

# Generalized Predictive Controller Design for Ship Track Keeping

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## Summary

A Generalized Predictive Controller (GPC) for ship track keeping is introduced in this paper. A predictive model for desired heading angles is deduced, the controller is designed based not only on the prediction of the outputs (heading angles), but also on the prediction of the desired heading angles. A recursive algorithm for matrix inversion in the controller is also presented. Applied to cargo ship's track keeping, simulation results show efficiency of this new control scheme. The computational time required by the recursive algorithm for matrix inversion is only 1/3 as general matrix inversion algorithms.

## Key words:

*Generalized Predictive Control (GPC), ship control, recursive algorithms, matrix inversion.*

## 1. Introduction

Generalized Predictive Control (GPC) (Clarke, *et al.*, 1987) has been proved to be efficient in various discrete-time systems as well as in ship control (Hu, *et al.*, 2007). The control strategy is based on the model of the process, and by using past inputs and outputs to predict future outputs over a finite output horizon; it chooses future control increments according to some optimal criteria.

Ship steering is a complex nonlinear control process with several hydrodynamic parameters that vary in wide ranges such as ship's load condition, wind, wave, current, and so on. In ship course keeping the autopilot is desired to maintain a fixed heading while minimizing the rudder activity, hence it can shorten the length of the journey, and reduce the fuel consumption with respect to steering. Track keeping is different from course keeping in that the autopilot automatically determines the heading that the ship should follow in order to stay on course. The autopilot takes the ship's position information and uses it to calculate heading corrections so that the ship follows a predetermined course made up of waypoints (Fossen, 1994; Healey and Lienard, 1993). For this purpose the controller should offset low frequency disturbances such as wind, loading, sea wave and current. To achieve these objects, various adaptive autopilots were developed such as Genetic Algorithm (McGookin, *et al.*, 1998), fuzzy control (Yang and Ren, 2003), internal model control

(IMC) (Velasco, *et al.*, 2002), predictive control (Velasco, *et al.*, 2000) and so on.

However, in the track keeping controller design, the desired heading angle is decided by the current position of the ship and the waypoint. With the position of the ship changes, the desired heading is time-varying. In this paper, a model which can "predict" the future desired heading is deduced. By predicting the future outputs of system (heading angles) and desired outputs (desired heading angles), a new Generalized Predictive Control scheme is presented, it can adjust the desired heading angles in advance, thus lead to better performance. Furthermore, to reduce the computational time of the controller, a recursive algorithm is introduced which can save 2/3 computational time.

The paper is organized as follows. A nonlinear model which accurately represents the characteristics of the ship is described in Section 2; The simplified model for controller design is introduced and the basic GPC controller for track keeping is given in Section 3; In Section 4, the model for ship's desired heading angles is deduced, and a new GPC controller is designed based on this model; In Section 5, noticed the special array in the matrix needed to be inverted, a recursive algorithm is presented which needs only 1/3 computational time as general algorithms need; Simulation experiments are given in Section 6.

## 2. Ship Model

The ship is normally regarded as a rigid body with six degrees of freedom. Here a mixed nonlinear model (Yang, 1996) is used to simulate the dynamics of ship, which accurately represents all the relevant characteristics of the ship, including not only hydrodynamic force, rudder force, and propeller thrust acting on the ship hull, but also the effects of normal and low speed, small and large drift angle, shaft torque, shallow water, wind, current and wave. This model is described as follows:

$$\begin{aligned} (m+m_x)\dot{u}-(m+m_y)vr &= X_H+X_P+X_R+X_A+X_W+X_C \\ (m+m_y)\dot{v}+(m+m_x)ur &= Y_H+Y_P+Y_R+Y_A+Y_W+Y_C \\ (I_z+J_{zz})\dot{r} &= N_H+N_P+N_R+N_A+N_W+N_C \end{aligned} \quad (1)$$

where  $u$ ,  $v$  are surging velocity and swaying velocity of ship,  $r$  is the angular rate of yaw angle with respect to time;  $m$ ,  $m_x$ ,  $m_y$ ,  $I_z$  and  $J_{ZZ}$  are mass, added mass, inertia moment and added inertia moment of ship respectively.  $X$ ,  $Y$  are hydrodynamic forces acting on ship in the body-fixed axis system;  $N$  is a moment by the aforementioned forces. In Eqn. (1), the subscript H denotes the bare hull, P denotes screw, R denotes rudder, A denotes wind, W denotes wave and C denotes current respectively. The formulas of calculating the above forces and moments have been proposed by Yang (1996).

The above model is used to simulate the real performances of the ship in Section 6, but it is too complicated and is not suitable for autopilot design. In GPC controller design the ship is always simplified as a discrete CARIMA model, as shown in section 3.

### 3. Basic Generalized Predictive Controller Design For Track Keeping

#### 3.1 Predictor of Heading Angle

Heading angles and rudder angles of surface ship can be related by the following discrete CARIMA model:

$$\psi(t+1) = \sum_{i=1}^n a_{1,i} \psi(t+1-i) + \sum_{i=0}^m b_{1,i} \Delta \delta(t-d-i) + \sum_{i=0}^{r_1} c_{1,i} \xi_1(t+1-i) \quad (2)$$

where  $\psi(t)$  is the output variable(heading angle),  $\delta(t)$  is the input variable(commanded rudder angle) of the system,  $\xi_1(t)$  is a disturbance term which is assumed to be a white Gaussian noise with zero-mean,  $(d+1)$  is the time delay of the system,  $\Delta=1-q^{-1}$  is the differencing operator,  $a_{1,i}$ ,  $b_{1,i}$  and  $c_{1,i}$  are coefficients of polynomials with degrees  $n$ ,  $m$  and  $r_1$ .

The  $k$ -step ahead minimum variance predictor of heading angle,  $\psi(t+k|t)$ , can be derived recursively from Eqn. (2) as follows, where the future noises are omitted:

$$\psi(t+k|t) = \sum_{i=1}^n a_{k,i} \psi(t+1-i) + \sum_{i=1}^m b_{k,i} \Delta \delta(t-d-i) + \sum_{i=1}^{r_1} c_{k,i} \xi_1(t+1-i) + \sum_{i=0}^m b_{k-i,0} \Delta \delta(t-d+i) \quad (3)$$

The coefficients  $a_{k,i}$ ,  $b_{k,i}$  and  $c_{k,i}$  in Eqn. (3) can be derived from  $a_{1,i}$ ,  $b_{1,i}$  and  $c_{1,i}$  (Jin and Gu, 1990).

Eqn. (3) can be rewritten as:

$$\psi(t+k|t) = \begin{cases} \psi_m(t+k) & k < d+1 \\ \psi_m(t+k) + \sum_{i=d}^{k-1} b_{k-i,0} \Delta \delta(t-d+i) & k \geq d+1 \end{cases} \quad (4)$$

where  $\psi_m(t+k)$  is defined as the future output at time  $(t+k)$  in case the future control increments are zero, it depends entirely on the past known commanded rudder angles, measured heading angles and noise signals:

$$\psi_m(t+k) = \sum_{i=1}^n a_{k,i} \psi(t+1-i) + \sum_{i=1}^m b_{k,i} \Delta \delta(t-d-i) + \sum_{i=1}^{r_1} c_{k,i} \xi_1(t+1-i) + \sum_{i=0}^m b_{k-i,0} \Delta \delta(t-d+i) \quad (5)$$

$\psi_m(t+k)$  can also be computed recursively as follows:

$$\begin{aligned} \psi_m(t+k) &= \sum_{i=1}^n a_{1,i} \psi_m(t+k-i) + \sum_{i=0}^{r_1} c_{1,i} \xi_1(t+k-i|t) \\ &+ \sum_{i=0}^m b_{1,i} \Delta \delta(t+k-d-i-1|t) \\ \xi_1(t+i|t) &= \begin{cases} 0 & i > 0 \\ \xi_1(t+i) & i \leq 0 \end{cases} \end{aligned} \quad (6)$$

Equations (4) and (5) can be rewritten as the following vector form:

$$\begin{aligned} \Psi &= \Psi_m + G \Delta \delta \quad (7) \\ \Psi &= [\psi(t+d+1|t), \psi(t+d+2|t), \dots, \psi(t+d+p|t)]^T \\ \Psi_m &= [\psi_m(t+d+1), \psi_m(t+d+2), \dots, \psi_m(t+d+p)]^T \\ \Delta \delta &= [\Delta \delta(t), \Delta \delta(t+1), \dots, \Delta \delta(t+p-1)]^T \\ G &= \begin{bmatrix} b_{1,0} & \dots & 0 \\ \dots & \ddots & \\ b_{p,0} & \dots & b_{1,0} \end{bmatrix} \end{aligned}$$

#### 3.2 Desired Heading

In track keeping, the reference heading angle that ship should follow in order to stay on course is determined automatically. The system takes the ship's position information and uses it to calculate the heading corrections so that the ship follows a predetermined course set out prior to autopilot activation. See Fig.1, A and B are waypoints (from A to B), C is the current position of the ship, then the reference heading can be obtained as follows:

$$\psi_{ref} = \tan^{-1} \left( \frac{y_{wp} - y_p}{x_{wp} - x_p} \right) \quad (8)$$

where  $(x_p, y_p)$  are the coordinates of C obtained from a Global Positioning System (GPS),  $(x_{wp}, y_{wp})$  are the coordinates of next waypoint position B (Fossen, 1994; Healey and Lienard, 1993), and we define the positive angles ( $0^\circ < \psi_{ref} \leq 180^\circ$ ) are to starboard and negative angles ( $-180^\circ < \psi_{ref} < 0^\circ$ ) are to port.

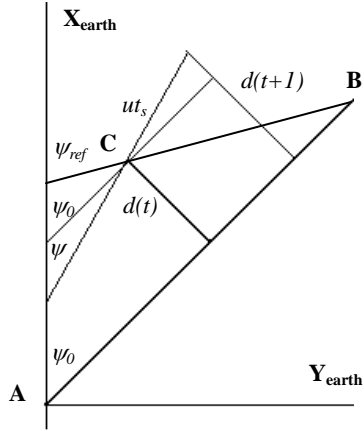


Fig.1. Reference heading angle and distance

### 3.3 Controller Design

In ship manoeuvring it is expected a smooth approach from the current heading angle to the desired heading angle  $\psi_{ref}$ , which is called reference trajectory, can be obtained from the first-order model with time delay:

$$\begin{aligned} \Psi_r &= [\psi_r(t+d+1), \psi_r(t+d+2), \dots, \psi_r(t+d+p)]^T \\ \psi_r(t+d) &= \psi_m(t+d) \\ \psi_r(t+d+j) &= \alpha\psi_r(t+d+j-1) + (1-\alpha)\psi_{ref}(t+d+j) \end{aligned} \quad (9)$$

where  $0 < \alpha < 1$  is a filter coefficient and the reference heading is a constant, i.e.  $\psi_{ref}(t+d+j) = \psi_{ref}$ .

The cost function is selected as follows which is desired to get minimum heading error:

$$J = \min \{ (\Psi_r - \Psi)^T (\Psi_r - \Psi) + \lambda \Delta \delta^T \Delta \delta \} \quad (10)$$

$\lambda$  is a positive weighting factor on control increment input. The optimal control input vector  $\Delta \delta$  can be got by minimizing  $J$ , and its first element is  $\Delta \delta(t)$ , so the increment of the control rudder at time  $t$  is:

$$\Delta \delta(t) = (1, 0, \dots, 0) [G^T G + \lambda I]^{-1} G^T (\Psi_r - \Psi_m) \quad (11)$$

## 4. Generalized Predictive Controller With Desired Heading Prediction

Noticed from Eqn. (8) that the desired heading is a function of the position of ship, with the position of ship changes, the desired heading is time-varying. If the position of ship can be predicted, then the desired headings can be adjusted in advance, which will lead to better performance.

### 4.1 Predict Model for Desired Heading

In Fig.1, waypoints A and B defined the predetermined course AB,  $\psi_0$  is the angle between North and AB. Define  $d(t)$  as the shortest distance from ship's current position C to the course AB, and  $l(t)$  as the distance from C to B, then we can get the following relations at time  $t$  and  $(t+1)$ :

$$\sin(\psi_{ref}(t) - \psi_0) = d(t) / l(t) \quad (12)$$

$$\sin(\psi_{ref}(t+1) - \psi_0) = d(t+1) / l(t+1) \quad (13)$$

Noticed that after each control period, the distance between C and B varies very little, which means  $l(t+1) - l(t) \ll l(t)$ . (Note: when  $l(t)$  is less than a specified distance which is always named as acceptance radii, the ship will head to the next waypoint).

Using Eqn. (13) subtract Eqn. (12), we can get:

$$\begin{aligned} & \sin(\psi_{ref}(t+1) - \psi_0) - \sin(\psi_{ref}(t) - \psi_0) \\ &= \frac{d(t+1)}{l(t+1)} - \frac{d(t)}{l(t)} \approx \frac{d(t+1) - d(t)}{l(t)} = \frac{\Delta d(t+1)}{l(t)} \end{aligned} \quad (14)$$

Eqn. (14) can be rewritten as:

$$\begin{aligned} & 2 \sin\left(\frac{\psi_{ref}(t+1) - \psi_{ref}(t)}{2}\right) \cos\left(\frac{\psi_{ref}(t+1) + \psi_{ref}(t)}{2} - \psi_0\right) \\ &= \Delta d(t+1) / l(t) \end{aligned} \quad (15)$$

Noticed that the reference heading corrections is small at each control period, using  $\sin \theta \approx \theta$  when  $\theta$  is around 0, then Eqn. (15) may be approximated as:

$$\psi_{ref}(t+1) \approx \psi_{ref}(t) + \Delta d(t+1) / k_l \quad (16)$$

$$k_l = l(t) \cos(\psi_{ref}(t) - \psi_0) \quad (17)$$

Now we deal with  $\Delta d(t)$ . The position of the ship is mainly affected by two factors: the ship moves at speed  $u$  with heading  $\psi(t)$ , the current moves the ship at speed  $V_c$  with direction  $\psi_c$ . See Fig. 1, when ship moves at speed  $u$  with direction  $\psi$ , then after time  $t_s$  (control period), the distance difference will be:

$$\Delta d_1(t+1) = ut_s \sin(\psi_0 - \psi) \quad (18)$$

In Eqn. (18) the direction  $\psi$  can be approximated by the heading angle  $\psi(t)$  or  $\psi(t+1)$ , here we select  $\psi(t+1)$  related to  $\Delta d(t+1)$ . Similarly, the distance difference caused by current, which is almost a constant, can be expressed as:

$$\Delta d_2(t+1) = V_c t_s \sin(\psi_0 - \psi_c) \quad (19)$$

Considering the unmodeled dynamics and rounding errors, the predict model for distance  $d(t)$  can be got from Eqn. (18) and (19) as:

$$d(t+1) = d(t) + k_u(\psi_0 - \psi(t)) + k_c + \sum_{i=0}^{r_2} h_{1,i} \xi_2(t+1-i) \quad (20)$$

$$k_c = V_c t_s \sin(\psi_0 - \psi_c)$$

$$k_u = ut_s$$

where  $\xi_2(t)$  is assumed to be a white Gaussian noise with zero-mean. Taking Eqn. (20) into (16), we can get the predict model for the desired headings as:

$$\psi_{ref}(t+1) = \psi_{ref}(t) + \frac{k_c + k_u(\psi_0 - \psi(t+1)) + \sum_{i=0}^{r_2} h_{1,i} \xi_2(t+1-i)}{k_l} \quad (21)$$

### 4.2 Predictor of Desired Heading

The minimum variance predictor of desired heading angles at time  $(t+k)$  can be derived recursively from Eqn. (21) as follows:

$$\psi_{ref}(t+k|t) = \psi_{ref}(t) + \frac{k_u}{k_l} \cdot \sum_{i=0}^{k-1} (\psi_0 - \psi(t+i+1|t))$$

$$+ \frac{k_c k}{k_l} + \sum_{i=1}^{r_2} \frac{h_{k,i}}{k_l} \xi_2(t+1-i)$$

$$h_{k,i} = h_{1,i+k-1} + h_{k-1,i} \quad i=1,2,\dots,r_2 \quad (22)$$

Using Eqn. (4), Eqn. (22) can be rewritten as:

$$\psi_{ref}(t+k|t) = \begin{cases} \psi_{ref}^m(t+k) & k \leq d \\ \psi_{ref}^m(t+k) - \frac{k_u}{k_l} \sum_{i=d+1}^k \sum_{j=0}^{i-1} \Delta\delta(t-d+j) & k > d \end{cases} \quad (23)$$

where  $\psi_{ref}^m(t+k)$  is the desired heading at time  $(t+k)$  in case the future control increments are zero, it depends entirely on the past known information:

$$\psi_{ref}^m(t+k|t) = \psi_{ref}(t) + \frac{k_u}{k_l} \cdot \sum_{i=0}^{k-1} (\psi_0 - \psi_m(t+i+1))$$

$$+ \frac{k_c k}{k_l} + \sum_{i=1}^{r_2} \frac{h_{k,i}}{k_l} \xi_2(t+1-i) \quad (24)$$

Using Eqn. (7), equations (23) and (24) can be rewritten as the following vector form:

$$\Psi_{ref} = \Psi_{ref}^m - \frac{k_u}{k_l} FG\Delta\delta \quad (25)$$

$$\Psi_{ref} = [\psi_{ref}(t+d+1|t), \dots, \psi_{ref}(t+d+p|t)]^T$$

$$\Psi_{ref}^m = [\psi_{ref}^m(t+d+1), \dots, \psi_{ref}^m(t+d+p)]^T$$

$$F = \begin{bmatrix} 1 & & 0 \\ \dots & \ddots & \\ 1 & 1 & \dots & 1 \end{bmatrix}$$

### 4.3 Controller design

Using the same reference trajectory as Eqn. (9) defined and cost function as Eqn. (10) defined, from Eqn. (9) we can get:

$$\Psi_r = \begin{bmatrix} \alpha \\ \alpha^2 \\ \dots \\ \alpha^p \end{bmatrix} \Psi_m(t+d) + \begin{bmatrix} 1-\alpha & & & \\ \alpha(1-\alpha) & 1-\alpha & & \\ \dots & \dots & \ddots & \\ \alpha^{p-1}(1-\alpha) & \dots & 1-\alpha & \end{bmatrix} \begin{bmatrix} \psi_{ref}(t+d+1) \\ \psi_{ref}(t+d+2) \\ \dots \\ \psi_{ref}(t+d+p) \end{bmatrix}$$

$$= K_{\alpha 1} \Psi_m(t+d) + K_{\alpha 2} \Psi_{ref} \quad (26)$$

Taking Equations (7), (25) and (26) into Eqn. (10) and minimizing  $J$ , we can get the optimal control input vector  $\Delta\delta$ :

$$\Delta\delta = [H^T H + \lambda I]^{-1} H^T \{K_{\alpha 1} \Psi_m(t+d) + K_{\alpha 2} \Psi_{ref}^m - \Psi_m\} \quad (27)$$

$$H = G + \frac{k_u}{k_l} K_{\alpha 2} FG \quad (28)$$

The first element of  $\Delta\delta$  is  $\Delta\delta(t)$ , so the increment of the control rudder at time  $t$  is given by:

$$\Delta\delta(t) = (1, 0, \dots, 0) \Delta\delta \quad (29)$$

In application,  $\psi_{ref}^m(t+k)$  will be calculated using the following recursive equations:

$$\Delta d_m(t+k) = k_u(\psi_0 - \psi_m(t+k)) + k_c$$

$$+ \sum_{i=0}^{r_2} h_{1,i} \xi_2(t+k-i|t) \quad (30)$$

$$\xi_2(t+i|t) = \begin{cases} 0 & i > 0 \\ \xi_2(t+i) & i \leq 0 \end{cases}$$

$$\psi_{ref}^m(t+k) = \psi_{ref}^m(t+k-1) + \frac{\Delta d_m(t+k)}{k_l} \quad (31)$$

### 4.4 GPC for Track Keeping

The above controller design can be summarized as the following algorithm:

1. Set initial values of  $n, m, r_1, r_2, d$  in the model (2) and (20), as well as predictive range  $p$ , filter coefficient  $\alpha$ , weighting factor  $\lambda$ , and initial values of parameters needed to be estimated in Equations (2) and (20);
2. Compute matrix  $K_{\alpha 1}$ ,  $K_{\alpha 2}$  and  $FK_{\alpha 2}$ ;
3. Sample the true heading  $\psi(t)$  and position of ship  $(x_p, y_p)$ ;
4. Use  $\psi(t)$  to estimate the coefficients in Eqn. (2);
5. Compute  $\psi_m(t+k)$  by Eqn. (6);

6. Compute  $d(t)$ , estimate the coefficients in Eqn.(20);
7. Compute distance  $l(t)$  from ship's current position to waypoint, if  $l(t)$  less than the acceptance radii, then change to the next waypoint;
8. Compute  $\psi_{ref}(t)$  by Eqn. (8) and  $\psi_0$ ;
9. Compute  $k_l$  by Eqn. (17);
10. Compute  $\psi_{ref}^m(t+k)$  by Equations (30) and (31);
11. Compute the control increment  $\Delta\delta(t)$  by Equations (27) and (29);
12. Return to step 3.

## 5. Recursive Algorithm For Matrix Inversion

To reduce the computational time on matrix inversion in Eqn. (27), a recursive algorithm is presented which considered the special arrangement of the matrix  $H$ . The algorithm is based on the special arrangements of the matrix  $K_{\omega}$ ,  $F$  and  $G$ :

1. They are lower triangular;
2. The elements of each diagonal are same.

Taking matrix  $K_{\omega}$ ,  $F$  and  $G$  into Eqn. (28), we can find that matrix  $H$  has the above two properties too:

$$S = (I + \frac{k_u}{k_l} K_{\alpha 2} F) = \begin{bmatrix} s_1 & & 0 \\ \cdots & \ddots & \\ s_p & s_{p-1} & \cdots s_1 \end{bmatrix} \quad (32)$$

$$s_1 = 1 + (1 - \alpha)k_u / k_l$$

$$s_i = (1 - \alpha)^i k_u / k_l \quad i = 2, \dots, p$$

$$H_p = SG = \begin{bmatrix} h_1 & & 0 \\ \cdots & \ddots & \\ h_p & h_{p-1} & \cdots h_1 \end{bmatrix} \quad (33)$$

$$h_i = \sum_{j=1}^i s_j h_{i+1-j,0} \quad i = 1, \dots, p \quad (34)$$

Denote the matrix which needed to be inverted as:

$$Q_p = H_p^T H_p + \lambda I_p \quad (35)$$

Define the vector  $T_p$  as the last row of  $Q_p$ :

$$T_p = [h_p, h_{p-1}, \dots, h_1] \quad (36)$$

Then matrix  $Q_p$  can be decomposed as:

$$Q_p = \begin{bmatrix} Q_{p-1} & 0 \\ 0 & \lambda \end{bmatrix} + T_p^T T_p \quad (37)$$

Using formula  $[A + BC^T]^{-1} = A^{-1} - A^{-1}B[I + C^T A^{-1}B]^{-1}C^T A^{-1}$  to Eqn. (37), we can get:

$$Q_p^{-1} = \begin{bmatrix} Q_{p-1}^{-1} & 0 \\ 0 & \lambda^{-1} \end{bmatrix} - W_p W_p^T / (1 + T_p W_p) \quad (38)$$

$$W_p = \begin{bmatrix} Q_{p-1}^{-1} & 0 \\ 0 & \lambda^{-1} \end{bmatrix} T_p^T \quad (39)$$

Equations (38) and (39) are the recursive formula to compute the inverse of matrix in Eqn. (27), with initial value is:

$$Q_1^{-1} = (h_1^2 + \lambda)^{-1} \quad (40)$$

The recursive formula can be summarized as follows:

1. Compute  $s_i$  and  $h_i$  in Equations (32) and (34);
2.  $j=1$ , compute initial value  $Q_1^{-1}$  as Eqn. (40);
3. Compute  $W_{j+1}$  as Eqn. (39) and  $Q_{j+1}^{-1}$  as Eqn. (38);
4.  $j=j+1$ , return to step 3 until  $j=p$ .

The general algorithms for matrix inversion need to carry out about  $2p^3$  times addition and subtraction, as well as about  $2p^3$  times multiply and division (here the symmetry of the matrix has been considered). However, the recursive algorithm needs about  $p^3/2$  times addition and subtraction, as well as about  $2p^3/3$  times multiply and division.

## 6. Simulation

### 6.1 Parameters of the Simulated Ship

The model introduced in Section 2 is used to simulate the real dynamics of the ship. The principal data of the simulated cargo ship 'hongqi177' in fully loaded status are listed in Table 1. We select the model of rudder engine as  $\delta(s) = e^{-4s} / (4s + 1)$ , and define the course as  $0^\circ$  if the ship heads to North, define the rudder angle as positive if the rudder angle is turn right.

Table 1: Parameters of the cargo 'hongqi177'

Ship length	Block coefficient	Depth	Draft
99m	0.703	9m	6.5m
Breadth	Cruising Speed	Engine speed	
16m	15knot	175 rpm	

### 6.2 Operation conditions

Assume the ship is navigating under the wind speed 12.28m/s with direction 180°, the wave 4 meters high with direction 180°, and the current 2knot with direction 135°. The ship started from point 1(123°28'08"E, 25°37'01"N), via waypoint 2(123°29'28"E, 25°42'21"N), point 3(123°31'17"E, 25°44'34"N) and point 4(123°33'18"E, 25°44'58"N), arrived point 5(123°34'30"E, 25°46'20"N). When the distance of the ship to the waypoint is less than 0.2nmile, the ship will change to the next waypoint. The parameters of the controller are selected as:  $n=2$ ,  $m=2$ ,  $r_1=2$ ,  $r_2=2$ ,  $\alpha=0.9$ ,  $\lambda=5$ ,  $p=19$ ,  $d=1$ , control period  $t_s=4s$ .

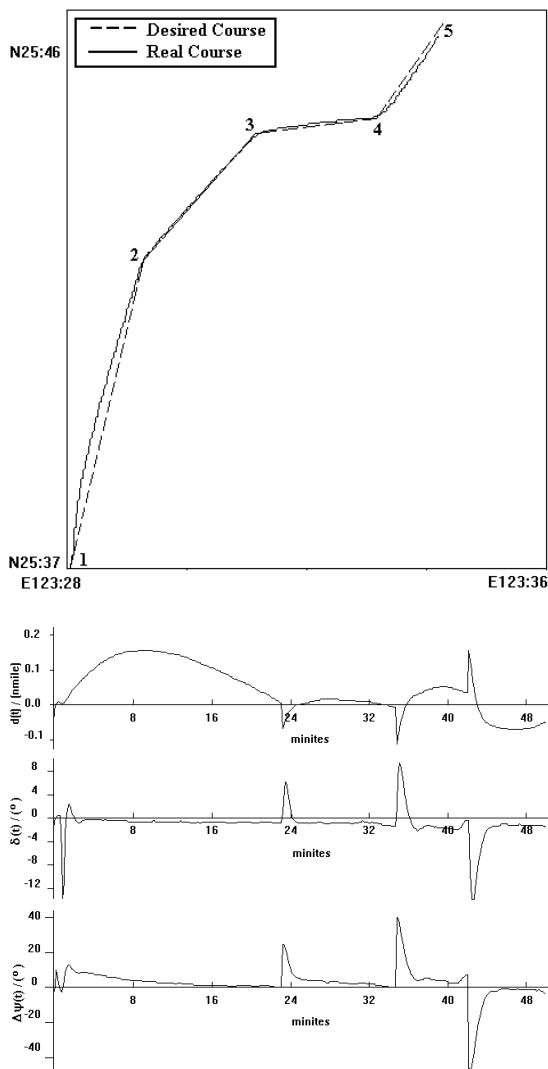


Fig. 2. Results by basic GPC controller.  
 (a) Desired course and real course  
 (b) The distance from ship to reference course  
 (c) Rudder angle  
 (d) Heading angle error

### 6.3 Results

Fig. 2 shows the simulation results by the basic controller introduced in Section 3; and Fig. 3 shows the simulation results by the new controller introduced in Section 4. We can see that both controllers are efficient for track keeping. However, by predicting the desired heading angles, the new controller can adjust the commanded rudder angle in advance, thus the actual course by the new controller is closer to the reference course, and has shorter length of the journey.

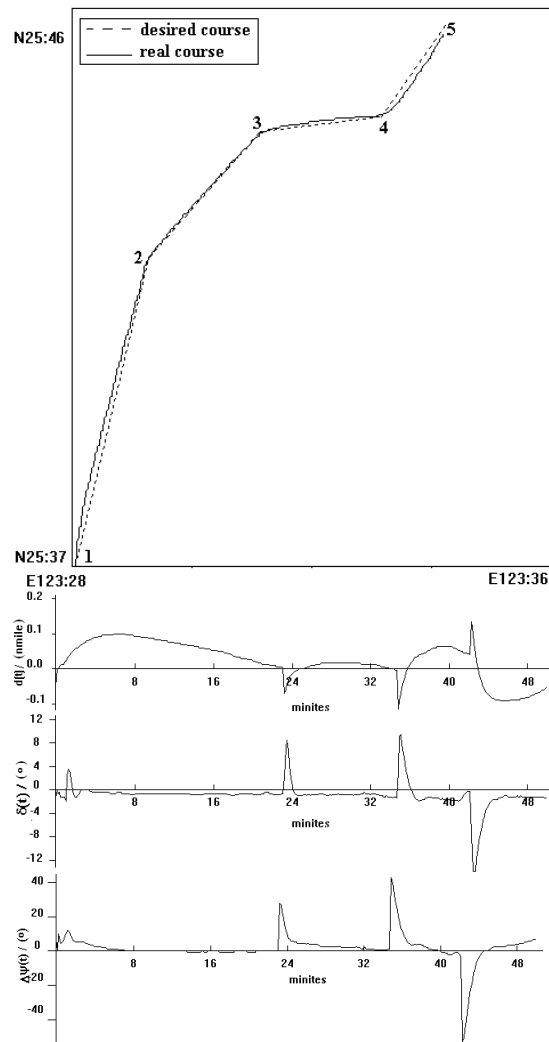


Fig. 3. Results by GPC with desired heading prediction  
 (a) Desired course and real course  
 (b) The distance from ship to desired course  
 (c) Rudder angle  
 (d) Heading angle error

## 7. CONCLUSION

In this paper, we introduced a basic generalized predictive controller for ship track keeping, then deduced a model to predict the desired heading angles and presented a new GPC controller for track keeping which based on the prediction of the future heading and desired heading angles. A recursive formula to compute the inverse of the matrix, which need only as 1/3 computational time as that of general formula, is also presented. The simulation results show that the controller is efficient and the course is closer to the predetermined course.



Technology.

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