

Solving Transportation Problem Using Object-Oriented Model

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Summary

This paper is about solving transportation problem using Operation Research (OR) approach in analysis and design phases and we use C++ programming language to model the problem. The results obtain from both solutions are compared in order to make analysis and prove the object-oriented model correctness. We proved that both results are identical and have the same results when solving the problem using the five methods: northwest corner method; minimum cost method; row minimum cost method; column minimum cost method, and Vogel's approximation method.

Key words:

Transportation problem, Linear Programming (LP), object-oriented programming

1. Introduction

The first main purpose is solving transportation problem using five methods of transportation model by linear programming (LP). The second main purpose is solving transportation problem by object-oriented programming. C++ programming language is used to get the solution. The results obtain from both LP and object-oriented programming solutions are compared.

The five methods for solving Transportation problem are:

1. Northwest Corner method
2. Minimum cost method
3. Vogel's approximation method
4. Row Minimum Method
5. Column Minimum Method.

This paper introduces methods for solving transportation problem by C++ programming language; we use flow chart, algorithms and consider the importance of defining a problem sufficiently and what assumptions we may consider during the solution. Solving transportation problem by computer involves serves of steps: define the problem, analysis the problem and formulate a method to solve it, describe the solution in the form of an algorithm, draw a flow chart of the algorithm, write the computer program, compile and run the program, test the program and interpretation of results.

We design Object-Oriented Model as decision support tool to evaluate the solution for the five methods using C++ language. After designing the five models (the five programs) we compare between each solution using C++ programs and LP solution which have the same result. Comparison different solutions is done for choosing less value of the objective function so that the user will be able to make decision.

TRANSPROTATION MODEL

Transportation model is a special type of networks problems that for shipping a commodity from source (e.g., factories) to destinations (e.g., warehouse).

Transportation model deal with get the minimum-cost plan to transport a commodity from a number of sources (m) to number of destination (n).

Let s_i is the number of supply units required at source i ($i=1, 2, 3, \dots, m$), d_j is the number of demand units required at destination j ($j=1, 2, 3, \dots, n$) and c_{ij} represent the unit transportation cost for transporting the units from sources i to destination j .

Using linear programming method to solve transportation problem, we determine the value of objective function which minimize the cost for transporting and also determine the number of unit can be transported from source i to destination j .

If x_{ij} is number of units shipped from source i to destination j . the equivalent linear programming model will be [5]

The objective function

$$\text{Minimize } Z = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij}$$

Subject to

$$\sum_{j=1}^n x_{ij} = s_i \quad \text{for } i=1,2,\dots,m.$$

$$\sum_{i=1}^m x_{ij} = d_j \quad \text{for } j=1,2,\dots,n.$$

And

$$x_{ij} \geq 0 \quad \text{for all } i \text{ to } j.$$

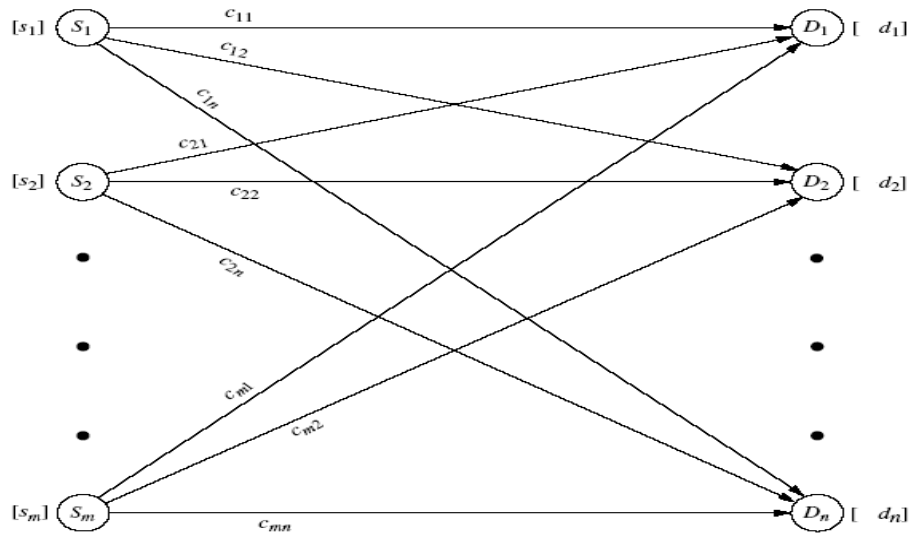


Fig.4.1 Network representation of the transportation problem

A transportation problem is said to be balanced if the total supply from all sources equals the total demand in all destinations

$$\sum_{i=1}^m s_i = \sum_{j=1}^n d_j.$$

Otherwise it is called unbalanced.

METHODS FOR SOLVING TRANSPORTATION PROBLEM

There are five methods to determine the solution for balanced transportation problem:

1. Northwest Corner method
2. Minimum cost method
3. Vogel's approximation method
4. Row Minimum Method
5. Column Minimum Method

The five methods differ in the "quality" of the starting basic solution they produce and better starting solution yields a smaller objective value.

We present the five methods and an illustrative example is solved by these five methods.

1. North West-Corner Method

The method starts at the northwest-corner cell (route) of the tableau (variable x_{11})

(i) Allocate as much as possible to the selected cell and adjust the associated amounts of supply and demand by subtracting the allocated amount.

(ii) Cross out the row or Column with zero supply or demand to indicate that no further assignments can be

made in that row or column. If both a row and a column net to zero simultaneously, cross out one only and leave a zero supply (demand in the uncrossed-out row (column)).

(iii) If exactly one row or column is left uncrossed out, stop .otherwise, move to the cell to the right if a column has just been crossed out or below if a row has been crossed out .Go to step (i). [2]

2. Minimum-Cost Method

The minimum-cost method finds a better starting solution by concentrating on the cheapest routes. The method starts by assigning as much as possible to the cell with the smallest unit cost .Next, the satisfied row or column is crossed out and the amounts of supply and demand are adjusted accordingly. If both a row and a column are satisfied simultaneously, only one is crossed out, the same as in the northwest –corner method .Next ,look for the uncrossed-out cell with the smallest unit cost and repeat the process until exactly one row or column is left uncrossed out . [2]

3. Vogel's Approximation Method (VAM)

Vogel's Approximation Method is an improved version of the minimum-cost method that generally produces better starting solutions.

(i) For each row (column) determine a penalty measure by subtracting the smallest unit cost element in the row (column) from the next smallest unit cost element in the same row (column).

(ii) Identify the row or column with the largest penalty. Break ties arbitrarily. Allocate as much as possible to the variable with the least unit cost in the selected row or column .Adjust the supply and demand and cross out the

satisfied row or column. If a row and column are satisfied simultaneously, only one of the two is crossed out, and the remaining row (column) is assigned zero supply (demand).

- (iii) (a) If exactly one row or column with zero supply or demand remains uncrossed out, stop.
- (b) If one row (column) with positive supply (demand) remains uncrossed out, determine the basic variables in the row (column) by the least-cost method .stop.
- (c) If all the uncrossed out rows and columns have (remaining) zero supply and demand, determine the zero basic variables by the least-cost method .stop. .)[2]
- (d) Otherwise, go to step (i).

4. Row Minimum Method

Row minimum method start with first row and choose the lowest cost cell of first row so that either the capacity of the first supply is exhausted or the demand at jth distribution center is satisfied or both. Three cases arise:

- (i)If the capacity of the first supply is completely exhausted, cross off the first row and proceed to the second row.
- (ii)If the demand at jth distribution center is satisfied, cross off the jth column and reconsider the first row with the remaining capacity.
- (iii)If the capacities of the first supply as well as the demand at jth distribution center are completely satisfied, make a zero allocation in the second lowest cost cell of the first row .cross off the row as well as the jth column and move down to the second row.

Continue the process for the resulting reduced transportation table until all the rim conditions (supply and demand condition) are satisfied. [6]

5. Column Minimum Method

Column minimum method starts with first column and chooses the lowest cost cell of first column so that either the demand of the first distribution center is satisfied or the capacity of the ith supply is exhausted or both .three cases arise:

- (i)If the demand of the first distribution center is satisfied, cross of the first column and move right to the second column.
- (ii)If the capacity of ith supply is satisfied, cross off ith row and reconsider the first column with the remaining demand.
- (iii)If the demands of the first distribution center as well as the capacity of the ith supply are completely satisfied, make a zero allocation in the second lowest cost cell of the first column. Cross of the column as well as the ith row and move right to the second column. [6]

Continue the process for the resulting reduced transportation table until all the rim conditions are satisfied.

ILLUSTRATIVE EXAMPLE (SUNRAY TRANSPORTATION)

Sun Ray Transportation Company ships truckloads of grain from three silos to four mills [2]. The supply (in truckloads) and the demand (also in truckloads) together with the unit transportation costs per truckload on the different routes are summarized in the transportation model in table.1.

	1	2	3	4	Supply
1	10	2	20	11	15
2	12	7	9	20	25
3	4	14	16	18	10
Demand	5	15	15	15	=50

Table 1. Transportation model of example (SunRay Transportation)

The model seeks the minimum-cost shipping schedule between the silos and the mills. This is equivalent to determining the quantity x_{ij} shipped from silo i to mill j ($i=1, 2, 3; j=1, 2, 3, 4$)

1. Northwest-Corner method

The application of the procedure to the model of the example gives the starting basic solution in table.2.

	1	2	3	4	Supply
1	10	2	20	11	15/10/0
2	12	7	9	20	25/20/5/0
3	4	14	16	18	10/0
Demand	5/0	15/5/0	15 /0	15/10/0	=50

Table 2. The starting solution using Northwest-corner method

The Starting basic Solution is given as
 $x_{11}=5, x_{12}=10, x_{22}=5, x_{23}=15, x_{24}=5, x_{34}=10$

The objective function value is
 $Z=5 \times 10 + 10 \times 2 + 5 \times 7 + 15 \times 9 + 5 \times 20 + 10 \times 18 = \520

2. Minimum Cost Method

The minimum-cost method is applied to Example (SunRay Transportation) in the following manner:

1. Cell (1,2) has the least unit cost in the tableau ($=\$2$).the most that can be shipped through (1,2) is $x_{12}=\min(15,15)=15$ which happens to satisfy both row 1 and column 2 simultaneously, we arbitrarily cross out column 2 and adjust the supply in row 1 to 0.

2. Cell (3,1) has the smallest uncrossed-out unit cost ($=\$4$).Assign $x_{31}=5$, cross out column 1 because it is satisfied and adjust the demand of row 3 to $10-5=5$ truckloads.

3.continuing in the same manner ,we successively assign 15 truckloads to cell (2,3),0 truckloads to cell (1,4),5 truckloads to cell(3,4) and 10 truckloads to cell (2,4).

The resulting starting solution is summarized in this table.3.

The arrows show the order in which the allocations are made.

	1	2	3	4	Supply
1	10	2	20	11	15/0
2	12	7	9	20	25/10/0
3	4	14	16	18	10/5/0
Demand	5/0	15/0	15 /0	15/5/0	=50

Table 3. The starting solution using minimum-cost method

The starting solution (consisting of 6 basic variables) is $x_{12}=15, x_{14}=0, x_{23}=15, x_{24}=10, x_{31}=5, x_{34}=5$

The associated objective function value is $Z = 15 \times 2 + 0 \times 11 + 15 \times 9 + 10 \times 20 + 5 \times 4 + 5 \times 18 = \475

The quality of the least cost starting solution is better than of the northwest –corner method because it yields a smaller value of Z (\$475 versus \$520 in the northwest-corner method).

	1	2	3	4	Supply	Row penalty
1	10	2	20	11	15	10-2=8
2	12	7	9	20	25	9-7=2
3	4	14	16	18	10	14-4=10
Demand	5	15	15	15	=50	
Column penalty	10-4=6	7-2=5	16-9=7	18-11=7		

Table 4. Step1 to determine the starting solution using (VAM)

3. Vogel’s Approximation Method (VAM)

VAM is applied to Example (SunRay Transportation) in the following manner:-

1. We compute from row 3 because row 3 has the largest penalty =10 and cell (3,1) has the smallest unit cost in the row, the amount 5 is assigned to x_{31} . Column 1 is now satisfied and must be crossed out. Next, new penalties are recomputed in the table. 5.

2. Shows that row 1 has the highest penalty(=9).hence ,we assign the maximum amount possible to cell (1,2),which yields $x_{12}=15$ and simultaneously satisfies both row 1 and column 2 .we arbitrarily cross out column 2 and adjust the supply in row 1 to zero as table. 6.

	1	2	3	4	Supply	Row penalty
1	10	2	20	11	15	10-2=8
2	12	7	9	20	25	9-7=2
3	4 5	14	16	18	10/5	14-4=10 ←
Demand	5/0	15	15	15	=50	
Column penalty	10-4=6	7-2=5	16-9=7	18-11=7		

Table 5. Step2 to determine the starting solution using (VAM)

	1	2	3	Supply	Row penalty
1	2 15	20	11	15/0	11-2=9 ←
2	7	9	20	25	9-7=2
3	14	16	18	10/5	16-14=2
Demand	15	15	15	=50	
Column penalty	7-2=5	16-9=7	18-11=7		

Table 6. Step3 to determine the starting solution using (VAM)

3. Continuing in the same manner, row 2 will produce the highest penalty (=11) and we assign $x_{23}=15$, which crosses out column 3 and leaves 10 units in row 2. Only column 4 is left, and it has a positive supply of 15 units. Applying the least-cost method to that column, we successively assign $x_{14}=0$, $x_{34}=5$ and $x_{24}=10$ (verify!) as table. 7.

The associated objective function value is $Z=15 \times 2 + 0 \times 11 + 15 \times 9 + 10 \times 20 + 5 \times 4 + 5 \times 18 = \475 . VAM produces a better starting Solution.

4. Row Minimum Method

Row minimum method is applied to Example (SunRay Transportation) in table. 8.

The associated objective function value is $Z = 15 \times 2 + 12 \times 5 + 9 \times 15 + 20 \times 5 + 10 \times 18 = \505 . This cost is less than northwest-corner method

5. Column Minimum Method

Column minimum method is applied to Example (SunRay Transportation) in table. 9.

The associated objective function value is $Z = 4 \times 5 + 15 \times 2 + 9 \times 15 + 10 \times 20 + 18 \times 5 = \475 . This cost is less than northwest-corner method.

	1	2	Supply	Row penalty
1	9 15	20	25/10	20-9=11 ←
2	16	18	10/5	18-16=2
Demand	15/0	15	=50	
Column penalty	16-9=7	20-18=2		

Table 7. Step4 to determine the starting solution using (VAM)

	1	2	3	4	Supply
1	10	2 15	20	11	15/0
2	12 5	7	9 15	20 5	25/20/5/0
3	4 5	14	16	18 10	10/0
Demand	5/0	15/0	15 /0	15/10/0	=50

Table 8. The starting solution using row minimum method

	1	2	3	4	Supply
1	10	2 15	20	11	15/0
2	12	7	9 15	20 10	25/10/0
3	4 5	14	16	18 5	10/5/0
Demand	5/0	15/0	15 /0	15/10/0	=50

Table 9.the starting solution using column minimum method

COMPARISON BETWEEN THE FIVE METHODS

North-west corner method is used when the purpose of completing demand No. 1 and then the next and is used when the purpose of completing the warehouse No. 1 and then the next. Advantage of northwest corner method is quick solution because computations take short time but yields a bad solution because it is very far from optimal solution.

Vogel's approximation method and Minimum-cost method is used to obtain the shortest road. Advantage of Vogel's approximation method and Minimum-cost method yields the best starting basic solution because gives initial solution very near to optimal solution but the solution of Vogel's approximation methods is slow because computations take long time. The cost of transportation

with Vogel's approximation method and Minimum-cost method is less than north-west corner method.

Row-minimum method is used when the purpose of completing the warehouse No. 1 and then the next. Row minimum cost is useful in small number of supply and when the cost of transportation on supply.

The cost of transportation is less than North-west corner method.

Column minimum method is used when the purpose of completing demand No. 1 and then the next .Column minimum cost is useful in small number of demand and when the cost of transportation on demand . The cost of transportation is less than North-west corner method.

OBJECT-ORIENTED PROGRAMMING

Object-Oriented Programming (OOP) is method of implementation in which programs are organized as cooperative collections of objects and each object is an instance of some classes, classes are related to one another via inheritance relationship. [1]

In Object-Oriented programming, the data and functions are integrated. An object is like a box containing its data and its functions which can operate on the data. [4]

Object-Oriented programming languages Provides great flexibility, clarity and reusability through inheritance. It leads to faster software development, increased quality, easier maintenance, and flexible modifiability.

Objects are the basic elements for executing object-oriented programs while classes are the basic elements for defining object-oriented programs. If any of these elements is missing, it is not an object-oriented program. [4] object-oriented programming languages such as java, C# and C++.

SOLVING TRANSPORTATION PROBLEM USING C++ LANGUAGE

We need to describe the five methods (mentioned above) of transportation model in LP using the five algorithms and we draw a flow chart for each algorithm.

After designing algorithms for the five methods we develop C++ program for each one. We used C++ language to facilitate getting the result and the complex problems which take long time using LP solution. After running these programs we compared between each solution using C++ program and LP solution which show that have the same result and compare between different solutions for choosing less value of the objective function.

The main ideas from design five C++ programs are save time, money, and effort.

- In the example (*Sun Ray Transportation*) we use the five C++ programs to minimize the cost of transportation and determine the number of units transported from source i to destination j .

The results are shown as follows.

- The result of northwest-corner method program by C++ language is the cost of transportation = \$520

The number of units transported from source i to destination j .

```
we transport
supply [0] to demand[0] =5
supply [0] to demand[1] =10
supply [1] to demand[1] =5
```

```
supply [1] to demand[2] =15
supply [1] to demand[3] =5
supply [2] to demand[3] =10
Press any key to continue
```

- The result of minimum-cost method program by C++ language is
the cost of transportation = \$475
The number of units transported from source i to destination j

```
we transport
supply [0] to demand[1] =15
supply [1] to demand[2] =15
supply [1] to demand[3] =10
supply [2] to demand[0] =5
supply [2] to demand[3] =5
Press any key to continue
```

- The result of vogel approximation method program by C++ language is
the cost of transportation = \$475
The number of units transported from source i to destination j

```
we transport
supply [0] to demand[1] =15
supply [1] to demand[2] =15
supply [1] to demand[3] =10
supply [2] to demand[0] =5
supply [2] to demand[3] =5
Press any key to continue
```

- The result of row minimum method program by C++ language is
the cost of transportation = \$505
The number of units transported from source i to destination j

```
we transport
transport supply [0] to demand[1] =15
transport supply [1] to demand[0] =5
transport supply [1] to demand[2] =15
transport supply [1] to demand[3] =5
transport supply [2] to demand[3] =10
Press any key to continue
```

- The result of column minimum method program by C++ language is
the cost of transportation = \$475
The number of units transported from source i to destination j

```
we transport
supply [0] to demand[1] =15
supply [1] to demand[2] =15
supply [1] to demand[3] =10
supply [2] to demand[0] =5
```


supply [2] to demand[3]=5
Press any key to continue

Minimum-cost method, Vogel's approximation method and column minimum methods are having the same objective value are equal to \$ 475 and give less value from other methods. We choose less result from these results to reduce the cost of transportation and we transport

supply [0] to demand[1] =15
supply [1] to demand[2] =15
supply [1] to demand[3] =10
supply [2] to demand[0] =5
supply [2] to demand[3] =5

The results of the five programs using C++ language are equal to LP solution but the solution using C++ language faster and easier than LP solution.

CONCLUSION

Running the five C++ programs for solving transportation problem show that the results of the five C++ programs is equal to the result of the LP solution. But the result the five program are defferent.the decision maker may choose the optimal result of the running of the five program (minimum) and determined the number of units transported from source i to destination j.

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