EEG Source Localization Using the Inverse Problem Methods

Rafik KHEMAKHEM[†], Wassim ZOUCH[†], Ahmed BEN HAMIDA[†], Abdelmalik TALEB-AHMED[††], and Imed FEKI[†††]

[†]Unité de Recherche en Traitement de l'Information et en Electronique Médicale ENIS, Sfax, Tunisie, C.P. 3038
[††]Laboratoire LAMIH UMR CNRS UVHC 8530, Université de Valenciennes, France, C.P. 59300
[†††]Service de Neurologie Clinique, Centre Hospitalier Régional Universitaire de Sfax, Tunisie, C.P. 3029

Summary
An Electroencephalography (EEG) inverse solution technique can be seen as a way to add spatial information to extra-cranial measurements. In other words, it is a mathematical/physical way to expand the dimensionality of scalp measurements so as to embed intra-cranial spatial information. This paper presents the new sLORETA-FOCUSS approach estimating the current density distribution in the brain. A comparative study of the sLORETA, FOCUSS, sLORETA-FOCUSS and its recursive version is also performed using the ROC curve analysis. The results demonstrate that the recursive sLORETA-FOCUSS method gives good solutions in terms of localization error, simulation time, and reconstruction precision in 3D.

Key words:
EEG, Inverse Problem, Localization, sLORETA-FOCUSS.

1. Introduction

EEG is a recording of electrical potentials at a set of electrodes placed on the scalp. Though it is a measurement at the surface of the head, EEG can still be used in an attempt to infer the location of the neural electrical sources within the brain, such as epileptic spikes or somatosensory evoked potentials [1, 2]. This localization problem is commonly referred to as the inverse problem [3-5] of electroencephalography. The inverse problem is ill-posed as there are an infinite number of source configurations that can produce the same and exact potential at the surface of the head.

The resolution methods can be classified into two main categories: dipolar model which assumes that the electric sources are equivalent to one or few dipoles, and distributed source model which consider that the dipoles are distributed regularly in cerebral volume according to a 3D grid. The positions of the dipoles are then fixed and their amplitudes should be estimated.

In order to solve the inverse problem, it is necessary to perform the forward problem [6] to construct the lead field matrix that would be inversed to compute the potential over the scalp surface from simulated dipoles whose position and strength are known.

Several methods of the resolution of the inverse problem, such as the LOw-Resolution Electromagnetic TomogrAphy algorithm LORETA [7, 8], the standard form of LORETA method: sLORETA [9], the focal underdetermined system solver FOCUSS [10-12] can be found. The latter repeats the procedure of the WMN [13-15] method, recursively adjusting the weighting matrix until most elements of the solution become nearly zero. However, the final solution depends, to some degree, on the assumed initial current distribution. Moreover, we present our method named sLORETA-FOCUSS witch represent a combination method between the sLORETA and FOCUSS [16] can be found. Finally, an amelioration of the sLORETA-FOCUSS method is given by the recursive sLORETA-FOCUSS that gives best results in terms of sources reconstruction and computing time.

This paper is organized as follows. Section 2 presents the mathematical steps, the advantages and the disadvantages of each inverse problem method. Results and discussion are presented in section 3. Section 4 provides a comparative study.

2. Technical details

Consider the problem:

$$\min_x X^t X \text{, under constraint: } Y = AX \text{.} \qquad (1)$$

Where X is an M-vector, Y is an N-vector, M>N, and $Rank(A) = H \leq N$

Matrix A has singular value decomposition (SVD):

$$A = L\Lambda R^t \text{, with } \Lambda \text{ is a diagonal } H*H \text{ matrix} \qquad (2)$$

Then:

Manuscript received April 5, 2009
Manuscript revised April 20, 2009

$$A^t = R\Lambda L^t . \quad (3)$$

And:

$$A^+ = A^t(AA^t)^+ = R\Lambda L^t(L\Lambda^{-2}L^t) = R\Lambda^{-1}L^t . \quad (4)$$

The solution is:

$$\hat{X} = A^+Y = R\Lambda L^t(L\Lambda^{-2}L^t)Y . \quad (5)$$

Note that "+" denotes the Moore-Penrose pseudo-inverse, and "t" indicates the matrix transpose.
We consider here that:

- V(N, 1): equivalent in equation (5) to Y : is a vector containing scalp electric potentials measured at N cephalic electrodes:

$$V = K * J . \quad (6)$$

- J(3M, 1): equivalent in equation (5) to X : is a vector containing current densities at M sources in the brain:

$$J = V * K^{-1} . \quad (7)$$

- K(N, 3M): equivalent in equation (5) to A : is a Lead Field matrix, containing the relation between the position of the point where the measure of the potential is taken in scalp surface, and the position of the sources in the brain:

$$K = \frac{1}{4\pi\sigma}.\frac{(r_s - r_v)}{|r_s - r_v|^3} . \quad (8)$$

Where r_s and r_v represent the coordinates of the measurement points in the scalp surface and the coordinates of the source points within the brain volume respectively; σ is the electrical conductivity.

The final expressions of the estimated current density J with the minimum norm "MN" [17, 18], the weighted minimum norm "WMN" and the "LORETA" methods are:

- For the MN algorithm, using equations (4) and (5), we obtain:

$$J_{MN} = K^t(KK^t)^+V . \quad (9)$$

- For the WMN algorithm, we obtain:

$$J_{WMN} = W^{-2}K^t(KW^{-2}K^t)^+V . \quad (10)$$

Where W is a diagonal 3Mx3M matrix.

- For the LORETA algorithm, we obtain:

$$J_{LORETA} = (C)^{-1}K^t[K(C)^{-1}K^t]^+V . \quad (11)$$

Where C is the weighting matrix: $C = W^tB^tBW$.
In the following equations:

- the EEG measurements, the current density distribution, and the lead field matrix are termed V, J and K respectively
- W is a weighting diagonal matrix, used in order to affect the same weight to the deep and the superficial sources:

$$w_i = \left(\frac{1}{Ne}\right).\sqrt{\sum_{j=1}^{Ne} K_{ij}^2} . \quad (12)$$

- B represents the discrete operator Laplacian:

$$B = \frac{6}{d^2}(A - I_{3M}) . \quad (13)$$

With:

$$A = A_0 \otimes I_3$$

$$A_0 = \frac{1}{2}(I_M + [diag(A_1 1_M)]^{-1})A_1$$

$$[A_1]_{\alpha\beta} = \begin{Bmatrix} (1/6), if \|v_\alpha - v_\beta\| = d \\ 0, otherwise \end{Bmatrix},$$

$$\forall \alpha, \beta = 1...M$$

In the next stage, we present the mathematical details of the sLORETA, FOCUSS, sLORETA-FOCUSS and the recursive sLORETA-FOCUSS algorithms.

2.1 sLORETA Approach

The sLORETA is a new tomographic method for electric neuronal activity which employs the current density estimated by the minimum norm solution. The minimum norm inverse solution is inconvenient for its incapability of correct localization of deep point sources. This problem is solved by the standardization of the minimum norm inverse solution, basing localization inference on these standardized estimates.

$$J_{MN} = K^t (KK^t)^+ .V \,. \quad (14)$$

The standardization of the estimate *J* requires an estimate of its variance. In this view, the actual source variance:

$$S_J = I, I \in IR^{(3M)\times(3M)} \,. \quad (15)$$

The electric potential variance is due to noisy measurements:

$$S_V^{noise} = \alpha H \,. \quad (16)$$

$\alpha \geq 0$ is the regularization parameter.

$H \in IR^{N_e \times N_e}$ is the average reference operator.

We obtain the electric potential variance that depends on the linear relation V=KJ and the measurement noise:

$$S_V = KS_J K^t + S_V^{noise} = KK^t + \alpha H \,. \quad (17)$$

The variance of the estimated current density is:

$$S_{\hat{J}} = TS_V T^t = T(KK^t + \alpha H)T^t \,. \quad (18)$$

Where $T = K^t [KK^t + \alpha H]^+$

Finally:

$$J_{sLORETA} = \hat{S}_j * J_{MN} \,. \quad (19)$$

The problem of the sLORETA is that it provides good results only in the absence of noise.

2.2 FOCUSS Approach

FOCUSS is a recursive approach which has two integral parts: a low-resolution initial estimate of the real signal and the iteration process that refines the initial estimate to the final localized energy solution. The iterations are based on adjusting the weighting matrix C:

$$C = (W^{-1})^t W^{-1} \,. \quad (20)$$

Where W is a diagonal 3Mx3M matrix which is recursively refined.

By including (20) in (10), we obtain the following equation:

$$\hat{J}_{FOCUSS} = WW^t K^t (KWW^t K^t)^+ V \,. \quad (21)$$

For each iteration in FOCUSS, the matrix W is updated based on the current density estimate of the previous iteration. The recursive version of (21) can be written as:

$$\hat{J}_{i_{FOCUSS}} = W_i W_i^t K^t (KW_i W_i^t K^t)^+ V \,. \quad (22)$$

In each step, W_i is updated as follows:

$$W_i = PW_{i-1}[diag(\hat{J}_{i-1_{FOCUSS}}(1),...,\hat{J}_{i-1_{FOCUSS}}(3M))]. \quad (23)$$

Where $\hat{J}_{i-1_{FOCUSS}}(n)$ represents the n^th^ elements of vector $\hat{J}$ at the (i-1)^th^ iteration, and P is a diagonal matrix for deep source compensation, where:

$$P = diag[1/\|K_1\|, 1/\|K_2\|, ..., 1/\|K_{3M}\|] \,. \quad (24)$$

The FOCUSS algorithm will converge to a localized solution with zero on most elements. The problem is that the simulation time is very high.

2.3 sLORETA-FOCUSS Approach

FOCUSS is a recursive method using the solution of the previous iteration to reconstruct the weighting matrix. Then, it requires a good initialization to produce the best reconstruction of the cerebral activity. The sLORETA method gives images of standardized current density with zero localization error. For these reasons, we have proposed to combine sLORETA and FOCUSS to develop the sLORETA-FOCUSS method.

sLORETA-FOCUSS is a solution to improve the 3D reconstruction of the neuronal activity in the brain. Since the sLORETA algorithm produces a smooth result of source distribution, the iterative FOCUSS algorithm enhances the strength of some significant dipoles in the solution, and decreases the strength of other dipoles. Therefore, the sLORETA-FOCUSS method suppresses the dipoles whose current densities are close to zero, and authorize only the dipoles of high density.

sLORETA-FOCUSS can be developed according to the following steps. Firstly, we calculated the current density using sLORETA; $\hat{J}_{sLORETA}$. Secondly, we constructed the weighting matrix W using the current density obtained by the sLORETA method, and therefore the initial value of W is given by $W_0 = diag(\hat{J}_{sLORETA}(i))$. Thirdly, we calculated the iterative form of the weighting matrix W with reference to the previously mentioned equation (23). After that, we calculated the current density distribution of the sLORETA-FOCUSS method using the recursive expression of the FOCUSS method in (22). Finally, we repeated the last two steps until the convergence.

Although sLORETA-FOCUSS gives good results to reconstruct the cerebral activity in 3D, it contains some problems; for example the computation time is still high. Hence, the algorithm must repeat the calculation with big matrices, whereas the majority of the elements in these matrices are equal to zero. A second problem is that no criteria are used to limit the error between two iterations.

2.4 Recursive sLORETA-FOCUSS Approach

In order to address these problems given by the sLORETA-FOCUSS algorithm, we developed a new algorithm named recursive sLORETA-FOCUSS to ameliorate the reconstruction of the cerebral activity in 3D. The recursive sLORETA-FOCUSS is based on the recursively FOCUSS algorithm. It tends to eliminate the non-active zones and choose the zones of high activity in order to obtain a gain in terms of simulation times and localization precision.

The algorithm can be summarized as follows. We started by estimating the current density using sLORETA method. Afterwards, we reconstructed the weighting matrix W using equation (23) and we computed the current density using equation (22). Then, we reduced the space of the active zone in the brain by choosing nodes having high current density. Subsequently, we repeated the previous steps while eliminating the non-active zones of the cerebral volume and we redefined the new space of work in the cerebral volume, i.e., by choosing the active zones in the brain. Eventually, we repeated the last three steps until convergence.

The recursive sLORETA-FOCUSS has offered good results of localization and amelioration of simulation time, but we have always found the problem of false dipoles in the case of the increasing number of active sources.

3. Results and discussion

To carry out the reconstruction of the active sources in the brain in 3D, we used measures consisting of 2319 nodes in the brain and 127 electrodes on the scalp surface. Data are obtained from EEG recordings provided by the Neurophysiology Clinical Service, CHRU Lille, FRANCE. The first step is to compute the forward problem to obtain the gain matrix K using the Brainstorm Matlab Toolbox [19]. The gain matrix describes the physical relations between electrodes distributed on the scalp surface, with sources distributed on cerebral volume. The gain matrix depends on the electrode and source position, and the electrical conductivity.

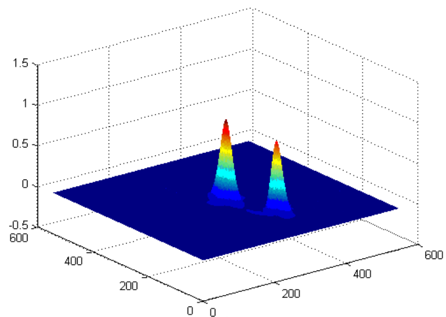
In this section, we present two cases: the first one is an example for reconstruction of two simulated sources, and the second is an example of the reconstruction of three simulated sources.

3.1 Reconstruction of the amplitude of two simulated dipoles in 3D

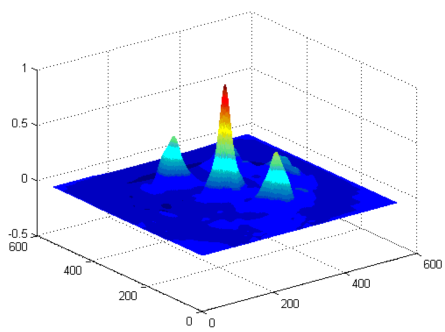
Four different inverse methods; sLORETA, FOCUSS, sLORETA-FOCUSS and the Recursive sLORETA-FOCUSS method, are evaluated.

In figure1, we simulated the presence of two currents dipoles in the brain and we tried to find it by the different methods in order to provide some comprehensive information about the performance of these algorithms. Two simulated dipoles are presented in figure1.a. Figures1.b, 1.c, 1.d and 1.e present the reconstruction of these dipoles using the sLORETA, FOCUSS, sLORETA-FOCUSS and the recursive sLORETA-FOCUSS methods respectively.

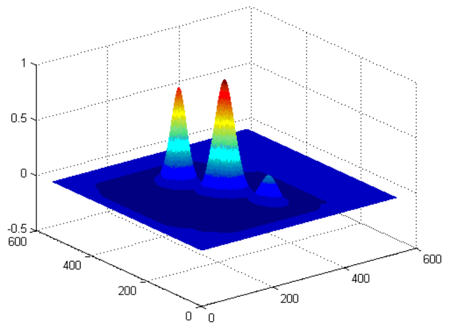
According to figure1.b, the sLORETA method gives smooth results, so it is not able to offer good reconstruction of the simulated dipoles. However, in figure1.c, FOCUSS presents a sparse result, but unable to reconstruct the good position of the simulated dipole. sLORETA-FOCUSS in figure1.d is a combination between the two previous methods in order to profit their advantages.



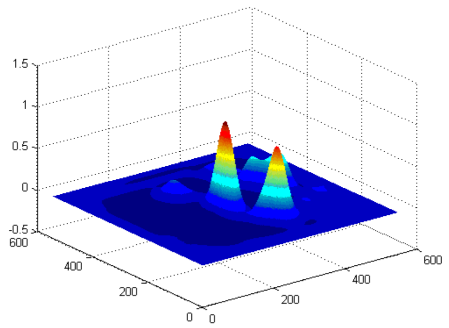
(a)



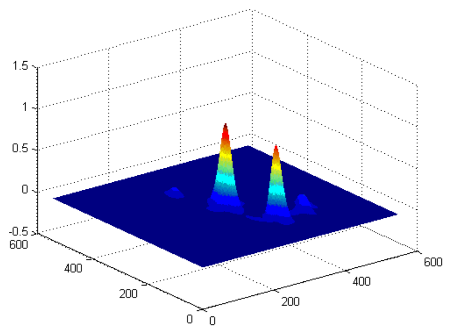
(b)



(c)



(d)



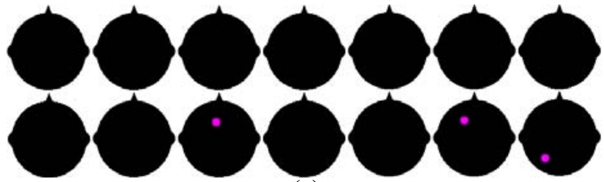
(e)

Fig.1. Comparison of the amplitude of two active dipoles in the brain. (a) Test dipoles; (b) Solution of the sLORETA method; (c) Solution of the FOCUSS method; (d) Solution of the sLORETA-FOCUSS method; (e) Solution of the Recursive sLORETA-FOCUSS method.

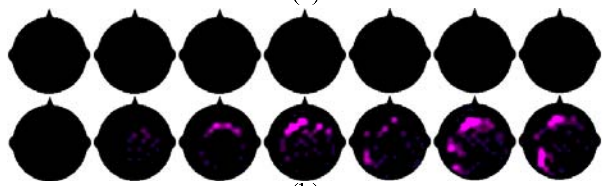
In the case of dipole configuration, it gives an exact convergence to the real dipole with zero localization error, but the problem is that some small spurious sources are generated on the space solution. In figure1.e, the recursive sLORETA-FOCUSS method gives exactly the same result as the simulated dipole in term of amplitude of the simulated dipoles. So, the presented results demonstrate that in 3D, the sLORETA-FOCUSS method and its recursive version have good localization properties.

3.2 Reconstruction of the position of three simulated dipoles in 3D

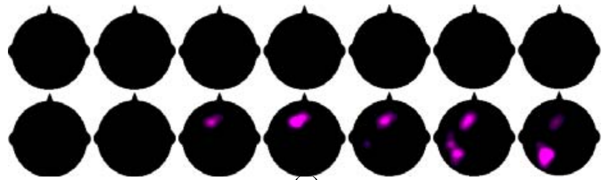
We can see the reconstruction results on 14 sagittal slices of the head. To elaborate this study, we have simulated three sources in the brain volume, and we have tried to locate these sources by each method. Figure2 shows results of source reconstructions in the brain.



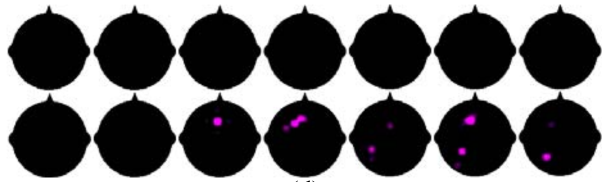
(a)



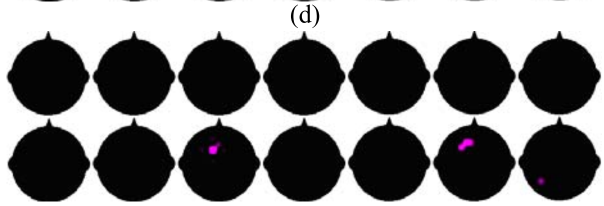
(b)



(c)



(d)



(e)

Fig.2. Comparison of the position of three active sources in the brain: (a) Simulated sources, (b) Source reconstruction with sLORETA, (c) Source reconstruction with FOCUSS, (d) Source reconstruction with sLORETA-FOCUSS, (e) Source reconstruction with recursive sLORETA-FOCUSS

The simulated dipoles are presented in Figure2.a. Figures2.b, 2.c, 2.d and 2.e represent the reconstruction of these dipoles using the sLORETA, FOCUSS, sLORETA-FOCUSS and the recursive sLORETA-FOCUSS methods respectively. According to figure2.b, we notice that the sLORETA method provides a smooth and diffused reconstruction of the original dipoles. However, FOCUSS in figure2.c provides a sparse solution, which does not necessarily reflect the good solution. In figure2.d, we see an improvement of the source reconstruction compared to the FOCUSS but it is not the best reconstruction. According to figure2.e, we observe that the recursive sLORETA-FOCUSS method is able to give the best reconstruction in term of position of the simulated dipoles. In fact, it converges towards the real source without traces of any false ones.

4. Evaluation using ROC curve and computing times metrics

To validate the results of these algorithms, we used the Receiver Operating Characteristic (ROC) [20] graphs. So, we classified our results in four categories (TP: True Positive, TN: True Negative, FP: False Positive and FN: False Negative). This approach is used to obtain two parameters for statistical analysis: sensibility and specificity.

Sensibility is the ability to reconstruct the active dipole on the right position among the false position, as shown in following:

$$sensibility = \frac{TP}{TP + FN}. \quad (25)$$

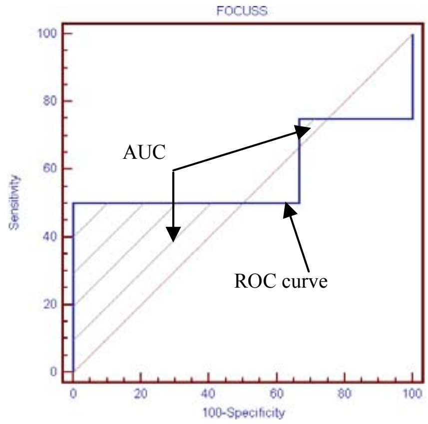
Specificity is the ability of non-reconstructing the right dipole, or the ability of reconstructing a false dipole, given in the next equation:

$$specificity = \frac{TN}{TN + FP}. \quad (26)$$

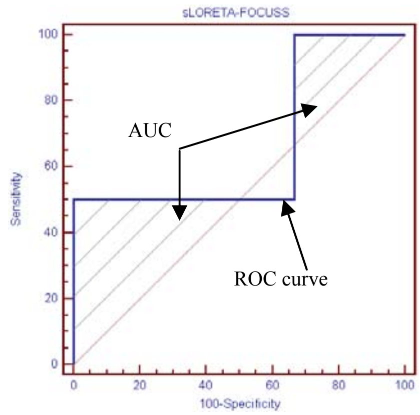
The performance of each algorithm to reconstruct the good position of active dipoles can be presented by the ROC curve. ROC curve is used to compare the performance of two or more methods in the diagnosis of cerebral activity. It is the relation between the sensibility and specificity value. The obtained results are plotted on a Cartesian graph, on which the true positive rate (sensitivity) is assigned to the abscissa, and the false positive rate (100-specificity) is assigned to the ordinate (Figure3).

In order to perform ROC curve analysis, we used the measurements of current density in the brain for each algorithm. Figure3 (a), 3(b), and 3(c) show ROC curves for FOCUSS, sLORETA-FOCUSS and the recursive sLORETA-FOCUSS algorithms.

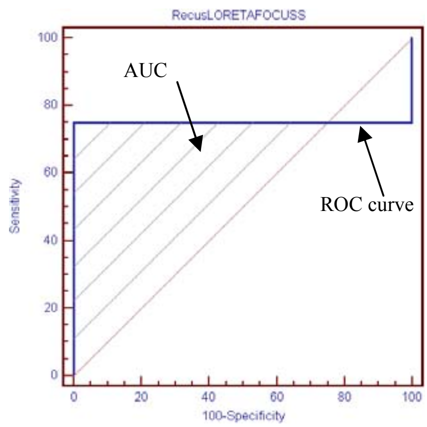
Figure3 presents an improved performance for the recursive sLORETA-FOCUSS in comparison with the other methods.



(a)



(b)



(c)

Fig.3. ROC curve: (a) FOCUSS, (b) sLORETA-FOCUSS, (c) Recursive sLORETA-FOCUSS

We can compare the efficiency of these algorithms by studying the area under the ROC curve "AUC" [21, 22]. The AUC is the area between the diagonal and the ROC curve. It gives an improvement of the advantages and disadvantages of each algorithm to reconstruct the best position of the tested sources. The AUC has an important statistical property, which is equivalent to the probability to give the good reconstruction of the source distribution in the brain.

Figure3 shows that the recursive sLORETA-FOCUSS gives the big area under the ROC curve. We can demonstrate that the recursive sLORETA-FOCUSS presents the best result of the 3D source reconstruction in the brain.

In table 1, we present the comparison in terms of the simulation time of each method. We notice that the recursive sLORETA-FOCUSS method is faster than the other methods. We obtain a gain in terms of simulation time about of 54,6 %.

Table 1: Computing Times

Method	FOCUSS	sLORETA-FOCUSS	Recursive sLORETA-FOCUSS
Computing Times	501,0625 sec	485,82 sec	265,42 sec

This paper has presented an evaluation of the 3D neuronal activity reconstruction algorithms. It proved that the good result to reconstruct the best position of the simulated dipole is given by the recursive sLORETA-FOCUSS method. We have validated our work using the ROC curve in which we have used the area under the ROC curve to measure the efficiency of each algorithm. Furthermore, we have studied the difference between each algorithm in terms of simulation time, the recursive sLORETA-FOCUSS method presents an amelioration of the simulation time of about 54,6 % compared with the sLORETA-FOCUSS. In conclusion, the recursive sLORETA-FOCUSS is an excellent method for study of the neuronal activity in the brain.

Acknowledgment

We would like to thank the Service de Neurophysiologie Clinique de Lille (France) directed by Pr. Philippe DERAMBURE and the Service de Neurologie Clinique de Sfax (Tunisia) directed by Pr. Chokri MHIRI for their support. We also extend our thanks to Mme Leila MAHFOUDHI for proofreading this manuscript.

References

[1] P. Nunez et al., “A theoretical and experimental study of high resolution EEG based on surface laplacians and cortical imaging,” Electroenceph. Clin. Neurophysiol., vol. 90, no. 1, pp. 40–57, 1994.

[2] B. He, Y. Wang, and D. Wu, “Estimating cortical potentials from scalp EEG’s in a realistically shaped inhomogeneous head model by means of the boundary element method,” IEEE Trans. Biomed. Eng., vol. 46, pp. 1264–1268, Oct. 1999.

[3] Albert Tarantola, “Inverse Problem Theory and Methods for Model Parameter Estimation”, Institut de Physique du Globe de Paris, Université de Paris 6, Paris, France, 2005

[4] J. Sarvas, “Basic mathematical and electromagnetic concepts of the biomagnetic inverse problem,” Phys. Med. Biol., vol. 32, pp. 11–22, 1987.

[5] R. D. Pascual-Marqui, “Source localization: Continuing discussion of the inverse problem,” ISBET Newslett., vol. 6, pp. 16–28, 1995. Reply to comments by Hämäläinen, Ilmoniemi and Nunez.

[6] D. Rubio, M.I. Troparevsky, “The EEG forward problem: theoretical and numerical aspects”, Latin American Applied Research, pp 87-92, 2006”

[7] R. D. Pascual-Marqui, C. M. Michel, and D. Lehmann, “Low resolution electromagnetic tomography: A new method for localizing electrical activity in the brain,” Int. J. Psychophysiol., vol. 18, pp. 49–65, 1994.

[8] R. D. Pascual-Marqui, “Source localization: Continuing discussion of the inverse problem,” ISBET Newslett., vol. 6, pp. 16–28, 1995. Reply to comments by Hämäläinen, Ilmoniemi and Nunez.

[9] R.D. Pascual-Marqui. Standardized low resolution brain electromagnetic tomography (sLORETA): technical details. Methods & Findings in Experimental & Clinical Pharmacology 2002, 24D:5-12.

[10] I. F. Gorodnitsky, J. S. George, and B. D. Rao, “Neuromagnetic source imaging with FOCUSS: A recursive weighted minimum norm algorithm,” Electroenceph. Clin. Neurophysiol., vol. 95, pp. 231–251, 1995.

[11] I. F. Gorodnitsky and B. D. Rao, “Sparse signal reconstruction from limited data using FOCUSS: A re-weighted minimum norm algorithm,” IEEE Trans. Signal Processing, vol. 45, pp. 600–616, Mar. 1997

[12] I.F. Gorodnitsky, B.D. Rao, and J. George, “Source localization in magnetoencephalography using an iterative weighted minimum norm algorithm,” in Proc. Asilomar Conf Signal System Computation, Pacific Groo ve, CA, 1992, pp. 167–171.

[13] M. S. Hämäläinen, “Magnetoencephalography Theory, instrumentation and applications to noninvasive studies of the working human brain,” Rev. Mod. Phys., vol. 65, no. 2, pp. 413–497, 1993.

[14] J. Sarvas, “Basic mathematical and electromagnetic concepts of the biomagnetic inverse problem,” Phys. Med. Biol., vol. 32, pp. 11–22, 1987.

[15] J. Z.Wang, S. J.Williamson, and L. Kaufman, “Magnetic source images determined by a lead-field analysis: The unique minimum-norm leastsquares estimation,” IEEE Trans. Biomed. Eng., vol. 39, pp. 665–675, July 1992.

[16] R. KHEMAKHEM, A. BEN HAMIDA, A. TALEB-AHMED, P. DERAMBURE, “New hybrid method for the 3D reconstruction of neuronal activity in the brain”, 15th International Conference on Systems, Signals and Image Processing June 25-28, 2008, Bratislava Slovakia.

[17] J. Z.Wang, S. J.Williamson, and L. Kaufman, “Magnetic source images determined by a lead-field analysis: The unique minimum-norm leastsquares estimation,” IEEE Trans. Biomed. Eng., vol. 39, pp. 665–675, July 1992.

[18]M. S. Hämäläinen and R. J. Ilmoniemi, “Interpreting Measured Magnetic Fields of the Brain: Estimates of Current Distributions,” Helsinki Univ. Technol., Helsinki, Finland, Tech. Rep. TKK-F-A559, 1984.

[19] Open source Matlab toolbox « brainstorm » http://neuroimage.usc.edu/brainstorm (Baillet and al., 2004).

[20] Tom Fawcett, “ROC Graphs: Notes and Practical Considerations for Researchers“,HP Laboratories, MS 1143, 1501 Page Mill Road, Palo Alto, CA 94304, March 16, 2004

[21] Hanley, J. A. and B. J. McNeil: 1982, `The Meaning and Use of the Area under a Receiver Operating Characteristic (ROC) Curve'. Radiology 143, 29-36.

[22] Bradley, A. P.: 1997, `The use of the area under the ROC curve in the evaluation of machine learning algorithms'. Pattern Recognition 30(7), 1145-1159.