Linear Programming and Sensitivity Analysis in Production Planning

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Summary
Production planning process consists of three stages, namely, 1) manufacturing and marketing data preparation, 2) generation of production items and selling alternatives, and 3) production plan formulation. In this paper, Integer Linear Programming IP model in used for optimization of the produced and soled items under manufacturing constraints that will maximize the profit. Sensitivity Analysis SA presents a post optimality investigation of how a change in the model data changes the optimal solution. SA allows decision makers to determine how "sensitive" the optimal solution is to changes in data values.

Key words:
Linear programming, Integer Programming, Sensitivity analysis, production planning

1. Introduction

Integer Linear Programming (IP) played an important role is a problem solving and analysis tool. Researchers have addressed a variety of important problems through integer linear programming. IP [2,5,8] is usually combined with Sensitivity Analysis (SA) of input data to explore how changes in this data might change the optimal solution, for example, how a change in production costs or demand projections might affect a production schedule. Because large-scale planning efforts often rely on large amounts of data, much of which represents best-guess estimates, the ability to undertake such SA is critical to the acceptance of the methodology. Sensitivity analysis (or post-optimality analysis) is used to determine how the optimal solution is affected by changes, within specified ranges, in the objective function coefficients or the right-hand side (RHS) values. SA is important since IP models generally do not consider the times at which decisions are made. IP also not distinguish between what will be known, and what will remain uncertain when the decisions are made.

This paper presents how in production planning under uncertainty of input data, it is critical to properly consider the results of both IP optimal solution and SA that reflect the relationship between input data changes and the optimal solution.

The remainder of this paper is organized as follows: in section 2 IP production planning problem formulation that maximizes profit under manufacturing and selling constrains is presented. The SA formulation that determines how the optimal solution may be affected by changes in the objective function coefficients or the right-hand side (RHS) values is also given. For illustration, a production planning example is given in section 3, where the IP optimal solution is determined using the WINSQB solver. The SA for the production planning under different demand scenarios is presented in section 4 where each scenario is given a probability of occurrence. To capture the relationship between the times at which decisions are made and the time at which the demands are known, in section 5 demands are classified into three scenarios: low, most likely, and high. Section 6 discusses the production and selling decisions using the IP model where. Section 7 contributes on the IP problem formulation and optimal solutions for each demand-scenario. Section 8 contains the conclusions.

2. IP and SA Problem Formulation for Production Planning:

Production planning process consists of three stages, namely, manufacturing and marketing data preparation, generation of production items and selling alternatives, and production plan formulation. [1,3,4]. IP model in used for optimization of the produced and soled items under manufacturing constraints that will maximize the profit.

2.1. General IP Formulation for Production Planning:

The general problem of IP is the search for the optimal (minimum or maximum) of a linear function of variables constrained by linear relations (equations or inequalities) [2,5,7,8]. The IP optimizes a linear objective function subject to a set of linear equalities and/or inequalities. The general production planning maximization models is:

Objective Function:
Maximize $Z= c_1 x_1+ c_2 x_2 + \dots\dots + c_n x_n$
Subject to:
$a_{11} x_1 + a_{12} x2+ + \dots\dots + a_{1n} x_n \le r_1$
$a_{21} x_1 + a_{22} x2+ + \dots\dots + a_{2n} x_n \le r_2$

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$a_{m1} x_1 + a_{m2} x2+ + \ldots\ldots + a_{mn} x_n \le r_m$

And

$x_j \ge 0$

$j = 1, 2, 3 \ldots.n; i = 1, 2\ldots m$

Where:

Z= objective function that maximized selling profits

x_j = choice variable (production item) for which the problem solved

c_j = coefficient measuring the contribution of the j^{th} choice variable to the objective function.

r_i = constraint or restrictions placed upon the problem

a_{ij} = coefficient measuring the effect of the i^{th} constraint on the j^{th} choice variable.

The above problem can be solved using Software packages such as WINQSB and MS-Excel which provide following LP information:

1. Information about the objective function:
   a. objective function optimal value
   b. coefficient ranges (ranges of optimality). The range of optimality for each coefficient provides the range of values over which the current solution will remain optimal. Managers should focus on those objective coefficients that have a narrow range of optimality and coefficients near the endpoints of the range.
2. Information about the decision variables:
   a. their optimal values
   b. their reduced costs
3. Information about the constraints:
   a. the amount of slack or surplus
   b. the dual prices that represent the **improvement** in the value of the optimal solution per unit **increase** in the right-hand side.
   c. Right-hand side ranges (ranges of feasibility) that represent the range over which the dual price is applicable. As the RHS increases, other constraints will become binding and limit the change in the value of the objective function.

2.2. Sensitivity Analysis Rules: [6]

- For the objective function coefficients):

If $\sum_j \frac{\delta c_j}{\Delta c_j}$ <= 1, the optimal solution will not change,

Where:

δc_j is the actual increase (decrease) in the coefficient, and

Δc_j is the maximum allowable increase (decrease) from the sensitivity analysis.

* For the RHS constants:

If $\sum_j \frac{\delta b_j}{\Delta b_j}$ <= 1, the optimal basis and product mix will not change,

Where

δb_j is the actual increase (decrease) in the coefficient, and

Δb_j is the maximum allowable increase (decrease) from the sensitivity analysis.

3. Production Planning Illustrative Example:

An Electronics Firm currently has resources to produce two items: Laptop PC and Desktop PC. Now it is required to know if it will be profitable to produce a third item (LCD TV) which can be sold separately or with the Desktop PC. The Laptop PC sells for \$600, the Desktop PC sells for \$400, and the LCD sells for \$140. The manufacture of each item requires resources including: production line space and three types of skilled labors: Assemble, Quality Control/Quality Assurance, and Packing. Table 1 shows required production line space and labor (Assembly, Quality control, and packing) to produce the three items: Laptop PC, Desk PC, and LCD. The cost of these resources varies.

Current resources are sufficient for producing the amount of Laptop PC's and Desktop PC's but not for the LCD's.

The maximum production constraints and demands for the 3 products are also shown in Table 1.

From table 1, it is possible to determine how much of each item to produce and the resources required to meet this production in a number of ways. Perhaps the easiest method is a simple per-item profit analysis. A Laptop PC costs \$470 to produce and sells for \$600, for a net profit of \$130. A Desktop PC costs \$280 to produce and sells for \$400, for a profit of \$120.

On the other hand, an LCD costs \$110 to produce and sells for \$140.00, for a net profit of \$30. That is, Laptop PC and desktop PC seem to be profitable compared to LCD TV.

In the absence of constraints on resource availability, to maximize profit the Firm should produce as many of these items as it can sell according to its production constraints.

To produce 16500 Laptop PC, 13750 Desktop PC, and 11000 LCD the Firm needs the resources given in Table 2:

Resource	**Cost$**	**Production requirements**			**Production Resources constraints**
		Laptop	**Desktop**	**LCD**	
Production line space (board meter)	15	8	6	1	95 %
Assembly	30	5	3	1,5	88 %
Quality control (hours)	40	3	1.5	.5	80 %
Packing (hours)	20	4	2	1.5	92 %
Production Demand		15000+10% =16500	12500+10% =13750	10000+10% =11000	The 10% difference between produced and soled items are used for testing and customer support

Table 1: Firm manufacturing data

Total Resources	**Production requirements**			**Total Resources**	**Production Resources constraints**
	Laptop	Desktop	LCD		
Production line space (board meter)	132000	82500	11000	225500	95 %
Assembly	82500	41250	16500	140250	88 %
Quality control (hours)	49500	20625	5500	75625	92 %
Packing (hours)	66000	27500	16500	110000	85 %
Total Resources	330000	171875	49500	551375	

Table 2: Total Production Resources

As shown in table 2, the maximum resources needed are:

- 225500 Production line space (board meter)
- 140250 Assembly (hours)
- 75625 Quality control (hours)
- 110000 Packing (hours)

I.e. total Resources= 551375
The maximum anticipated profit (from selling the 15000 Laptop PC and 12500 Desktop PC, and producing 16500 Laptop PC and 13750 Desktop PC) = ($15.400.000 (selling of 100% products) - $11.726.000 (resources cost) – 0.279.000 (cost of the 10% stocked products)) = $2.395.000.
Since the available resources are not sufficient to produce the quantities of the three items, one preliminary decision from above data is that the Firm should not produce LCD's unless the number of demanded products increased above 10000, or the LCD selling price is better than $140.
The available resources are not enough to produce all the quantities since the available resources are 456250 divided as follows:

- 195000 Production line space (board meter)
- 112500 Assembly (hours)
- 63750 Quality control (hours)
- 85000 Packing (hours)

4. IP and SA model and solutions for the Production Planning: [2,5]

Production planning is required to settle the following decisions:

1) What is the maximum profit?
2) How much of each resource should it acquire?
3) How many of each item should it produce?
4) How many of each item should it sell?

The model behind this analysis does not consider these issues separately. Given the data in table 1, figure 1 draws the correspondences between these issues and ensure that production of those items is activated depending on what can be soled and acquire only the resources needed to produce them. The just-on-time-production strategy relates production to selling to ensures that produced products are those that can be soled and acquire only the resources needed to produce them. Production of new products starts automatically if the number of products in the firm stock is less than 10% of the products.

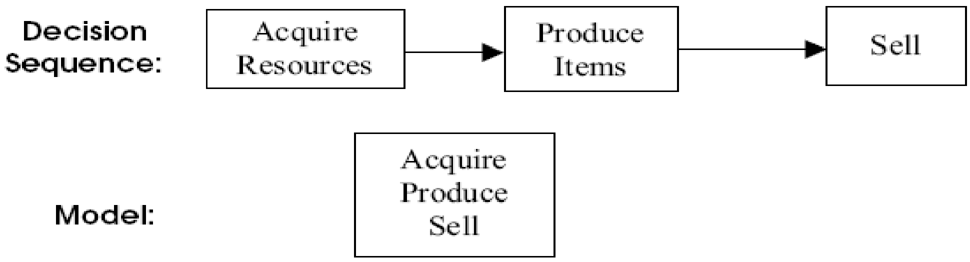

Figure 1: the correspondences between manufacturing resources, production, and selling

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As shown in figure 1, the Firm is actually faced with a sequence of three related decisions: How many resources to acquire, how many items to produce, and how many items to sell. The model represents these decisions as being made simultaneously, not sequentially.

4.1. IP Problem model:

The IP model and analysis exploit the structural advantages that accompany deterministic data and avoid representing potentially costly errors. In reality, the decisions occur sequentially over time. This manufacturing problem is straightforward. In the following, let

x_1 = number of Laptop PC to produce,
x_2 = number of Desk PC to produce,
x_3 = number of LCD to produce,
x_4 = number of board feet of production line space to acquire,
x_5 = number of labor hours for Assembly,
x_6 = number of labor hours for Quality control,
x_7= number of labor hours for packing,
x_8 = number of Laptop PC to sell,
x_9 = number of Desktop PC to sell, and
x_{10} = number of LCD to sell.

With these variables, it is possible to formulate the manufacturing problem with the following IP:

Objective function: (P.0)

Maximize $Z= -15x_4 -30x_5 -40x_6 -20x_7 +600x_8 +400x_9 +140x_{10}$ (1)

Subject to:

$$
\begin{array}{lllllllllll}
8x_1 & +6x_2 & +x_3 & -x_4 & & & & & & & \le 0 \\
5x_1 & +3x_2 & +1.5x_3 & & -x_5 & & & & & & \le 0 \\
3x_1 & +1.5x_2 & +.5x_3 & & & -x_6 & & & & & \le 0 \\
4x_1 & +2x_2 & +1.5x_3 & & & & -x_7 & & & & \le 0 \\
 & & & & & & & x_8 & & & \le 15000 \\
 & & & & & & & & x_9 & & \le 12500 \\
 & & & & & & & & & x_{10} & \le 10000 \\
-.9x_1 & & & & & & & +x_8 & & & \le 0 \\
 & -.9x_2 & & & & & & & x_9 & & \le 0 \\
 & & -.9x_3 & & & & & & & x_{10} & \le 0 \\
 & & & +x_4 & & & & & & & \le 195000 \\
 & & & & +x_5 & & & & & & \le 112500 \\
 & & & & & x_6 & & & & & \le 63750 \\
 & & & & & & x_7 & & & & \le 85000 \\
x_{1,} & x_{2,} & X_{3,} & x_{4,} & x_{5,} & x_{6,} & x_{7,} & x_{8,} & x_{9,} & x_{10} & \ge 0
\end{array}
$$

Solving this problem using e.g. WINQSB package will result:

Net profit Z=$2.096.825

X1 = 13810	X2 = 13889
X3 = 1190	X4 = 152407
X5 = 195000	X6 = 62857
X7 = 84802	X8 = 12429
X9 = 12500	X10 =1071

The results conclude that the optimal decision is not to produce LCD or to decrease the produced number from 10000 to 1190, slightly decrease number of produced LAPTOP PC, and to produce all Desktop PC.

4.2. SA of the input data:

If the data in table 1 change, the structure of the model remains the same. For example, to study the effect of producing certain product such as the LCD can be done based on following scenarios:

- Increase selling price by e.g. 10% (i.e. from $140 to e.g. $155)
- Decrease number of this product

Through SA, it is possible to change the corresponding coefficient in the objective function and resolve the IP problem once more.
The same can be considered for the other products, taking into consideration that increasing the price would decrease the selling probability Ps whereas decreasing the price will increase Ps.
These observations give rise to the investigation of the SA. Knowing that the structure of the problem does not change, it is possible to investigate how changes in individual data elements change the optimal solution as follows:

- If nothing else changes except the objective function value when slightly change the price of a product, producing becomes profitable, and the nature of the solution changes considerably.
- On the other hand, if the selling price is kept fixed, and the demands increase or drop by e.g. 10% and there would be no major impact on the solution, Firm would still not produce all quantities of the products and take the initial IP problem solution into consideration.

5. Sensitivity Analysis for the Production Process: [2]

Production demands may be uncertain, but low, most likely, and high values may be available. Assume that:

- low values of demand for Laptop PC, Desktop PC, and LCD equal 75% i.e. (11750, 9375, and 7500) occur with probability $p_l = 0.3$
- most likely values equal 90% (13500, 1125, and 9000) occur with probability $p_m = 0.4$
- high values equal 100% (15000, 12500, and 10000) will occur with probability $p_h = 0.3$.

The possible demand scenarios and the corresponding probabilities form a distribution that can be used to describe future demands. The demand scenario presented in table 4 is the expected value associated with the distribution in table 3.

Product	**Production requirements**		
Scenario	I	II	III
LAPOP PC	11750	13500	15000
DESKTOP PC	9375	11250	12500
LCD TV	7500	9000	10000
Expected Probability of occurrence	.3	.4	.3

Table 3: Production possible demand scenarios

Analysis of the sensitivity of the solution to (P.0) indicates that decision “produce as many Laptop PC and Desktop PC as can be sold, but do not produce high quantity of LCD” will remain valid for any set of non-negative demands. Table 4 shows the optimal response to each of the individual demand scenarios.

Product	**Production requirements**		
Scenario	I	II	III
LAPTOP PC	8500	13500	12429
DESKTOP PC	9375	11250	12500
LCD TV	7500	0	1071
production line space to acquire	174907	195000	152407
labor hours for Assembly	108472	112500	195000
labor hours for Quality control	58773	62750	62857
labor hours for packing	85000	85000	84802
Profit	1.874.629	2.050.000	2.069.825

Table 4: Profit optimal solution corresponding to each scenario.

Table 4 shows that production quantities and the resource acquisitions vary widely across the scenarios. In all cases, firm would produce part of (Laptop PC and LCD) and all Desktop PC. The firm needs to acquire resources to satisfy production schedule. The production and resource quantities in the expected-value column are the expected

values of the corresponding quantities in the remaining columns. (This is a property of the simplicity of the example; in general, the expected value of the data does not correspond to the expected value of the solutions.)
Given the stability of the structure of the solution and the relationship among the various solutions, we might think that the solution with the expected demand is an appropriate response for the manufacturing problem. However, if Firm produces 15000 Laptop PC and 12500 Desktop PC, to meet the mean demand constraint, it has a 30% chance of producing Laptop PC and a 70% chance of producing desktop PC.
If it produces 15000 Laptop PC and 12500 Desktop PC and the low-demand scenario occurs, the Firm's profit will be much lower than $1,874.629. The costs for resources at this level are $0,427.152 and for a net loss of $195,196.
If the Firm produced l5000 Laptop PC and l2500 Desktop PC and experienced the most likely demand, its net gain would be $2.050.000. Although not a loss, this amount is well below the projected profit of Z=$2.096.825 suggested by the original IP solution.

6. Production and Selling Decisions Using IP Model:

When faced with uncertainty in the demand for products, a more thoughtful approach to model development is needed. In this case, capture the relationship between the times at which decisions are made and the time at which demands are known is needed. The objective is to adapt decisions made after the demand is known to the specific demand scenario. Something we cannot do for decisions made before we know the demand. Logically, three potential information timings are of concern (Figure 2).

When demand is uncertain, it is important to know when it will be revealed to the decision maker. Will it be known before resources are acquired, between acquisition and production, or after production decisions are made? That is, to determine the point during the decision sequence at which demands are known and to have complete information about the demand before making any decisions.
At the other extreme, demands might not be known until after resources are acquired. The demand determines the actual sales quantities and consequently revenues.
An intermediate possibility is that resources are acquired while firm is uncertain about the demand, but the production schedule is set only after knowing the demand and thus have adapted to it. These three possibilities give rise to three different types of models.

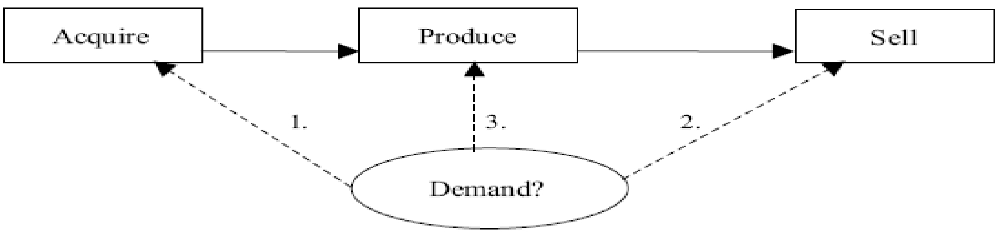

Figure 2: When will demand be known?

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At the other extreme, demands might not be known until after resources are acquired. The demand determines the actual sales quantities and consequently revenues.
An intermediate possibility is that resources are acquired while firm is uncertain about the demand, but the production schedule is set only after knowing the demand and thus have adapted to it. These three possibilities give rise to three different types of models.

In the rest case, we know demand at the start and can base decisions about acquiring resources, production, and sales on whether demand is low, most likely, or high (Figure 3). As shown in figure 3, if demand will be known before any decision is made, the decision tree contains the deterministic model depicted in figure 1. It is replicated for each demand scenario. If demand is known at the start, our decisions are not exposed to uncertainty, and we need no cross-scenario evaluation. Because all uncertainty is resolved before we make any decisions, we adapt any decision to the specific scenario realized, and the problem collapses into a collection of deterministic problems; only the origin remains uncertain.
To formulate this problem, we need three separate sets of variables, one for each possible demand scenario (low, most likely, and high). An IP model for this problem will

be separable by scenario. Working from (P.0), and letting X_8ds denote the demand for Laptop PC under demands scenarios (with low l, medium m, and high h demands with probabilities Ps defined), we obtain:

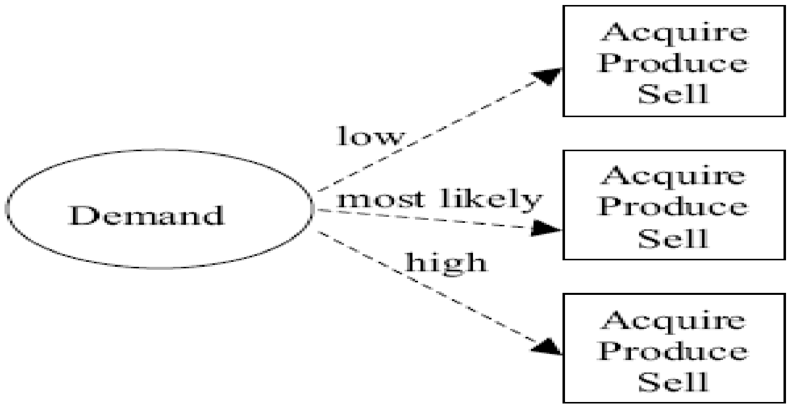

Figure 3: Production Planning demands and decisions

Objective function: (P.1)
Maximize
$Z=\sum_{(l,m,h)}(-15X_4-30X_5-40X_6-20X_7+(600\ X_8ds+400X_9+140X_{10}))\ Ps$ (2)
Subject to:

$8x_1$	$+6x_2$	$+x_3$	$-x_4$							≤ 0
$5x_1$	$+3x_2$	$+1.5x_3$		$-x_5$						≤ 0
$3x_1$	$+1.5x_2$	$+.5x_3$			$-x_6$					≤ 0
$4x_1$	$+2x_2$	$+1.5x_3$				$-x_7$				≤ 0
							x_8			$\leq$ ds for LAPTOP
								x_9		$\leq$ ds for DESKTOP
									x_{10}	$\leq$ ds for LCD
$-.9x_1$							$+x_8$			≤ 0
	$-.9x_2$							x_9		≤ 0
		$-.9x_3$							x_{10}	≤ 0
			$+x_4$							≤ 195000
				$+x_5$						≤ 112500
					x_6					≤ 63750
						x_7				≤ 85000
$x_1,$	$x_2,$	$X_3,$	$x_4,$	$x_5,$	$x_6,$	$x_7,$	$x_8,$	$x_9,$	x_{10}	≥ 0

Based on this equation and table 3 it is possible to calculate RHS for each case:
ds for LAPTOP = .3x11750+.4x13500+.3x15000=13425
ds for DESKTOP = .3x9375+.4x11250+.3x12500=11063
ds for LCD = .3x7500+.4x9000+.3x10000=8850
Solving the IP problem using WINQSB will result:
Z=$2.035.533
$X_1 = 14917$ $X_2 = 12292$
$X_3 = 499$ $X_4 = 193586$
$X_5 = 112209$ $X_6 = 62438$
$X_7 = 85000$ $X_8 = 13425$
$X_9 = 11063$ $X_{10} = 449$

As indicated, equation (2) is separable by scenario. We can consider each demand scenario separately, and we can obtain scenario-specific solutions independently. Only in calculating the objective value we combine them. At the other extreme, we determine both acquisition and production before we know the demand (Figure 4).

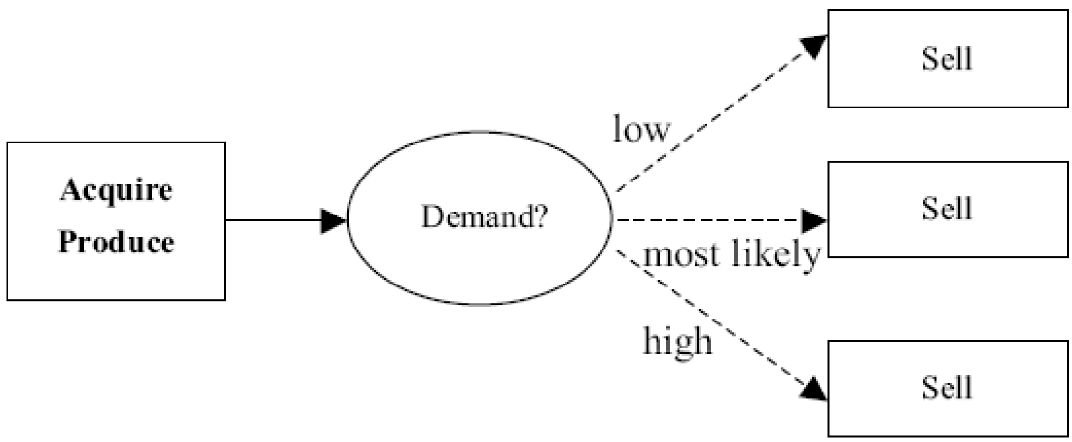

Figure 4: Decisions of Production planning demands

As shown in Figure 4: If demand is known after acquisition and production are determined, it will affect only the amount of product that is sold. Once made, the decisions about acquisition and production are fed into the demand uncertainty. Only the sales levels respond to the acquisition and production levels and the manner in which the demand uncertainty is resolved.

Any IP model of this problem must capture the fact that the initial decisions must be weighed against all possible demand scenarios. To accomplish this, we use three separate sets of the sell variables, and only one set of the acquisition and production variables. As before, we work from (P.0) to develop our model. To connect Figure 4 and the IP model, we use a bold font to identify decisions made before demand is known.

Objective function: (P.2)

Maximize

$$Z=-15X_4-30X_5-40X_6-20X_7+\sum_{(l,m,h)}(600X_8ds+400X_9+140X_{10}))\ Ps \qquad (3)$$

Subject to:

$8x_1$	$+6x_2$	$+x_3$	$-x_4$							≤ 0
$5x_1$	$+3x_2$	$+1.5x_3$		$-x_5$						≤ 0
$3x_1$	$+1.5x_2$	$+.5x_3$			$-x_6$					≤ 0
$4x_1$	$+2x_2$	$+1.5x_3$				$-x_7$				≤ 0
							x_8			$\leq$ ds for LAPTOP
								x_9		$\leq$ ds for DESKTOP
									x_{10}	$\leq$ ds for LCD
$-.9x_1$							$+x_8$			≤ 0
	$-.9x_2$							x_9		≤ 0
		$-.9x_3$							x_{10}	≤ 0
			$+x_4$							≤ 195000
				$+x_5$						≤ 112500
					x_6					≤ 63750
						x_7				≤ 85000
$x_1,$	$x_2,$	$X_3,$	$x_4,$	$x_5,$	$x_6,$	$x_7,$	$x_8,$	$x_9,$	x_{10}	≥ 0

Let assume the case for changing the selling price by 10%.for one product: LABTOP PC

Objective function:

Maximize

1) $Z=-15X_4-30X_5-40X_6-20X_7+660X_8ds+400X_9+140X_{10}$, where P_s=.2 as the probability of selling if the price increases

2) $Z=-15X_4-30X_5-40X_6-20X_7+540X_8ds+400X_9+140X_{10}$, where P_s=.5 as the probability of selling if the price decreases

Let assume the case for changing the selling price by 10% for LCD:
Objective function: (P.3)

Maximize
$Z=-15X_4-30X_5-40X_6-20X_7+600X_8ds+400X_9+155X_{10}$,
where P_s =.2 as the probability of selling if the price increases. Solving the IP will yield the following table 5

Product	**Production requirements for LAPTOP PC**			**Production requirements for LCD**	
Scenario	I	II	III	II	II
LAPTOP PC price	660	600	540	600	600
LCD price	140	140	140	140	155
Selling Probability	.2	.3	.5	.8	.2
LAPTOP PC	15000	13500	9125	13500	9025
DESKTOP PC	12500	12500	12500	12500	12500
LCD TV	0	0	10000	0	100000
production line space to acquire	195000	195000	175556	195000	109028
labor hours for Assembly	112500	112500	109027	112500	109028
labor hours for Quality control	63750	63750	56806	63750	56806
labor hours for packing	85000	85000	85000	85000	85000
Profit	2.860.000	2.050.000	1.451.111	2.050.000	2.148.611

Table 5: Production and selling items if LCD price change by 10%

The decisions from table 5 would be:

- Not to produce LCD if its price is $140
- Produce LCD if LAPTOP price is dropped by 10% or LCD price increased 10%

In contrast to (P.1), (P.2) is not separable by scenario. Acquisition and production, represented by x and y, are determined before demand is known and are held constant across all scenarios. The second set of constraints models the manner in which sales depend on the combination of production and demand. The lack of reparability arises because of the interaction of the two types of variables in these constraints. Finally, in the remaining case (3 in Figure 3) we determine acquisition before we know the demand and production and sales afterward (Figure 5).

As we work from (P.0) to develop an IP model for this problem, we have a single set of acquisition variables, and three sets of production and sales variables:

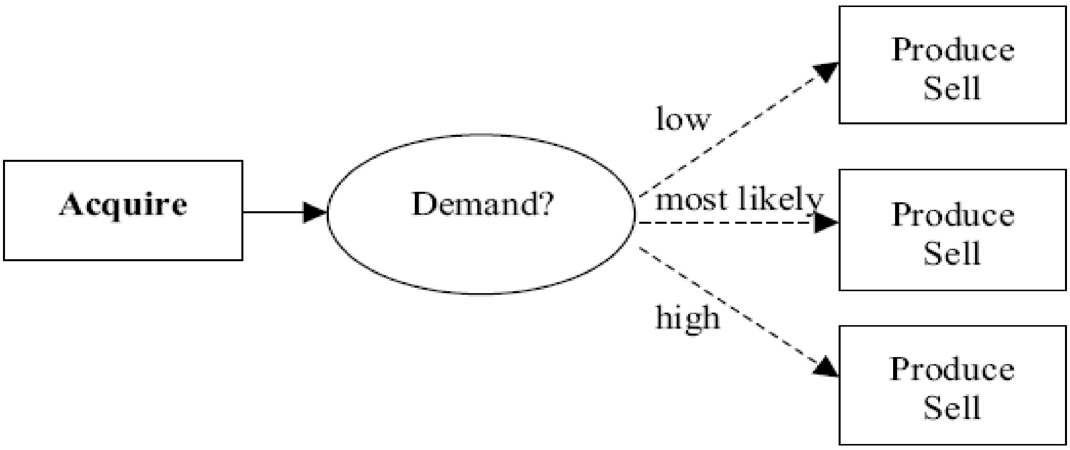

Figure 5: Case demands known after resources are acquired but before production levels are determined

Similar to (P.2), (P.3) lacks segmentation. In general, segmentation does not occur when the IP model includes uncertainty within the midst of the decision sequence.

7. Contribution on Problem Formulations and Solutions

The IP models, (2) and (3) can be traced back to the original model, (1), but they differ. They represent three different models of the problem. We have little need for a model such as (2). Because we know the demand before making any decisions, we do not need to solve (2). That is, we can wait until we know the demand and solve the appropriate scenario problem. As presented, the output of (2) provides the optimal solution and

objective values for all possible demand scenarios. For planning, this information might be helpful.
The second model, (3), provides a proper mechanism for determining the expected revenues when we must determine production before we know the demand. This model accounts for the possibility that production might exceed demand. In particular, when we set production levels (which in turn determine the levels of resource acquired), we base them upon a model of the revenues that we can expect from selling them. Although the output to (2) is structurally similar to that of the individual scenario problems in (1), the values are different. In (2), the firm produces items prior to knowing the demand. Unlike (1), the production levels suggested by (2) do not match any of the demand scenarios. In (2), production levels are set in a manner that balances the potential sunk cost of producing items that cannot be sold against the potential revenue available from selling a larger number of items.
The various objective values differ as well. It is well known that solving an IP in which random variables in the right-hand sides of the constraints are replaced by their expected values yields an optimistic objective value, as indicated in (1) compared to the rest. Indeed, in this case, (P.0) is as optimistic as (2), in which the decision maker knows all information before making any decisions. That the objective value for (2) exceeds that of (1) is no surprise; delaying decisions until one has information usually brings economic advantages. To determine the appropriate model, one must identify the point at which information about demand will be available.

8. Conclusions

Managing data when constructing IP models can be challenging. Planning models often address decisions to be made in the future. The data used in IP models is often clouded with uncertainty.
Sensitivity Analysis SA (or post-optimality analysis) is used to determine how the optimal solution is affected by changes, within specified ranges, in the objective function coefficients and the right-hand side (RHS) values. Sensitivity analysis is important to the manager who must operate in a dynamic environment with imprecise estimates of the coefficients. SA allows asking certain what-if questions about the problem. SA is appropriate when the basic structure of the model is not altered by the presence of uncertainty, for example, when all uncertainties will be resolved before any decisions are made. When the decisions are to be made, a deterministic model will be appropriate. In this situation, SA can help us to appreciate the impact of uncertainty. In all other cases, we cannot count on it to do so.
When information is obtained during a decision sequence, we have the opportunity to adapt to it. Whether the adaptation is imposed, as when sales are constrained by demand, or advantageous, as when production decisions can be delayed until after demand is known, adaptation causes changes in the IP model. The constraint matrix changes considerably, affecting both the number of constraints and the number of variables.

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