Estimating the weight of inputs in combined multiple truth values to enhance decision making process

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Summary

Every decision is made within its special environment, which is consisting of all available alternatives and values. Having a mechanism that enables us to estimate the weight or the impact of values on the final decision will help us to enhance our decision depending on our need.
Based on truth value and truth-functional structure, this paper introduces new method and algorithm to estimate the weight of values. The method is discussed in detail and the algorithm is tested and verified.

Key words:
Truth value, Truth-functional structure, Functional interrelation, Decision making, Recognizing function

1. Introduction

All people make decisions every day of varying importance, so the idea that decision making can be a rather sophisticated art may at first seem strange. However, studies have shown that most people are much poorer at decision making than they think. An understanding of what decision making involves, together with a few effective techniques, will help us produce better decisions [7].
Decision making can be regarded as an outcome of mental processes, leading to the selection of a course of action among several alternatives [12]. The selection between alternatives can be achieved by using conditional statements with a series of conditions connected by logical operations (AND, OR and NOT). Every condition can be evaluated to true or false. At the end, the conditional-statement evaluated to false or true depending on the logical relations between conditions.
It is clear that there is a logical process on the inputs which leads to the final output, which means that there is a functional interrelation between the inputs and the outputs.
As a result of the functional interrelation, any change in the inputs can lead to some change in the outputs, the amount of changes in the outputs depend on the inputs that have been changed, where inputs varies in their impact on the output [1]. So, it is not necessarily that significant change in one or two inputs (from the whole set of inputs) will lead to significant change in the outputs; because not all inputs have the same weight or impact.

2. Truth values and conditions

Each statement having the property of being either true or false is called proposition [22] or a boolean-valued function, which is defined as:
$f : X \rightarrow B$, where X is an arbitrary set and B is a boolean domain {0 = false, 1 = true } [3].
The truth value of a statement does not depend on when it is true or false, only whether it is true or false right now [14, 15].
If we ask ourselves whether the temperature greater than 35, and look at the thermometer.
The statement "the temperature greater than 35", will still either be true (1), or be false (0).
When conditions need to take more than one thing into account the logical operators are used to combine multiple truth values into one single truth value. By using combinations of AND/OR logical operators, we can express any condition we wish, no matter how many things our condition depends on.
Binary decision diagrams are used to tell the user in terms of diagram for a defined logical function, how to determine the output value of the function by examining the values of the inputs [21].

3. Binary Decision Diagrams

Large Boolean functions can be represented efficiently using BDDs, which were originally invented for hardware verification to efficiently store a large number of states that share many commonalities [18, 14].
A Boolean function can be represented as a rooted, directed, acyclic graph, which consists of decision nodes and two terminal nodes called 0-terminal and 1-terminal.

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Each decision node is labeled by a Boolean variable and has two child nodes called low child and high child. The edge from a node to a low (high) child represents an assignment of the variable to 0 (1). Such a BDD is called 'ordered' if different variables appear in the same order on all paths from the root. A BDD is said to be 'reduced' if the following two rules have been applied to its graph [14]:

- Merge any isomorphic subgraphs.
- Eliminate any node whose two children are isomorphic.

If we have three variables (V_1, V_2, V_3), where $f(x) = V_1$ AND V_3 OR V_2 AND V_3.

V_1	V_2	V_3	$f(x)$
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

Figure 1 Truth table for $f(x) = V_1ANDV_3ORV_2ANDV_3$

The binary decision tree of where $f(x) = V_1$ AND V_3 OR V_2 AND V3, is shown in figure 2.

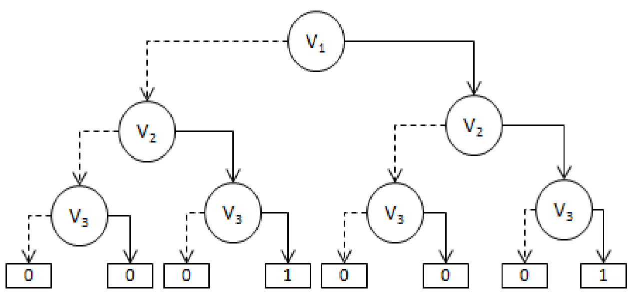

Figure 2 Decision tree for $f(x) = V_1ANDV3ORV_2ANDV3$

Binary decision tree can be transformed into a binary decision diagram by maximally reducing it according to the two reduction rules. The resulting BDD is shown in figure 3, and the reduced binary decision diagram is shown in figure 4.

As binary decision diagrams tells us the outputs through examining the inputs, but it can’t tell us any anything about the weight of the inputs, and which input has a maximum impact on the outputs.

In the following sections I will discuss the concept of variable weight, and introduce new algorithm to find the weight or impact of every variable (input) in both simple or complex combined multiple condition functions.

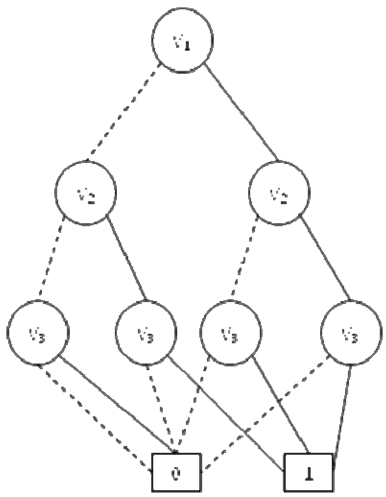

Figure 3 Binary decision diagram for $f(x) = V_1ANDV3ORV_2ANDV3$

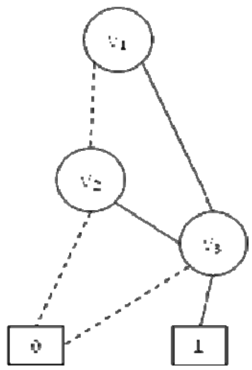

Figure 4 Reduced binary decision diagram for $f(x) = V_1ANDV3ORV_2ANDV3$

4. Variable weight

In combined multiple truth values that lead to one single truth value, every variable has its impact on the final value. This impact varies from value to value. In some cases variable has the same impact, which means that any variable is able to make $f(x)$ true [26, 1], without taking in consideration the value of other variables.

For example the sentence:

$$f(\text{x}) = \text{C}_1 \text{ OR } \text{C}_2 \text{ OR } \text{C}_3 \text{ OR } \text{C}_4 \quad (1)$$

But in other cases we can't know the impact of the variables because of the complex nature of combined multiple conditions, for example

$$f(\text{x}) = \text{w AND z OR x AND z AND x OR y} \quad (2)$$

In such case, if we want to change the value of *f*(x) from true to false or vice versa, we don't know which variable to change to get the desired result. By knowing the impact of every variable we will be able to customize *f*(x), so any change in heavy weight variables can lead to significant change in *f*(x) [8].
The ability of customizing *f*x(x), means the ability to enhance our decision making process; because the decision maker will take in consideration the heavy weight variable when he want to fine-tuning his decision [6, 20].

5. Estimating variable's weight

As mentioned previously, the only possible truth values are TRUE or FALSE, the combination of these values in compound conditional statement give us the truth-functional structure of this statement which can be represented by a truth table in which all possible truth values are displayed [2].
If we take the following statement as an example, to build the truth table, it is no matter how many variables used, but for simplicity we use four variables:

$$f(\text{x}) = \text{V}_1 \text{ AND } \text{V}_2 \text{ OR } \text{V}_3 \text{ OR } \text{V}_4 \quad (3)$$

Table 1 Truth table for formula 3

#	V_1	V_2	V_3	V_4	*f*(x)
1.	0	0	0	0	0
2.	0	0	0	1	0
3.	0	0	1	0	1
4.	0	0	1	1	1
5.	0	1	0	0	0
6.	0	1	0	1	0
7.	0	1	1	0	1
8.	0	1	1	1	1
9.	1	0	0	0	0
10.	1	0	0	1	0
11.	1	0	1	0	1
12.	1	0	1	1	1
13.	1	1	0	0	1
14.	1	1	0	1	1
15.	1	1	1	0	1
16.	1	1	1	1	1

Truth table can help us to identify the functional interrelation between the inputs and the outputs, in order to estimate the weight of every variable [2].
Assume that we have a sequence of combination length *m*. The importance of variables V_1 through V_n , where *n* represents the number of all possible combinations in the truth table, can be calculated by the following formula, and we will call this process **recognizing function** f_{REC}(x):

$$f_{REC}(V_i) = \sum_{j=0}^{n-1} \frac{\text{D}_j}{n} \cdot V_j , \quad (4)$$

Where

$$\text{V}_\text{i} = \text{MAX} \frac{p_{i,j}^m}{\text{D}_j};$$

D_j (j=0,1, ..., n–1) = number of all subsets in set m where V_i accept the value *j*;

$\frac{p_{i,j}^m}{\text{D}_j}$ = number of all subsets where variable V_i accept the value *j*, and f_{REC}(x) accept the value f_m(x), m = (1… n-1);
n = number of all subsets in set m;

Because we use binary representation for V_i, it will help us to simplify the process of finding the weight $f_{\text{REC}}(V_i)$ for every variable V_i.

Using formula 4, we can derive the following:

$$\text{WG}_1(V_i)_0 = \text{MAX}(\sum_0^{\text{n-1}} \text{D}_0^0, \sum_0^{\text{n-1}} \text{D}_0^1) \quad (5)$$

Where

D_0^0 = number of rows where V_i = 0 and $f(x_i)$ = 0;

D_0^1 = number of rows where $V_i = 0$ and $f(x_i) = 1$.

$$WG_2(V_i)_1 = MAX(\sum_0^{n-1} D_1^0, \sum_0^{n-1} D_1^1) \quad (6)$$

Where

D_1^0 = number of rows where $V_i = 1$ and $f(x_i) = 0$;

D_1^1 = number of rows where $V_i = 1$ and $f(x_i) = 1$.

By adding $WG_1(V_i)_0$ and $WG_2(V_i)_1$, and divide them by the number of rows that satisfied the condition of D at the current step we can get the weight of variable V_i, as follow:

$$f_{REC}(V_i) = \frac{WG_1(V_i)_0 + WG_2(V_i)_1}{L} \quad (7)$$

Where

L = number of all rows that satisfied the condition of D at the current step.

The weight or importance for every variable can be calculated sequentially using the pair (V_i, $f(V_i)$), so if we use a set of *m* variables the relation between $f(x)$ and V_i can be represented as (V_1, $f(V_1)$), (V_2, $f(V_2)$), (V_3, $f(V_3)$),…, (V_m, $f(V_m)$). Variables with highest $f_{REC}(V_i)$ considered essential for $f(x)$ because they have the highest importance than others.

As the calculations performed sequentially step by step, in every step we have to eliminate one variable with highest weight or importance, In case of equality of importance the variable is selected arbitrary.

6. Example of usage

For our example we will take formula 3 and table 1 to find the weight or importance for every variable, by applying the contents of table 1 to formula 7.

Step : #1
Path : unknown
L : 16
FIND $WG_1(V_i)_0$, with $V_i = 0$ and $f(x_i) = 0$. Then with $V_i = 0$ and $f(x_i) = 1$
$WG_1(V_1)_0$: 4
$WG_1(V_2)_0$: 4
$WG_1(V_3)_0$: 6
$WG_1(V_4)_0$: 3
$WG_1(V_1)_0$: 4
$WG_1(V_2)_0$: 4
$WG_1(V_3)_0$: 2
$WG_1(V_4)_0$: 4

$MAX(WG_1(V_i)_0)$

V_1	V_2	V_3	V_4
MAX(4,4)	MAX(4,4)	MAX(6,2)	MAX(3,4)
4	4	6	4

FIND $WG_2(V_i)_1$, with $V_i = 1$ and $f(x_i) = 0$. Then with $V_i = 1$ and $f(x_i) = 1$
$WG_2(V_1)_1$: 2
$WG_2(V_2)_1$: 2
$WG_2(V_3)_1$: 0
$WG_2(V_4)_1$: 3
$WG_2(V_1)_1$: 6
$WG_2(V_2)_1$: 6
$WG_2(V_3)_1$: 8
$WG_2(V_4)_1$: 5

$MAX(WG_2(V_i)_1)$

V_1	V_2	V_3	V_4
MAX(2,6)	MAX(2,6)	MAX(0,8)	MAX(3,5)
6	6	8	5

$f_{REC}(V_i) = (WG_1(V_i)_0 + WG_2(V_i)_1)/L$

V_1	V_2	V_3	V_4
(4+6)/L	(4+6)/L	(6+8)/L	(4+5)/L
10/16	10/16	14/16	9/16

MAX(10/16, 10/16, 14/16, 10/16)

	V_1	V_2	V_3	V_4
MAX	(10/16,	10/16,	14/16,	9/16)

Overall: V_1=10, V_2=10, V_3=14, V_4=9,

V_3 has the highest weight and highest overall, eliminate it and use it as recognition variable, other variables transferred to step 2.

Step : #2
Path : V_3
L : 8
FIND $WG_1(V_i)_0$, with $V_i = 0$ and $f(x_i) = 0$. Then with $V_i = 0$ and $f(x_i) = 1$, where V_3=0
FIND $WG_2(V_i)_1$, with $V_i = 1$ and $f(x_i) = 0$. Then with $V_i = 1$ and $f(x_i) = 1$, where V_3=0

Repeating the previous steps we get:

	V_1	V_2	V_4
MAX	(6/8,	6/8,	6/8)

Overall V_1=16, V_2=16, V_4=15,

V_4 has the minimum overall, we transfer it to next step, V_1 and V_2 has the same weight and overall, so, we choose one of them to be deleted (choose V_2).

Step : #3
Path : V_3, V_2
L : 4
FIND $WG_1(V_i)_0$, with V_i = 0 and $f(x_i)$ = 0. Then with V_i = 0 and $f(x_i)$ = 1, where V_3=0 and V_2=0
FIND $WG_2(V_i)_1$, with V_i = 1 and $f(x_i)$ = 0. Then with V_i = 1 and $f(x_i)$ = 1, where V_3=0 and V_2=0
FIND $WG_1(V_i)_0$, with V_i = 0 and $f(x_i)$ = 0. Then with V_i = 0 and $f(x_i)$ = 1, where V_3=0 and V_2=1
FIND $WG_2(V_i)_1$, with V_i = 1 and $f(x_i)$ = 0. Then with V_i = 1 and $f(x_i)$ = 1, where V_3=0 and V_2=1
FIND $WG_1(V_i)_0$, with V_i = 0 and $f(x_i)$ = 0. Then with V_i = 0 and $f(x_i)$ = 1, where V_3=1 and V_2=0
FIND $WG_2(V_i)_1$, with V_i = 1 and $f(x_i)$ = 0. Then with V_i = 1 and $f(x_i)$ = 1, where V_3=1 and V_2=0
FIND $WG_1(V_i)_0$, with V_i = 0 and $f(x_i)$ = 0. Then with V_i = 0 and $f(x_i)$ = 1, where V_3=1 and V_2=1
FIND $WG_2(V_i)_1$, with V_i = 1 and $f(x_i)$ = 0. Then with V_i = 1 and $f(x_i)$ = 1, where V_3=1 and V_2=1

Repeating the previous steps we get:

	V_1	V_4
MAX	(4/4,	3/8)

Overall: V_1=32, V_4=30,

V_1 has the highest weight and highest overall, eliminate it and use it as recognition variable, other variables transferred to step 4.

Step : #4
Path : V_3, V_2, V_1
L : 2

No more variables to test. So, the order of the elements by their weight is:

$V_3 \rightarrow V_2 \rightarrow V_1 \rightarrow V_4$

To check our result we can do that by inverting the value of V_i where $f(x_i)$=1, that is, the value will be changed to 0 if 1 and to 1 if 0.

$$\begin{cases} V_i = 0 \text{ if } V_i \text{ in the current row} = 1 \text{ and } f(x) = 1 \\ V_i = 1 \text{ if } V_i \text{ in the current row} = 0 \text{ and } f(x) = 1 \end{cases}$$

If the value of $f(x)$ is changed in the corresponding row as a result of the change of V_i, it means that V_i has some impact on $f(x)$. By calculating the impact average for every variable Vi,

$$\text{impact average} = \frac{\text{sum of changes in every column}}{\text{number of correspending rows}} \qquad (8)$$

The result must match our previous result.

depending on table 1, such case where $f(x)$=1 occurs in rows 3,4, 7, 8, 11, 12, 13, 14, 15 and 16, theses lines represent 10 of 16 rows. Table 2 through table 5 show the impact of every variable V_i on $f(x)$.

Table 6 shows the summary result and the amount of change in $f(x)$ for every variable, where $^{\sim}V_i$ represents modified V_i.

Table 2 shows the effect of V_1 on $f(x)$

#	V_1	$^{\sim}V_1$	$f(x)$	$f(^{\sim}V_1)$
3	0	1	1	1
4	0	1	1	1
7	0	1	1	1
8	0	1	1	1
11	1	0	1	1
12	1	0	1	1
13	1	0	1	**0**
14	1	0	1	**0**
15	1	0	1	1
16	1	0	1	1

Table 3 shows the effect of V_2 on $f(x)$

#	V_2	$^{\sim}V_2$	$f(x)$	$f(^{\sim}V_2)$
3	0	1	1	1
4	0	1	1	1
7	0	1	1	1
8	0	1	1	1
11	1	0	1	1
12	1	0	1	1
13	1	0	1	**0**
14	1	0	1	**0**
15	1	0	1	1
16	1	0	1	1

Table 4 shows the effect of V_3 on $f(x)$

#	V_3	$^{\sim}V_3$	$f(x)$	$f(^{\sim}V_3)$
3	0	1	1	**0**
4	0	1	1	**0**
7	0	1	1	**0**
8	0	1	1	**0**
11	1	0	1	**0**
12	1	0	1	**0**
13	1	0	1	1
14	1	0	1	1
15	1	0	1	1
16	1	0	1	1

Table 5 shows the effect of V_4 on $f(x)$

#	V_4	$^{\sim}V_4$	$f(x)$	$f(^{\sim}V_4)$
3	0	1	1	1
4	0	1	1	1
7	0	1	1	1
8	0	1	1	1
11	1	0	1	1
12	1	0	1	1
13	1	0	1	1
14	1	0	1	1
15	1	0	1	1
16	1	0	1	1

Table 6 Summary of f(x) after modification

#	$f(V_i)$	$f(\neg V_1)$	$f(\neg V_2)$	$f(\neg V_3)$	$f(\neg V_4)$
1	0	0	0	0	0
2	0	0	0	0	0
3	1	1	1	**0**	1
4	1	1	1	**0**	1
5	0	0	0	0	0
6	0	0	0	0	0
7	1	1	1	**0**	1
8	1	1	1	**0**	1
9	0	0	0	0	0
10	0	0	0	0	0
11	1	1	1	**0**	1
12	1	1	1	**0**	1
13	1	**0**	**0**	1	1
14	1	**0**	**0**	1	1
15	1	1	1	1	1
16	1	1	1	1	1
Changes		2	2	6	0
Impact average		≈%20	≈%20	≈%60	≈%0

It is clear that test result matches our previous result, where we got that:

- V_3 has the highest impact on f(x)
- V_1 and V_2 have medium impact on f(x) (*and have the same weight*)
- V_4 has the minimum impact on f(x)

This is confirming the effect of the proposed method to estimate the weight of variable in a compound conditional statement.

7. Conclusions

Knowing the importance of variables that affect the final decision is one of the key factors for decision making process. Therefore, the development of such algorithm that can help us measuring the weight for every variable will lead us to optimal situation, where we can make the right decision.

The discussed method and algorithm in this paper introduce a good solution for decision enhancement process, by providing us in advance with prior information about the impact of every variable depending on their weight.

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