A New Algorithm Using Hopfield Neural Network with CHN for N-Queens Problem

Wei Zhang† and Zheng Tang††,

Faculty of Engineering, Toyama University, Toyama-Shi, JAPAN

Summary
A model of neurons with CHN (Continuous Hysteresis Neurons) for the Hopfield neural networks is studied. We prove theoretically that the emergent collective properties of the original Hopfield neural networks also are present in the Hopfield neural networks with continuous hysteresis neurons. The network architecture is applied to the N-Queens problem and results of computer simulations are presented and used to illustrate the computation power of the network architecture. The simulation results show that the Hopfield neural network with CHN is much better than other algorithms for N-Queens problem in terms of both the computation time and the solution quality.

Key words:
Hopfield neural network, hysteresis, collective properties, N-Queen problem

1. Introduction

The optimization problems are encountered in various situation. There is a problem which has discrete determined variables is called "Combinatorial Optimization Problems." This problem is complicated more than a linear programming, and it is called "NP-Hard." N-Queens problem is one of the NP-Hard combinatorial optimization problems. N-Queens problem is that N chess queens must be placed on a square chessboard composed of N rows and N columns, in such a way that they do not attack each other for 8 directions.
The auto associative memory model proposed by Hopfield [1, 2] has attracted considerable interest both as a content address memory (CAM) and, more interestingly, as a method of solving difficult optimization problems [3-5]. The Hopfield neural networks contain highly interconnected nonlinear processing elements ("neurons") with two-state threshold neurons [1] or graded response neurons [2]. Takefuji and Lee proposed a two-state (binary) hysteretic neuron model to suppress the oscillatory behaviors of neural dynamics [14]. However, Tateshi and Tamura showed Takefuji and Lee's model did not always guarantee the descent of energy function [7], Wang also explained why the model may lead to inaccurate results and oscillatory behaviors in the convergence process [8]. Since their report, several modifications on the hysteretic function, for example Galán and Muñoz's binary [9] and Bharitkar and Mendel's multivalued [10] hysteretic functions.
In this paper, we propose a new Hopfield neural network algorithm for efficient solving N-Queens problem. Different to the original Hopfield neural network, our architecture uses continuous hysteresis neurons. We prove theoretically that the emergent collective properties of the original Hopfield neural network also are present in the Hopfield network with continuous hysteresis neurons. Simulations of randomly generated neural networks are performed on both networks and show that the Hopfield neural networks with CHN have the collective computational properties like the original Hopfield neural networks. What a more, it converges faster than the original Hopfield neural networks do. In order to see how well the architecture neurons do for solving practical combinatorial optimization problems, a large number of computer simulations are carried out for the N-Queens problem.

2. Hopfield Network with Continuous Hysteresis Neurons

2.1 Original Hopfield Neural Networks

For the original Hopfield neural networks, let the output variable y_i for neuron i have the range $y_i^0 \le y_i \le y_i^1$ and be a continuous and monotone-increasing function of the instantaneous input x_i to neuron i. The typical input-output relation $g_i(x_i)$ $y_i = 1/(1+e)$ shown in Fig.1(a) is sigmoid with asymptotes y_i^0 and y_i^1. as,

$$y_i = g(x_i) = 1/(1+e^{-r(x_i-\theta)}) \quad (1)$$

Where r is the gain factor and θ is the threshold parameter.

2.2 Continuous Hysteresis Neurous

If In biological system, x_i will lag behind the instantaneous outputs y_i of the other cells because of the input capacitance C of the cell membranes, the transmenmbrane resistance R, and the finite impedance w_{ij}^{-1} between the

Manuscript received April 5, 2009
Manuscript revised April 20, 2009

output y_i and the cell body of cell i. Thus there is a resistance-capacitance (RC) charging equation that determines the rate of change of x_i.

$$C_i(\frac{dx_i}{dt}) = \sum_{j=1}^{N} w_{ij} y_j - \frac{x_i}{R_i} + I_i \quad i = 1,2,\cdots,N$$
$$x_i = g_i^{-1}(y_i) \tag{2}$$

Where C_i is the total input capacitance of the amplifier i and its associated input lead. $w_{ij}y_j$ represents the electrical current input to cell i due to the present potential of cell j, and w_{ij} is thus the synapse efficacy. I_i is any other (fixed) input current to neuron i. In terms of electrical circuits, $g_i(x_i)$ represents the input-output characteristic of a nonlinear amplifier with negligible response time. It is convenient also to define the inverse output-input relation, $g_i^{-1}(y_i)$.

In order to improve the solution quality of N-Queens problem, we proposed a new neural network method for efficiently solving the N-Queens problem. In this method, an continuous hysteresis neuron is applied to the Hopfield neural network.

The continuous hysteresis neurons change the value of their output or leave them fixed according to a hysteretic threshold rule (Fig.1 (b)). Mathematically, the hysteretic neuron function is described as:

$$y(x_i(t) \big| \dot{x}_i(t-\delta t)) = g\left[x_i(t) - \theta(\dot{x}_i(t-\delta t)) \right] \tag{3}$$

where

$$\gamma(\dot{x}_i(t-\delta t)) = \begin{cases} \gamma_\alpha, & \dot{x}_i(t-\delta t) \geq 0 \\ \gamma_\beta, & \dot{x}_i(t-\delta t) \leq 0 \end{cases} \tag{4}$$

and

$$\theta(\dot{x}_i(t-\delta t)) = \begin{cases} -\alpha, & \dot{x}_i(t-\delta t) \geq 0 \\ \beta, & \dot{x}_i(t-\delta t) \leq 0 \end{cases} \tag{5}$$

$\beta > \alpha$, and $(\gamma_\alpha, \gamma_\beta) > 0$, and $\dot{x}_i(t-\delta t) \underline{\underline{\Delta}} \frac{dx_i(t-\delta t)}{dt}$.

Thus, there is a resistance-capacitance (RC) charging equation that determines the rate of change of x_i.

$$C_i(\frac{dx_i}{dt}) = \sum_{j=1}^{N} w_{ij} y_j - \frac{x_i}{R_i} + I_i + \frac{\theta_i(\dot{x}_i(t-\delta t))}{R_i} \quad i = 1,2,\cdots,N$$
$$x_i = g_i^{-1}(y_i) \tag{6}$$

Consider the energy:

$$E = -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} w_{ij} x_i y_j + \sum_{i=}^{1} \frac{1}{R_i} \int_0^{-1} g_i^{-1} \times \left[y_i(x_i \big| \dot{x}_i) \right] dy_i + \sum_{i=1}^{N} I_i y_i \tag{7}$$

Its time derivative for a symmetric W is:

$$\frac{dE}{dt} = -\sum_{i=1}^{N} \frac{dy_i}{dt} (\sum_{j=1}^{N} w_{ij} y_j - \frac{x_i}{R_i} + I_i + \frac{\theta_i(\dot{x}_i(t-\delta t))}{R_i}) \tag{8}$$

The parenthesis is the right-hand side of Eq.6, so

$$\frac{dE}{dt} = -\sum_{i=1}^{N} C_i(\frac{dy_i}{dt})(\frac{dx_i}{dt}) = -\sum_{i=1}^{N} C_i g_i^{-1'}(y_i)(\frac{dy_i}{dt})^2 \tag{9}$$

Since $g_i^{-1}(y_i)$ is a monotone increasing function and C_i is positive, each term in this sum is nonnegative. Therefore:

$$\frac{dE}{dt} \leq 0, \quad \frac{dE}{dt} = 0 \rightarrow \frac{dy_i}{dt} = 0 \quad \text{for all } i \tag{10}$$

Together with the boundedness of E, Eq.6 shows that the time evolution of the system is a motion in state space that seeks out minima in E and comes to a stop at such points. E is a Liapunov function for the system.

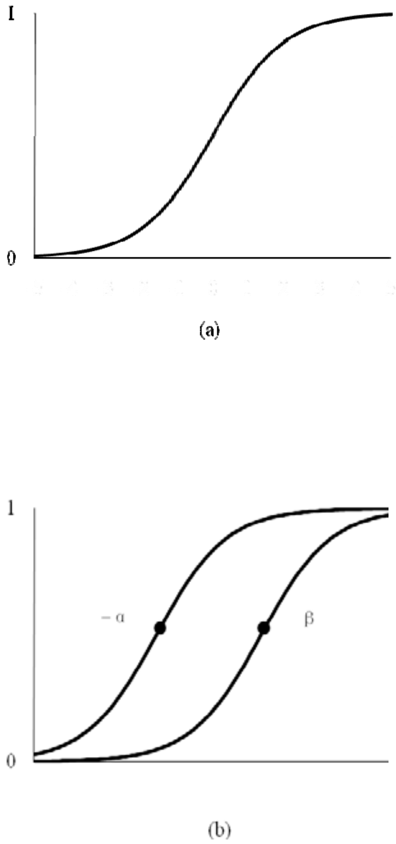

Fig. 1 Hysteresis functions.

3. Application to N-Queens Problem

N-Queens problem is classic of difficult optimization. The task is given a standard chessboard and N chess queens, to place them on the board so that no queen is on the line of attack of any other queen. The problem can be solved by constructing an appropriate energy function and minimizing the energy function to zero (E=0) using an N×N two-dimensional Hopfield neural networks [11~14].

The objective energy function of the N-Queens problem is given by:

$$E=\frac{A}{2}\left(\sum_{i=1}^{N}\left(\sum_{k=1}^{N}y_{ik}-1\right)^2+\sum_{j=1}^{N}\left(\sum_{k=1}^{N}y_{kj}-1\right)^2\right)$$
$$+\frac{B}{2}\left(\sum_{i=1}^{N}\sum_{j=1}^{N}y_{ij}\left(\sum_{\substack{1\le i-k,\,j-k\le N\\ k\ne 0}}y_{i-k,j-k}+\sum_{\substack{1\le i-k,\,j+k\le N\\ k\ne 0}}y_{i-k,j+k}\right)\right) \quad (11)$$

where A and B are coefficients, the output y_{ij}=1 represents that a queen is placed at i-th or j-th column on the chessboard, and output y_{ij}=0 represents no placement there. The first term becomes zero if one queen is placed in every row. The second term becomes zero if one queen is placed in every column and the third term becomes zero if no more than one queen is placed on any diagonal line. We can get the total input (x_{ij}) of neuron by using the partial derivation term of the energy function. Thus, the total input (x_{ij}) of neuron is given:

$$x_{ij}=-A\left(\sum_{k=1}^{N}y_{ik}-1\right)-A\left(\sum_{k=1}^{N}y_{kj}-1\right)$$
$$-B\left(\sum_{\substack{1\le i-k,\,j-k\le N\\ k\ne 0}}y_{i-k,j-k}+\sum_{\substack{1\le i-k,\,j+k\le N\\ k\ne 0}}y_{i-k,j+k}\right) \quad (12)$$

Using Eq.12, networks with continuous hysteresis neurons for a total 10 chessboard size instances from 10 to 300 queens problem were simulated on a digital computer. 100 simulation runs with different initial states were performed in each of these instances. In the simulations, Eq.6 was used as the input/output functions of neurons. The neuron's activation function has four parameters associated with it. We set $\alpha=0.5\,\beta=0.5$ and $\gamma_\alpha=\gamma_\beta=0.02$ for all neurons. The parameters A and B were set to 1. The maximum iteration steps were set to 1000. Using the same condition, the original Hopfield neural networks was also executed for comparison. The simulation results are shown in Table1.

In Table1, where the convergence rates and the average numbers of iteration steps required for the convergence were summarized. The simulation results show that the networks with continuous hysteresis neurons can almost find optimum solution to all N-Queens problems within short computation times; while the original Hopfield neural networks can hardly find any optimum solution to the N-Queens problems.

We also compared our results with that found by Takefuji's neural network [11] and the maximum neural network [13]. Table 1 shows the results by the four different networks, where the convergence rates and the average numbers of iteration steps required for the convergence are summarized. From Table 1 we can see that the Hopfield neural networks with continuous hysteresis neurons was very effective, and was better than other exiting neural networks in terms of the computation time and the solution quality for the N-Queens problem. Further, the average numbers of iteration steps indicated that the problem size did not strongly reflect the global minimum convergence rate and number of iteration steps. From the simulation results we can summarize that the Hopfield neural networks with continuous hysteresis neurons is very effective for solving the N-Queens problem.

Table 1: Simulation result

Queens	Proposed network		Hopfield		MaximumNN		Takefuji	
	Convergence	Step	Convergence	Step	Convergence	Step	Convergence	Step
10	99	8	9	609	20	85	42	178
20	100	19	8	701	37	121	53	260
30	100	24	0		49	146	53	259
50	100	43	0		68	152	86	331
100	100	52	0		79	162	99	324
150	100	81	0		89	161	98	410
200	100	99	0		96	165	90	521
300	100	146	0		98	159	83	631

4. Simulation Results

Experiments were first performed to show the convergence of the Hopfield neural networks with continuous hysteresis neurons. In the simulations, a 100-neuron Hopfield neural network with continuous hysteresis neurons (i = 1, 2, …, 100) was chosen. Initial parameters of the network, connection weights and thresholds were randomly generated uniformly between –1.0 and 1.0. Simulations on a randomly generated 100-neuron Hopfield network with continuous hysteresis neurons were carried out. We did 100 simulations with different randomly-generated weights and thresholds. Fig.2 shows the statistically compared convergence charactoristics of both networks with the average energy over 100 simulations. From this figure we can see that both the Hopfield neural networks with continuous hysteresis neurons ($\alpha = 0.5, \beta = 0.5$) and the original Hopfield neural networks converged to stable states that did not further change with time. It is worth to note that the Hopfield neural network with continuous hysteresis neurons ($\alpha = 0.5, \beta = 0.5$) seek out a smaller minimum at $E = -162.38$ than the original Hopfield neural network at $E = -136.90$.

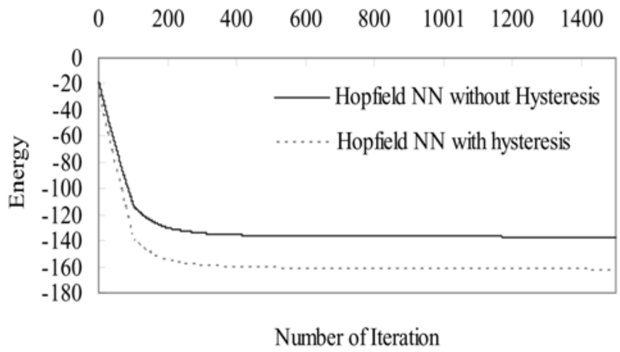

Fig. 2. The average convergence characteristic of a 100-neuron Hopfield network with and without the continuous hysteresis neurons.

In general, the performance of optimization problem using neural network depends on parameters. In the proposed algorithm, α and β are important parameters which influence the performance of the network. To study the appropriate range of α and β, we simulated two graphs using different α and β. We experimented with the following values of $\alpha = 0$, $\beta = 0, 0.5, 1, \cdots, 3$, respectively. Fig.3 shows the simulation results. From the simulation results we found that the networks with continuous hysteresis neurons ($1 \le \beta \le 1.5$, when $\alpha = 0$), had the best performance. But, when β was larger than 3.0 ($\alpha = 0$), the continuous hysteresis neurons tended to degrade the performance of the network. Using the same α and β set, we also simulated the average number of the iterations by the Hopfield neural networks with continuous hysteresis neurons. Fig.4 shows the simulation results.

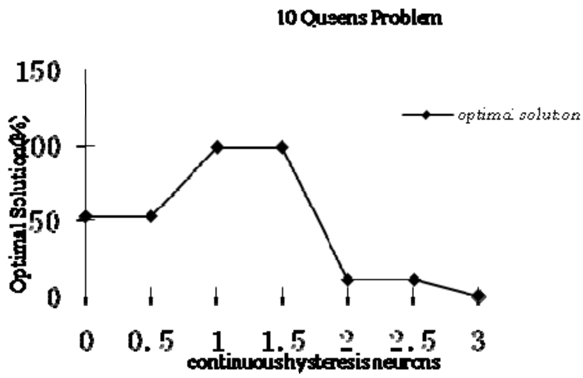

Fig.3. The percentage of optimal solution of the 10 Queens problem by the Hopfield neural networks with continuous hysteresis neurons (β - α = 0, 0.5, ,…, 3)

From these figures, we find that the appropriate range of α and β is almost as same as that in the first instance.

From these simulation results, we may conclude that the proposed architecture could improve the performance of the Hopfield neural network by selecting an appropriate value of continuous hysteresis neurons.

Fig.4. The percentage of optimal solution of the 10 Queens problem by the Hopfield neural networks with continuous hysteresis neurons (β - α = 0, 0.5，,…, 3)

In order to widely verify the proposed algorithm, we have also tested it with a few number of randomly generated queens defined in terms of two parameters, *A* and *B*. Fig.5 shows the simulation results. From this we can find that the when A=1 and B=1, it has the best performance for the problem.

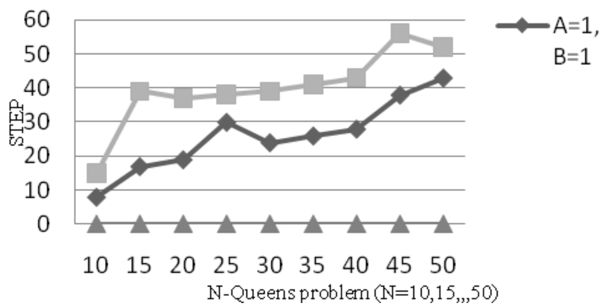

Fig.5. The relationship of the parameters *A*, *B* and with the continuous hysteresis neurons

5. Conclusions

In this paper, we have presented theoretical and experimental evidence showing that the Hopfield neural networks with continuous hysteresis neurons has the same collective computational properties as the original Hopfield neural networks. We have also compared the network with the original Hopfield neural networks. As theoretically predicted, it was found experimentally that the Hopfield neural networks with continuous hysteresis neurons converged faster than the original Hopfield neural networks did. In order to confirm the practical worth of the Hopfield neural networks with continuous hysteresis neurons, it was also applied to N-Queens problem. A large number of computer simulation have been carried out for N-Queens problem to verify the effectiveness of this network in combinatorial optimization problems. The simulation results showed that the Hopfield neural networks with continuous hysteresis neurons were better than the original Hopfield neural networks method and other existing neural network methods for solving N-Queens problem in terms of the computation time and the solution quality.

References

[1] J.J. Hopfield, “ Neural network and physical systems with emergent collective computational abilities.” Proc. Natl. Acad. Sci. USA. Vol.79 pp.2554-2558, 1982.
[2] J.J. Hopfield, “ Neurons with graded response have collective computational properties like those of two-state neurons.” Proc. Natl. Acad. Sci. USA. Vol.81 pp.3088-3092, 1984.
[3] J.J. Hopfield and D.W.Tank, “ ‘Neural’ computation of decisions in optimization problems.” Biol. Cybern., Vol.52 pp.141-152, 1985.
[4] J.J. Hopfield and D.W.Tank, “ Computing with neural circuits: a model.” Science, no.233, pp.625-633,1986.
[5] D.W. Tank and J.J. Hopfield, ”Simple neural optimization network: An A/D converter, signal decision circuit, and linear programming circuit.” IEEE Trans, Circuits & Systems, vol.CAS-33, no.5, pp.533-541,1986.
[6] Y. Takefuji and K.C. Lee, “An hysterisis binary neuron: A model suppressing the oscillatory behavior of neural dynamics,” Biol. Cybern., Vol.64 pp.353-356, 1991.
[7] M. Tateishi and S. Tamura, "Comments on `Artificial neural networks for four-coloring map problems and K-colorability problems'," IEEE Trans. Circ. Syst.-I: Fundamental Theory and Applications, vol. 41, no. 3, pp. 248-249, March, 1994.
[8] L. Wang, “Discrete-time convergence theory and updating rules for neural networks with energy functions”, IEEE Trans. Neural Networks, vol.8, no.5, pp. 445-447, 1997.
[9] G. Galán-Marín, J. Muñoz-Pérez, “A new input-output function for binary Hopfield neural networks,” Proceedings of the International Work-Conference on Artificial and Natural Neural Networks: Foundations and Tools for Neural Modeling, pp.311-320, June 02-04, 1999.

[10] S. Bharitkar, J. M. Mendel, "The hysteretic Hopfield neural network," IEEE Trans. Neural Networks, vol.11, no.4, pp. 879-888, 2000.
[11] Y. Takefuji, Neural Network Parallel Computing, Kluwer Academic Publishers, 1992.
[12] Y. Takenaka, N. Funabiki and S. Nishikawa, "Maximum neural network algorithms for N-Queens problem" J. IPSJ, vol.37, no.10, pp.1781-1788, 1996.
[13] Y. Takenaka, N. Funabiki and S. Nishikawa, "A proposal of competition resolution methods on the maximum neuron model through N-Queens problem" J. IPSJ, vol.38, no.11, pp.2142-2148, 1997.
[14] Y. Takenaka, N. Funabiki and T. Higashino, "A proposal of neuron filter: A constraint resolution scheme of neural networks for combinatorial optimization problems," IEICE Trans. Fundamentals, vol.E83-A, no.9, pp.1815-1823, 2000.

Wei Zhang received a B.S. degree from Zhejiang Normal University, Zhejiang, China in 2004. She received an M.S. degree from Toyama University, Toyama, Japan, in 2007. Form 2007 she is working toward the.D.E. degree at Toyama University, Japan. Her main research interests are neural networks and optimization problems.

Zheng Tang received a B.S. degree from Zhejiang University, Zhejiang, China in 1982 and an M.S. degree and a D.E. degree from Tsinghua University, Beijing, China in 1984 and 1988, respectively. From 1988 to 1989, he was an Instructor in the Institute of Microelectronics at Tsinghua University. From 1990 to 1999, he was an Associate Professor in the Department of Electrical and Electronic Engineering, Miyazaki University, Miyazaki, Japan. In 2000, he joined Toyama University, Toyama, Japan, where he is currently a Professor in the Department of Intellectual Information Systems. His current research interests include intellectual information technology, neural networks, and optimizations.