{2}$: moderate performance	4-7
$\tilde{3}$: good performance	6-9
$\tilde{4}$: Perfect performance	8-10

Table 3: States of the components

Component	States	Measurement range	Functional characteristic
oil tank	$\tilde{0}$: failure	0-4	airtight performance y_1
	$\tilde{1}$: Perfect	3-10	
tubing	$\tilde{0}$: failure	0-5	airtight performance y_2
	$\tilde{1}$: Perfect	3-10	
fuel filter	$\tilde{0}$: failure	0-2	impurity quantity

	$\tilde{1}$: moderate	1-4	after filter y_3
	$\tilde{2}$: Perfect	3-10	
fuel injection pump	$\tilde{0}$: failure	0-5	pressure of diesel y_4
	$\tilde{1}$: moderate	4-7	
	$\tilde{2}$: Perfect	6-10	
speed governor	$\tilde{0}$: failure	0-4	oil supply performance y_5
	$\tilde{1}$: moderate	3-7	
	$\tilde{2}$: Perfect	6-10	
fuel injector	$\tilde{0}$: failure	0-5	fuel utilization ratio y_6
	$\tilde{1}$: moderate	3-7	
	$\tilde{2}$: Perfect	6-10	
oil delivery pump	$\tilde{0}$: failure	0-2	oil flow y_7
	$\tilde{1}$: poor	1-5	
	$\tilde{2}$: moderate	4-7	
	$\tilde{3}$: Perfect	6-10	

There are 1296 component state vectors in the system, the equivalence classes are established through the method of lower boundary points which built up in 3.2 as Table 4:

Table 4: Lower boundary points

Equivalence class	Lower boundary points
S_0	NONE
S_1	0011121 0011211 011121
S_2	0112211 1211121
S_3	1111222 1122112
S_4	1112222

When the system state must remain $\tilde{2}$ i.e. $\phi(\tilde{x}) \geq \tilde{2}$, the minimal membership function $\mu_{\phi(\tilde{x})\geq\tilde{2}}$ is 0.55. Similarly, when the system state must remain $\tilde{3}$, $\tilde{4}$, the corresponding minimal membership function $\mu_{\phi(\tilde{x})\geq\tilde{2}}$ are 0.36 and 0.18. On the basis of 3.3 analyses, the membership functions of the components states must be equal to or greater than the minimum membership function of the system state. So, the feasible space of the functional characteristic of the components states can be determined by the Figure 2. Take the oil tank as the example, when the system state remain $\tilde{2}$, the feasible space of the functional characteristic of the oil tank in state $\tilde{0}$ is (0, 2.9), the feasible space of the functional characteristic of the oil tank in state $\tilde{1}$ is (4.93, 10), as Figure 4. Similarly, we can get all the feasible space of the functional characteristic of the components when the system states remain at $\tilde{2}$, $\tilde{3}$ and $\tilde{4}$, as Table 5.

The components states which result in minimum cost should be selected in Table 5, substitute it in formula of each component cost function. Then, we can get the total cost function i.e. the objective function. The cost functions of the components as Table 6.

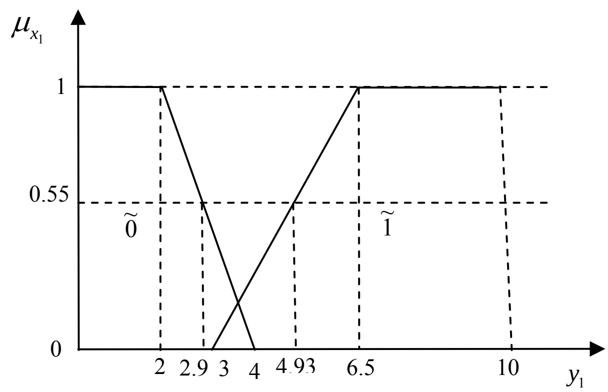

Fig. 5 Feasible spaces of the oil tank functional characteristic

Table 5: Feasible spaces of the components functional characteristic

Component	States	Space of functional characteristic		
		$\phi(\tilde{x}) \geq \tilde{2}$	$\phi(\tilde{x}) \geq \tilde{3}$	$\phi(\tilde{x}) \geq \tilde{4}$
oil tank	$\tilde{0}$	(0, 2.9)	(0, 3.28)	(0, 3.64)
	$\tilde{1}$	(4.93, 10)	(4.26, 10)	(3.63, 10)
tubing	$\tilde{0}$	(0, 3.63)	(0, 4.1)	(0, 4.55)
	$\tilde{1}$	(4.93, 10)	(4.26, 10)	(3.63, 10)
fuel filter	$\tilde{0}$	(0, 1.68)	(0, 1.96)	(0, 1.82)
	$\tilde{1}$	(1.83, 3.18)	(1.54, 3.46)	(1.27, 3.73)
	$\tilde{2}$	(4.93, 10)	(4.26, 10)	(3.63, 10)
fuel injection pump	$\tilde{0}$	(0, 3.63)	(0, 4.1)	(0, 4.55)
	$\tilde{1}$	(4.83, 6.18)	(4.54, 6.46)	(4.27, 6.73)
	$\tilde{2}$	(7.1, 10)	(6.72, 10)	(6.36, 10)

speed governor	$\tilde{0}$	(0, 2.9)	(0, 3.28)	(0, 3.64)
	$\tilde{1}$	(4.1, 5.9)	(3.72, 6.28)	(3.36, 6.64)
	$\tilde{2}$	(7.1, 10)	(6.72, 10)	(6.36, 10)
fuel injector	$\tilde{0}$	(0, 3.63)	(0, 4.1)	(0, 4.55)
	$\tilde{1}$	(4.1, 5.9)	(3.72, 6.28)	(3.36, 6.64)
	$\tilde{2}$	(7.1, 10)	(6.72, 10)	(6.36, 10)
oil delivery pump	$\tilde{0}$	(0, 1.45)	(0, 1.64)	(0, 1.82)
	$\tilde{1}$	(2.1, 3.9)	(1.72, 4.28)	(1.36, 4.64)
	$\tilde{2}$	(4.83, 6.18)	(4.54, 6.46)	(4.27, 6.73)
	$\tilde{3}$	(6.88, 10)	(6.4, 10)	(5.95, 10)

Table 6: The cost functions of the components

Component	Cost function ($c(y_i)$)	Space of functional characteristic (y_i)
oil tank	$15 + y_1 + y_1^2$	$y_1 \in (0, \quad 10)$
tubing	$e^{0.34 y_2}$	$y_2 \in (0, \quad 10)$
fuel filter	$5 + 3y_3 - 0.1y_3^2$	$y_3 \in (0, \quad 10)$
fuel injection pump	$10 + y_4 + y_4^2$	$y_4 \in (0, \quad 4.5)$
	$10 + y_4 + y_4^2 + 0.03y_4^3$	$y_4 \in (4.5, \quad 10)$
speed governor	$\sqrt{15 + y_5^2 + 0.285y_5^3}$	$y_5 \in (0, \quad 10)$
fuel injector	$\sqrt{20 - y_6^2 + 0.98y_6^3}$	$y_6 \in (0, \quad 10)$
oil delivery pump	$10 + 2y_7 + 1.6y_7^3$	$y_7 \in (0, \quad 4.5)$
	$10 + 2y_7 + 2.7y_7^2$	$y_7 \in (4.5, \quad 10)$

Each cost function has one minimum in the feasible space of the functional characteristic, and minimization of each component cost will minimize the total cost. At last, the results of the components states allocation in minimum cost are shown as Table 7.

Table 7: Allocation of the components states

system state	allocation of the components states	minimum cost(Yuan)
$\geq \tilde{2}$	0011121	115.97
$\geq \tilde{3}$	0112211	154.3
$\geq \tilde{4}$	1122122	214.44

5. Conclusions

The paper proposes a model to realize the fuzzy multi-state allocation of the diesel engine fuel supply system. In the presented model, the fuzzy concept and multi-state reliability theory are integrated based on the state detection technology of diesel engine. From the presented example of this paper we can see, the model can be used in many cases where the states of system and component are fuzzy and multi-state, and the results of the example tallies well with the reality.

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