Wavelets-based Multiresolution Surface as Framework for Editing the Global and Local Shapes

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Summary
In computer aided design (CAD), the availability of many types of surface editing which ease creating the geometric models is important. This paper describes a wavelets-based mulresolution representation for endpoint-interpolating cubic B-spline surface and algorithms to support the surface editing at different scales, allowing structural features (global shape) deformations as well as detail features (local shape) creation. The experiment results show that the multiresolution representation can be used as a single, unified framework for developing an interactive surface editor.

Keywords:
Wavelets, multiresolution surface, structural features, detail features.

1. Introduction

In current CAD systems there are many modeling methods, such as non-uniform rational B-splines (NURBS), constructive solid geometry (CSG), boundary representation (B-rep), and recently free-form deformation (FFD) [1,2], partial differential equation (PDE) [3], energy functional optimization [4]. However, all those modeling methods use a single three-dimensional vector space as the framework, so that it limits the modeling to simple objects.

The manipulating of the mesh of complex geometry objects which might be constructed from scratch (ab initio design) or be scanned-in either by hand or with automatic digitizing methods, can be difficult to be carried out in a single vector space, especially when they are to be edited or animated. Aside from considerations of economy, the choice of mesh representation is also guided by the need for multiresolution editing semantics. The representation of the mesh should provide control at a large scale, so that one can change the mesh in a global or in a local shape.

Wavelet is a mathematical tool that can be used to define multi-scale space and has found a wide variety of applications in recent years, including signal analysis [5], image processing [6], and numerical analysis [7]. Wavelet-based multiresolution analysis is based on decomposing a vector space into a set of nested vector spaces with different scales, and then analyzing the properties of functions in the time and frequency domains in those different scale spaces. Finkelstein [8] in 1994 first introduced wavelet-based multiresolution curve in the field of curve modeling to facilitate a variety of multiresolution editing operations. Wavelet-based multiresolution analysis brought a novel conception to the area of curve/surface modeling. With multiresolution editing, the curve can be smoothed and the overall form of the curve can be changed while preserving its details (sweep editing). The curve can be edited at any continuous level of detail (fractional editing). Additionally, the curve's character can be changed without affecting its overall shape.

Since the real objects modeling needs three dimensional representation, the goal of this research is to describe wavelet-based multiresolution representation for surface and investigate the capability of the multiresolution representation as framework that support editing of structural and detail features of surface. We propose the multiresolution representation for surface based on the extension of multiresolution curve proposed by Finkelstein [8]. The surface representation we used is cubic B-spline surface which interpolate endpoints and have uniform knots distance except knots at surface endpoints which have multiplicity 4. Furthermore, we refer this surface representation as *EI* B-spline surface. This surface is defined using patch scheme which is computed from tensor product of parametric basis function. The basis functions is polynomial functions with degree 3, continuity C2 at joint (meeting point between two series of surface patches) and have uniform knots distance.

The rest of this paper is organized as follows. The theoretical foundation of tensor product of endpoint-interpolating cubic b-spline surface is presented in Section 2. Our wavelet-based multiresolution representation of b-spline surface which is the extension of the work of Finkelstein[8] is then discussed in Section 3. Some

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experiments carried out in Matlab, in order to investigate capability of the framework of multiresolution surface, are presented in Section 4. Finally, conclusion and future research directions are suggested.

2. Tensor Product Cubic B-spline Surface

Two popular schemes of surface representations are subdivision scheme and patch scheme. In subdivision scheme, a surface is defined using mesh polyhedral. In patch scheme, a surface is defined using tensor product of parametric basis functions. A tensor product cubic B-spline surface is defined using the following formula:

$$Q_{ij}(u,v) = \sum_i \sum_j V_{ij} B_{i,4}(u) B_{j,4}(v) \qquad (1)$$

Vij is control points in three dimensional spaces. *Bij* is B-spline basis function. Fig. 1(a) shows the example of a mesh of control points for B-spline surface with 5 x 3 patches. Fig. 1(b) shows the corresponding surface computed using formula 1.

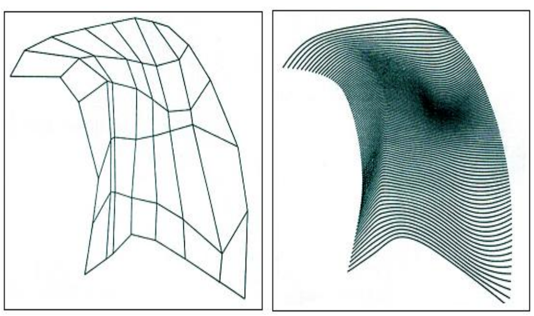


Fig. 1(a) The example of a mesh of control points of B-spline surface. (b) The corresponding B-spline surface.

Since discussion about the theory of B-spline surface is more easily carried out in the curve setting, the following two sub-sections would discuss the uniform cubic B-spline curve and the endpoint-interpolating cubic B-spline curve. All definition in the curve setting is also used in the surface setting.

2.1. Uniform Cubic B-spline Curve

Uniform cubic B-spline curve is a curve composed by cubic B-spline basis function with uniform knots distance. Cubic B-spline basis function is a parametric polynomial function with degree 3, piecewise, continuity *C2* at joint. Continuity *C2* means that the basis function has continuity at position, first derivative, and second derivative.

Fig. 2(a) shows the example of curve composed using uniform cubic B-spline basis function. One segment of uniform cubic B-spline curve is defined using 4 control points. The curve approximates the 4 control points which have index from 0 to 3. Since each curve segment is defined using 4 basis functions, the curve requires 3 more basis functions and 3 more control points than the number of curve segments. Each basis function has non zero value at 4 parametric intervals. The most left basis function adds 3 extra intervals in the left of curve, and the most right basis function adds 3 extra intervals in the right of curve. There are *m*+1 control points, *m*+1 basis functions, *m*-2 curve segments in the range of *m*-1 knots, and *m*-1+3+3=*m*+5 knots in total. Curve is drawn with $\bar{u}$ parameter running from $\bar{u}_3$ to $\bar{u}_{m+1}$.

The curve example shown in fig. 2(a) has the last index of control point *m*=9. The curve consists of 7 segments. Its definition requires 10 basis functions and 10 control points at $\bar{u}_3 \le \bar{u} < \bar{u}_{10}$ with parameter space $\bar{u}_0 \le \bar{u} < \bar{u}_{13}$. The uniform cubic B-spline basis function used to compose the curve in fig. 2(a) is shown in fig. 2(b).

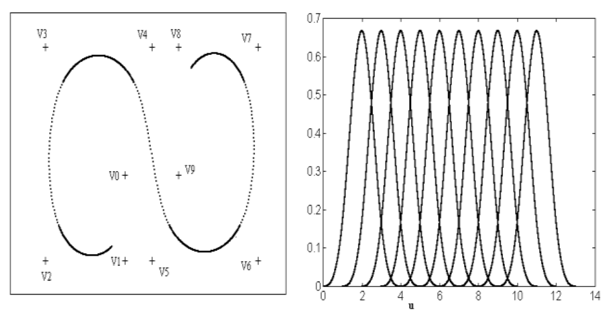


Fig. 2(a) The example of uniform B-spline curve. (b) The basis functions of uniform cubic B-spline.

In order to draw a curve, some control points *Vi* have to be defined first, and then, the control points are used to draw curve Q using the following formula:

$$\begin{aligned} Q_i(u) &= V_{i+r} b_r(u) \qquad (2) \\ &= V_{i-3} b_{-3}(u) + V_{i-2} b_{-2}(u) + V_{i-1} b_{-1}(u) + V_{i-0} b_{-0}(u) \end{aligned}$$

The basis functions $b_r(u)$ for cubic B-spline curve are shown in fig. 3 and have the following formula:

$$\begin{aligned} b_{i,-0}(\bar{u}) &= \frac{1}{6} u^3 \\ b_{i,-1}(\bar{u}) &= \frac{1}{6}(1 + 3u + 3u^2 - 3u^3) \\ b_{i,-2}(\bar{u}) &= \frac{1}{6}(4 - 6u^2 + 3u^3) \\ b_{i,-3}(\bar{u}) &= \frac{1}{6}(1 - 3u + 3u^2 - u^3) \end{aligned} \qquad (3)$$

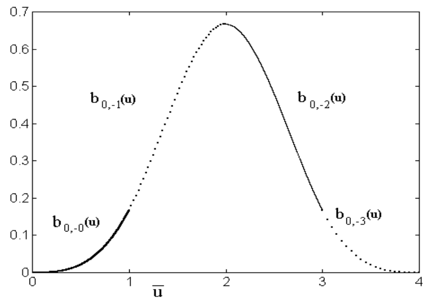


Fig. 3. Uniform cubic B-spline basis function $B_{0,4}(u)$

2.2. Endpoint-interpolating Cubic B-spline Curve

Basically, endpoint-interpolating cubic b-spline curve (*EI* B-spline curve) is defined using same basis functions as discussed in sub-section 2.1. The difference is if all knots in basis functions of uniform cubic B-spline curve has multiplicity 1, but all knots in basis function of *EI* B-spline curve has multiplicity 1 except 2 knots at endpoints which have multiplicity 4. Multiplicity 4 means that the knots have 4 other knots with same value.

The most striking difference between *EI* B-spline curve and uniform cubic B-spline curve can be seen at endpoints. In uniform cubic B-spline curve, all curve segments approximate all control points. While in *EI* B-spline curve, all curve segments approximate all control points except the first and the last control points which are interpolated. This condition eases controlling the endpoints of curve.

In uniform cubic B-spline curve, all curve segments is compiled using the same basis functions (which is shifted along knots). In *EI* B-spline curve, the basis functions compiling segments of curve have different forms, especially at two at the first segments and two at the last segments. The example of four segments of *EI* B-spline curve and the basis functions are shown in fig. 4(a) and (b), respectively.

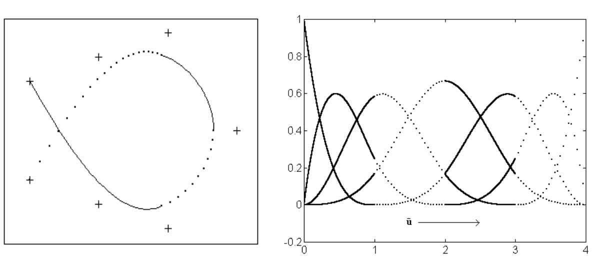


Fig. 4(a) The example of 4 segments of *EI* B-spline curve. (b) The basis functions of *EI* cubic B-spline.

3. Wavelets-based Multiresolution Representation

We divide the discussion in this section into two sub-sections. In sub-section 3.1, we discuss the theory of wavelets and multiresolution analysis, and how it can be applied to represent endpoint-interpolating B-spline curve, following the work by Finkelstein[8]. In sub-section 3.2, we extend the definition of wavelets-based multiresolution curve into surface, which is furthermore used as the framework to develop algorithms for editing the structural and detail features of surface.

3.1. Multiresolution Curve

Let S defines space of uniform cubic B-spline with knots $\bar{u}_i = i, i \in Z$, and distance of knots equal to 1. *Z* is positive integer. Multiresolution representation composes spaces S^j of uniform cubic B-spline with knots distance $2^{-j}, j \in Z$. Index *j* in S^j expresses the resolution level. Curve at resolution *j* is formed using (2^j+3)x1 control points. Therefore, a curve at the coarsest resolution (resolution 0) is formed using (2^0+3)x1 control points, consists of one segment and has knots distance equal to 1. Curve at resolution 1 is formed using (2^1+3)x1 control points, consist of 4 segments, and has knots distance equal to ½. Curve at resolution 2 is formed using (2^2+3)x1 control points, consist of 16 segments, and has knots distance equal to ¼, etc.

Since B-spline function at resolution *j1* with knots distance equal to 2^{-j1} is also B-spline function at resolution *j2* with knots distance equal to 2^{-j2} , and *j1<j2*, then there will be nested spaces as below :

$$S^0 \subset S^1 \subset S^2 \subset ...$$

If V^j express closed space $L^2(R)$ of $S^j \cap L^2(R)$, then there are closed B-spline subspaces dan nested of $L^2(R)$ as below:

$$V^0 \subset V^1 \subset V^2 \subset ....$$

In different resolutions, the same curve is expressed using different control points. For example, curve at resolution 2 which has 4 segments and 7x1 control points can be expressed using 2 segments and 5x1 control points at resolution 1.

Let C^n be a set of control points at resolution n. The number of control points in C^n is (2^n+3)x1. In order to arrange C^{n-1} (the approximation of C^n in one lower resolution) which has the number of control points $(2^{n-1}+3)$x1, it is required to do filtering to the control points of

C^n. This process is expressed using the following matrix equation:

$$C^{n-1} = A^n C^n \qquad (4)$$

A^n is a matrix with size $(2^{n-1}+3)$x$(^{2n}+3)$.

Since the number of control points C^{n-1} is smaller than the number of control points in C^n, there are some details missing during the filtering process. These missing details are captured in D^{n-1} which can be computed using the following matrix equation:

$$D^{n-1} = B^n C^n \qquad (5)$$

B^n is a matrix with size $[(2^n+3)-(2^{n-1}+3)]$x(2^n+3).

The matrix pair A^n and B^n are referred as analysis filters. The process to separate C^n into lower resolution C^{n-1} and detail D^{n-1} is referred as decomposition. The decomposition process can be carried out recursively to C^{n-1}, so that the control points C^n can be expressed as hierarchy of control points with lower resolution C^0, C^1, ..., C^{n-1} and details D^0, D^1, ..., D^{n-1}. This recursive decomposition process is known as filter bank and is illustrated in Fig. 5.

$$\begin{tikzcd}
C^n \arrow[r, "A^n"] \arrow[dr, "B^n"'] & C^{n-1} \arrow[r, "A^{n-1}"] \arrow[dr, "B^{n-1}"'] & C^{n-2} & \cdots \arrow[r, "A^1"] \arrow[dr, "B^1"'] & C^0 \\
& D^{n-1} & D^{n-2} & & D^0
\end{tikzcd}$$

Fig. 5. Filter bank

If the matrix pair A^n and B^n are selected carefully, the original control points C^n can be reconstructed from C^{n-1} and D^{n-1} using matrix pair P^n and Q^n with the following calculation:

$$C^n = P^n C^{n-1} + Q^n D^{n-1} \qquad (6)$$

Matrix P^n and Q^n are called synthesize filters. The reconstruction process is illustrated in Fig. 6.

$$\begin{tikzcd}
C^0 \arrow[r, "P^1"] & C^1 \cdots \arrow[r, "P^{n-1}"] & C^{n-1} \arrow[r, "P^n"] & C^n \\
D^0 \arrow[ur, "Q^1"] & D^1 \cdots \arrow[ur, "Q^{n-1}"] & D^{n-1} \arrow[ur, "Q^n"] &
\end{tikzcd}$$

Fig. 6. The reconstruction process

Since the original control points C^n can be reconstructed from C^0, D^0, D^1, ..., D^{n-1}, then the values of C^0, D^0, D^1, ..., D^{n-1}, are considered as the wavelets transformation of control points C^n. The wavelets transformation is computed by using pairs of the analysis and synthesize filter. Those filters are determined based on several choices below:

1. Selection of scaling functions $\Phi^j(u)$ which span V^j for all j in $[0,n]$. Scaling functions $\Phi^j(u)$ determine synthesize filter P^j.
2. Selection of inner product of two function f and g in V^j, to determine the norm function and the orthogonal complement space W^j of V^j in V^{j+1}.
3. Selection of wavelets $\Psi^j(u)$ which span W^j. Wavelets $\Psi^j(u)$ determine synthesize filter Q^j. Analysis filters A^j and B^j are computed from synthesize filters P^j and Q^j.

Fig. 7(b) shows plots of wavelets functions defined using basis functions of *EI* cubic B-spline curves as the scaling functions (fig. 7(a) and (c)); the standard form of inner product for any two functions f and g in $V^j, \langle f, g \rangle = \int f(u)g(u)du$, which determines the orthogonal complement spaces W^j; and the wavelets $\Psi^j(u)$ with minimally-supported functions that span W^j.

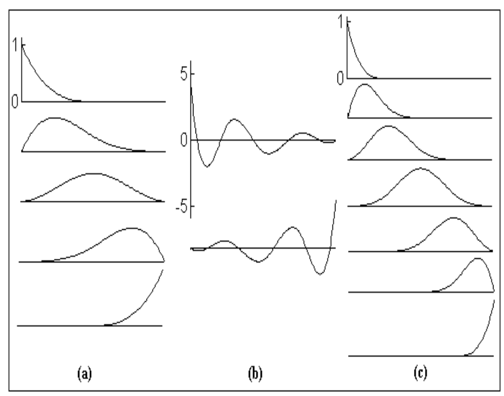


Fig. 7(a) The scaling functions at resolution 1. (b) The wavelets functions at resolution 1. (c) The scaling functions at resolution 2.

3.2. Multiresolution Surface

Multiresolution surface is extension of all definitions including scaling functions and filters that are used for multiresolution curve. Let multiresolution of tensor product b-spline surface *c(u,v)* be defined as an element of a certain functional space V^n, computed by:

$$c(u,v) = \sum_{i=1}^{2n+3} \sum_{j=1}^{2n+3} (x^n) \phi_i^n(u) \phi_j^n(v) = (x^n)(\Phi(u)^n)(\Phi(v)^n)$$

where x^n is a control points mesh with size mxm in R^2, $m = 2^n + 3$. In multiresolution surface, the extension of decomposition process specified by equation (4) and (5) would split the mesh of control points x^n into lower resolution mesh x^{n-1}, detail in row dx^{n-1} and detail in column dy^{n-1}. The extension of decomposition process is computed by using analysis filters A^n and B^n as follow:

$$\begin{aligned} xt^{n}_{(mxm')} &= \left(A^{n}_{(m'xm)} x^{n}_{(mxm)}\right)^{T} \\ dx^{n-1}_{(m''xm)} &= B^{n}_{(m''xm)} x^{n}_{(mxm)} \\ dy^{n-1}_{(m''xm')} &= B^{n}_{(m''xm)} xt^{n}_{(mxm')} \\ x^{n-1}_{(m'xm')} &= \left(A^{n}_{(m'xm)} xt^{n}_{(mxm')}\right)^{T} \end{aligned} \tag{7}$$

As the inverse of the decomposition process, the extension of reconstruction process specified by equation (6) which allows recovering the original mesh of control points x^n from x^{n-1}, dx^{n-1} and dy^{n-1}, is computed by using synthesis filters P^n and Q^n as follow:

$$\begin{aligned} xt^{n-1}_{(m'xm)} &= \left(P^{n}_{(mxm')} x^{n-1}_{(m'xm')} + Q^{n}_{(mxm'')} dy^{n-1}_{(m''xm')}\right)^{T} \\ x^{n}_{(mxm)} &= P^{n}_{(mxm')} xt^{n-1}_{(m'xm)} + Q^{n}_{(mxm'')} dx^{n-1}_{(m''xm)} \end{aligned} \tag{8}$$

4. Experiment

The objective of experiment is to investigate the capability of wavelets-based multiresolution surface to support surface editing which allows easy creation/modification of the global shape of surface as well as its local shape. The two scenarios used in the experiment are creating "head of cat" from scratch and modifying "a car". Both scenarios are run in the framework of wavelets-based multiresolution surface. If the first scenario is to examine the capability of wavelets-based multiresolution surface in creating the global and local shape, then the second scenario is to examine the capability in modifying the global shape while preserving its local shape. The experiments are carried out in Matlab 2008.

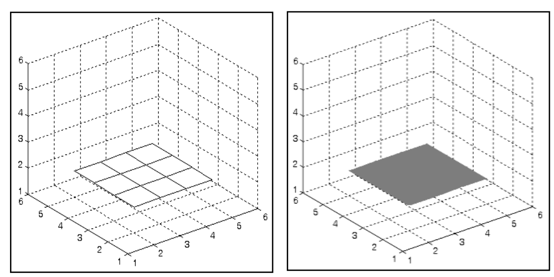
4.1. Scenario 1: Creating "head of cat"

The steps of creation "head of cat" are shown from fig. 8 to fig. 12. Creating/modifying the global shape is basically carried out by moving some control points of the mesh at the current resolution or at the higher resolution. Adding the local shape is usually carried out by first, increasing the resolution of the mesh and then, moving some control points of the mesh at the higher resolution.

In the framework of multiresolution surface, the reconstruction process which is computed by a set of equation (8) with x^{n-1} equal to the mesh of control point at current resolution, detail in row dx^{n-1} equal to 0, and detail in column dy^{n-1} equal to 0, is used to increase the resolution of surface. The more number of control points in the higher resolution allows modifying the global shape of surface and/or adding its local shape.

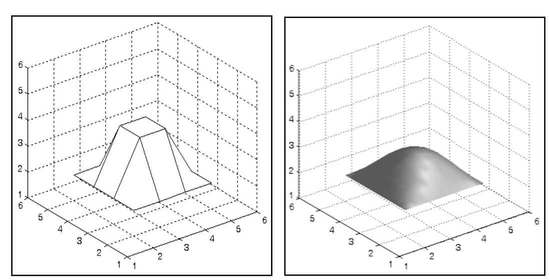
The creation of "head of cat" is started with one flat patch surface at resolution 0 as shown in figure 8. Subsequently, the resolution of mesh of control points is increased as shown in fig. 9, 10, 11, and 12. In each resolution, there is editing process to move some control points into new positions.

As shown in this experiment, it is easy to create various forms of surface in the framework of multiresolution surface which can increase/decrease the surface mesh into different resolution. Representation of a surface in a certain resolution determines the number of patches which are involved in the editing process. Wavelets multiresolution representation give a sufficient fast respond in increasing/decreasing the mesh resolution, which is important in developing interactive surface editor.



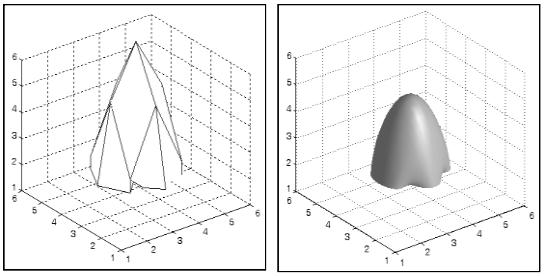
(a) (b)

Fig. 8(a). The mesh of control points at resolution 0. (b). The corresponding surface.



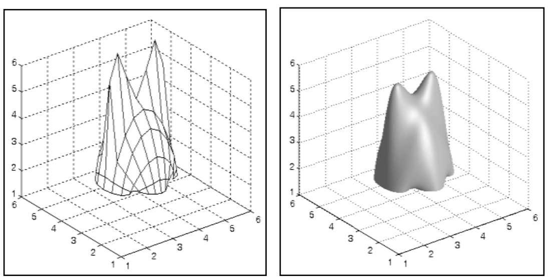
(a) (b)

Fig. 9(a). The mesh of control points after moving some of control points to a mesh shown in 8(a). (b). The corresponding surface after modifying its global shape.



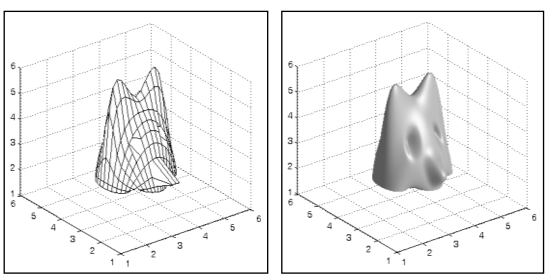
(a) (b)

Fig. 10(a). The mesh of control points after increasing the resolution level to 2 and moving some of control points. (b). The corresponding surface.



(a) (b)

Fig. 11(a). The mesh of control points after increasing the resolution level to 3 and moving some of control points. (b). The corresponding surface.



(a) (b)

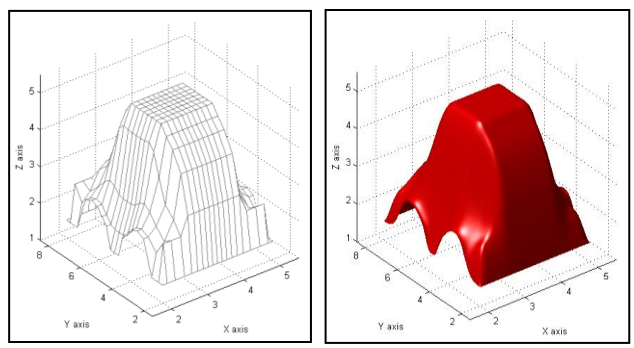
Fig. 12(a). The mesh of control points after increasing the resolution level to 4 and moving some of control points. (b). The corresponding surface after adding the local shape.

4.2. Scenario 2: Modifying "a car"

In this scenario, we start with the global shape of a car, as shown in fig. 13. The addition of the local shape on the car model is carried out by first, increase the resolution of mesh and then, move some control points at its higher resolution, as shown in fig. 14. The modification of the global shape of car model is then carried out by first, decrease the resolution of mesh, and move some control points at its lower resolution, as shown in fig. 15. Finally, the modification of the global shape of car model which should preserve its local shape is shown in fig. 16, when we move back to the high resolution.

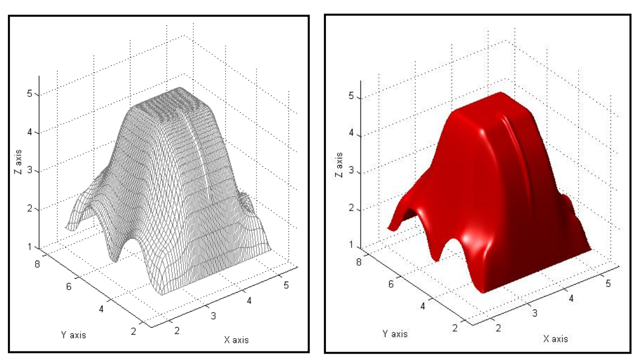
The capability to support the editing of global shape while preserve its local shape is important for the surface representation, since this kind of editing mostly used in the process of product designing. In wavelets-based multiresolution surface, this capability is supported by allowing the user to work at different resolution and by capturing the detail information in high resolution while the user works at the low resolution. The algorithm for editing the global shape of surface is listed below:

1. Move to low resolution by computing set of equation (7). The difference information between mesh at high resolution and mesh at low resolution is captured by a sequence of detail in row dx^{n-1} and detail in column dy^{n-1}.
2. Dispose some of control points at the low resolution to new positions in order to modify the global shape of the surface.
3. Move back to high resolution by computing set of equation (8) using new position of control points and the difference information captured in a sequence of dx^{n-1} and dy^{n-1} from the step 1.



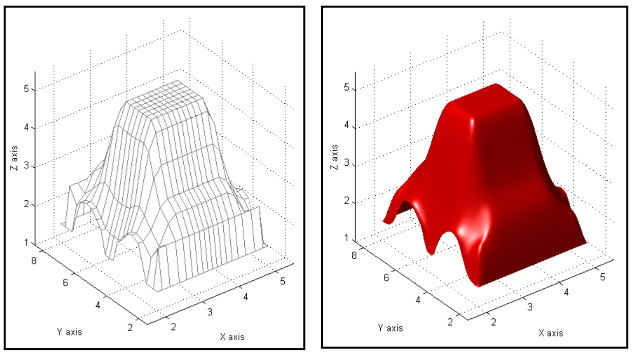
(a) (b)

Fig. 13(a). The mesh of control points at resolution 4. (b). The corresponding surface.



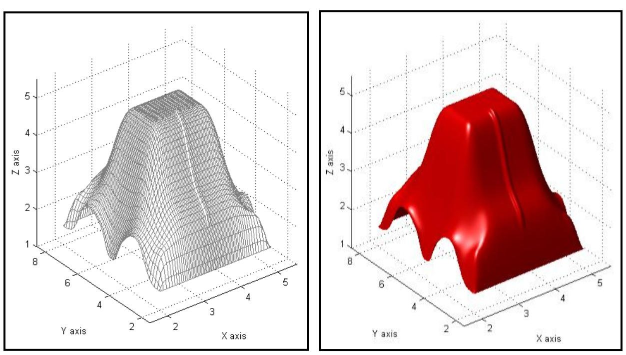
(a) (b)

Fig. 14(a). The mesh of control points after increasing the resolution level to 6 and moving some of control points. (b). The corresponding surface after adding the detail shape.



(a) (b)

Fig. 15(a). The mesh of control points after decreasing the resolution level to 4 and moving some of control points. (b). The corresponding surface after modifying the global shape.



(a) (b)

Fig. 16(a). The mesh of control points after going back to the resolution 6. (b). The corresponding surface which preserve the detail shape.

5. Conclusion

The wavelets-based multiresolution representation of *EI* B-spline surface gives a very good support toward the creation of global and local shape of surface and modification of global shape while preserving local shape. The wavelets-based multiresolution representation does not need extra memory except for saving the mesh of control points and it is also give sufficient fast respond in increasing/decreasing the resolution of mesh. Thus, it can be concluded that the wavelets-based multiresolution representation can be used as framework to build an interactive surface editor. The constraint of this representation is all surface has to be expressed using mesh of control points with size $(2^j+3)x(2^j+3)$, with j is the resolution level. In the next study, we plan to implement much more editing types on our multiresolution surface, such as features pasting [9], adding the local shape through drawing a curve directly on surface [10][11], etc.

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