Multi-dimensional Photonic Processing, A Discrete-Domain Approach: Part III – Realization

Le Nguyen Binh
Department of Electrical and Computer Systems Engineering,
Monash University, Clayton Victoria 3168 Australia.

Abstract
This paper, Part III of the series of three parts on multi-dimension photonic signal processing proposes techniques to integrate the fields of discrete signal processing and fiber-optic signal processing, integrated photonics and/or possibly nano-photonics to establish a methodology based on which physical systems which can be implemented. Using a combination of one-dimensional filter structures, 2-D fiber-optic filters can be constructed. The relationship between the fiber-optic model and the mathematical model has been linked to allow quick implementation. Using the developed methodologies, multi-dimensional coherent photonic signal processors can be designed. The technologies to support physical realization of such systems are not yet matured, therefore it is difficult to estimate the exact capabilities of such systems. However, theoretically the architectures provide the potential for all-optical signal processing and it is envisaged that the processing bandwidth can reach the Tera-Hz region. This paper proposes a number of multi-dimensional structures that would be realizable in the near future.
Key words:
Multi-dimensional, Discrete-Domain, Realization

1. Introduction to Photonic Signal Processing

In recent years, there has been a notable increase in the number of applications that require an extremely fast signal processing speed that cannot be met by current all-electronic technology. Photonic signal processing (PSP) opens the possibilities for meeting the demands of such high-speed processing by exploiting the ultra high bandwidth capability of lightwave signals with specific applications in the field of photonic communications and fiber optic sensor networks.
The demand for multi-dimensional photonic signal processing (M-D PSP) can be attributed to various factors due the growing feasibility of high-capacity digital transmission networks capable of transmitting ultra-high bit rate and time division multiplexing up to 160 Gb/s as well as fiber optical sensor networks.
A problem with the implementation of such systems is the lack of devices that are capable of processing an enormous amount of data associated with multi-dimensional signals. With photonic transmission networks becoming the transport infrastructure, PSP technique has become increasingly more desirable compared to O/E and E/O conversion techniques. As discussed in Part I [1], fiber-optic signal processing systems are ideal for such processing demands for several reasons: all-optical (or photonic) processing of photonic information of optical communication systems are possible using fiber-optic signal processing; 2-D signals usually require much higher bandwidth than 1-D signals and therefore must be processed by a high bandwidth system to allow real-time performance. It is likely that future telecommunication networks would be all fiber-optic.
Part I of the series [1] of three parts gave an introduction to multi-dimension signal processing and the fundamentals of 2D processing with mathematical representation and optical signal flow graphs so that the implementation using photonic structures can be carried out. Part II [2] deals with methods of decomposition and related photonic structures so as to simplify the practical implementation in photonic domain. This part, Part III gives some examples on the implementation of multi-dimension photonic signal processors. General concluding remarks are given to summarize all three parts of the series.

2. Photonic Implementation of 2-D Filters

In Part I [1] the fiber-optic signal processing technique was introduced. In this section, various structures of fiber-optic signal processing systems, some of them novel, are shown. Using these structures along with the 2-D filter design methods given in the Part II [2], 2-D filters can be implemented in photonic domain.

2.1 Photonic Filter Structures

In *Part II [2]*, various methods of developing 2-D transfer function of the filter with desired characteristics have been developed. To implement a 2-D transfer function in photonic domain, as with other implementations of digital filtering systems, one must consider what kind of structure the fiber-optic filter should have in order to reduce error and at the same time, be economical.

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Many of the structures used here are similar to 1-D fiber-optic filter structures and simply require cascading of the structures to construct a 2-D fiber-optic filter. However, for transfer functions not derived using matrix decomposition methods, novel structures must be devised to accommodate the requirements of non-separable transfer functions. Possible filter structures include binary tree structure, direct structure, lattice structure, parallel structure, and transversal structure all of which are described in the following sections.

2.2 Coherent System

The filters implemented in this paper assume coherent operation of the laser. As explained in Part II [2], this implies that the phase of incoming signal cannot be discarded and the constructive and destructive interference of signals from different paths combining must be taken into account when designing the filter. Although the restriction is somewhat harsh, it is noted in *Part II [2]* that there are important advantages of using coherent operation over incoherent operation. Furthermore, unless specifically noted otherwise, all filters implement 2-D FIR structures. The implementation methods for IIR filters are discussed in *Section 2.10*

2.3 2-D Direct Structure Filter

We first consider a filter structure that is suited to direct design methods of of Part II [2], i.e. no matrix decomposition. None of binary tree structure, transversal filter structure, or other 1-D structures which are introduced later this section is suitable for implementation of 2-D filters designed without using matrix decomposition. 2-D frequency sampling method for example does not generate a set of separable transfer functions and therefore cannot be implemented using 1-D structures. In such cases, two options exist for photonic implementation: (i) Break down the 2-D transfer function into a set of 1-D transfer functions by decomposing the filter transfer function using singular value decomposition and (ii) Use a 2-D direct filter structure.

2-D direct structure filter is derived from the signal flow diagram shown in of Part II [2]. Translating the signal flow diagram into photonic domain is a relatively easy task in this case and the result is shown in Figure 1.

In Figure 1, each box labeled A_{mn} represents a coefficient module which contains a one photonic attenuator (or some form of attenuation mechanism) and possibly a phase modulator. The coupler ports are arranged so that the number of cross coupling a signal path contains is four for all signal paths therefore ensuring that the phase of the signal entering each coefficient module is consistent over the whole network. The negative coefficients can then be realized by including a 180° phase shifter in each negative coefficient module.

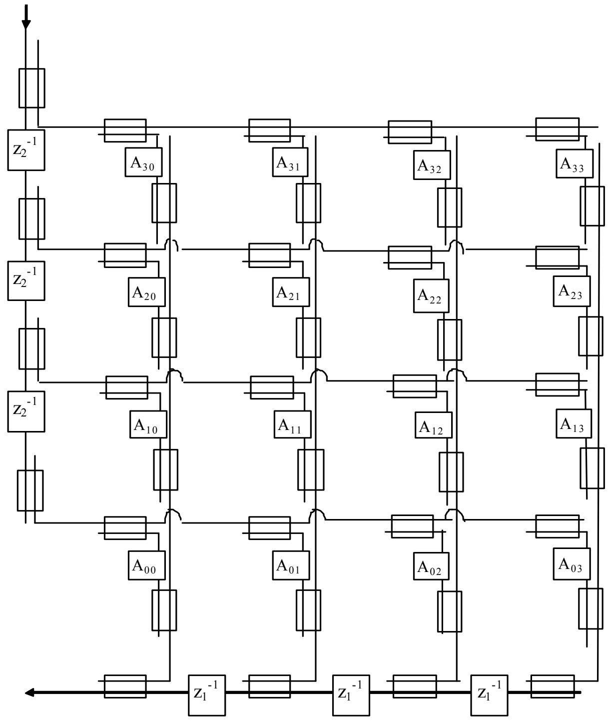

Figure 1: Fiber/integrated optic implementation of direct structure

It should be noted that there are two different kinds of delays. The implementation of the delays is quite simple if we are considering signal input of the form described in of Part II [2]. For the 2-D sequence of of Part II [2], z_1^{-1} can be implemented as just one unit delay, and z_2^{-1} can be implemented as 7 unit delays as this is the number elements in a row of the signal.

The quantities of photonic elements required for 2-D direct photonic implementation are given by

$$\begin{aligned} C_{2D} &= 2N_1N_2 + N_1 + N_2 \\ N_{PM2D} &= N_{OAt2D} = N_1 \times N_2 \\ N_{OA2D} &= 1 \end{aligned} \quad (1)$$

where $C_{2\text{-}D}$ = number of optical couplers required; $N_{PM2\text{-}D}$ = number of optical phase modulators required; $N_{OAt2\text{-}D}$ = number of optical attenuators required; N_1 = filter order in n_1 dimension; N_2 = filter order in n_2 dimension

2.4 2-D Separable Structure Filter

In the previous section, it is made clear that matrix decomposition methods of Part II [2] can be implemented in the photonic domain by cascading of 1-D fiber-optic structures since matrix decomposition methods basically generate sets of 1-D transfer functions. The 1-D structures however must be combined in a way so that the filter performs 2-D signal processing operation as intended. Essentially, the combined structure must implement a product of sum of 1-D transfer functions. Shown in Figure 2 is a fiber-optic structure that implements the required function.

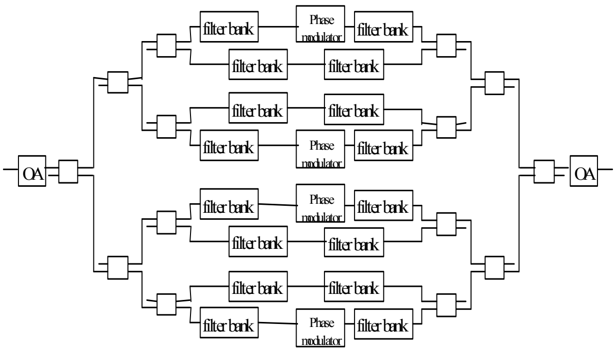

Figure 2: 2-D fiber-optic filter structure

Any 1-D filter implementation can be substituted for the filter banks as long as it is modular (the signal in and the signal out must have the same zero reference point). One can therefore regard the above structure as the general structure for a separable 2-D photonic filter. Specifically, Figure 2 implements a 2-D filter generated by matrix decomposition with eight significant singular values. The order of the resultant 2-D filter is defined by the order of 1-D filters in the filter banks. Even though the diagram does not make any distinctions, it should be kept in mind that the length of the delays z_1^{-1} and z_2^{-1} are also different for the two filter banks in each parallel branch.

In the following sections, a number of 1-D sub-structures that can be used to implement the filter banks are described. It is up to the filter designer to choose which structure is most suitable for the particular application as any one of the structures can be substituted into the filter banks.

The number of photonic elements required for the separable 2-D structure is given by

$$C = \sum_{i=0}^{\log_2 N_s} 2^{i+1}$$
$$N_{PM} = \frac{N_s}{2} \qquad (2)$$
$$N_{OA} = 2$$

where C = number of optical couplers required; N_{PM} = number of optical phase modulators required(maximum); N_{OA} = number of OAs required(maximum); N_s = number of parallel stages included

2.5 Binary Tree Filter

A 1-D FIR filter transfer function can be implemented in fiber-optic format by arranging filter elements in binary tree structure as shown in Figure 3. The particular filter shown in the figure is a 1-D 3rd order FIR filter. The extensions to higher order filters are obvious.

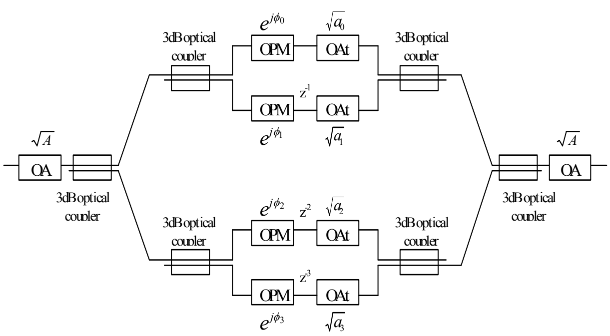

Figure 3: Binary tree structure [3]

The transfer function of the filter implemented with the structure can be expressed as

$$H_{az}(z) = \frac{E_{o,az}}{E_{i,az}} = (1-\gamma)^{\log_2(k+1)}(k+1)^{-1}A\sum_{d=0}^{k}(-1)^d \exp(j\varphi_d)\sqrt{a_d}\,z^{-d} \qquad (3)$$

where

$(1-\gamma)^{\log_2(k+1)}$ = common excess loss fctor

$(k+1)^{-1}$ = common 3 dB loss factor of the couplers

A, a_d =intensity attenuation of optical attenuators

φ_d = optical phase shift of the optical phase modulator

d=delay order

Because the system is not intensity based, the signal is assumed to have the form $E_o e^{j(\omega_o t+\phi(t)+\varepsilon)}$ which includes the optical phase ϕ. Negative coefficients of the transfer function can therefore be achieved by shifting the phase of the signal by 180° through phase modulators [3]. With appropriate manipulation of the optical coupler characteristic as a perfect -90° phase shifter, the use of optical modulators to achieve negative coefficients can be reduced or even completely eliminated as with transversal filter structure shown in the next section.

The advantage of the binary structure is that the number of 3 dB splitting stages the signal must travel through is the minimum achievable. The loss due to splitting is thus at minimum and the effect of noise due to splitting is also at minimum. The structure is used in [3] to implement a fiber-optic integrator in which the input and output terminals of couplers are selected so that the sign alternation in the numerator that is the characteristic of Newton's family of digital integrators is achieved without phase modulators. However, this (alternating sign) is not true for an arbitrary filter transfer function and therefore most filter transfer functions would necessitate the use of optical phase modulators (OPMs) as shown in Figure 3.

The number of photonic elements required for the binary tree structure is given by

$$\begin{aligned} C_{BT} &= \sum_{i=0}^{\log_2 N} 2^{i+1} \\ N_{PM_{BT}} &= N \\ N_{At_{BT}} &= N \\ N_{A_{BT}} &= 1 \end{aligned} \qquad (4)$$

where C_{BT} = number of optical couplers required; N_{PMBT} = number of optical phase modulators required(maximum); N_{AtBT} = number of optical attenuators required; N_{ABT} = number of OAs required(minimum) and N = order of the 1-D filter sections

2.6 Photonic Transversal Filter

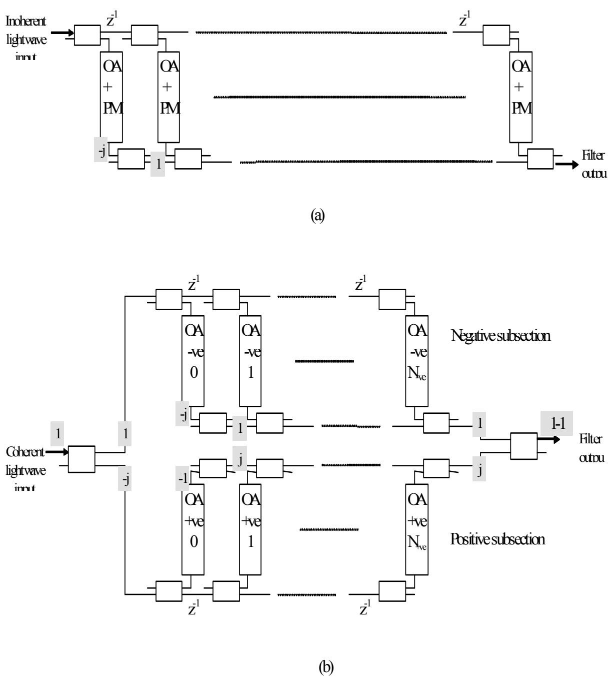

Figure 4 (a) The original transversal structure [7] and (b) The proposed structure

In the previous section, it is shown that the binary tree structure requires phase modulators except in cases when the signs are alternating or when there is equal number of positive coefficients as negative coefficients in the transfer function. Such cases are rare, and in most cases phase modulators will be needed to achieve the negativity. In this section, a structure that does not require any phase modulators is proposed. This structure, based on the transversal filter structure in [7, 8, 9] (shown in Figure 4*(a))* achieves 180° phase shift required by negative coefficients through appropriate arrangement of coupler ports. The signal paths for positive coefficients contain four cross couplings resulting in $-j\times-j\times-j\times-j = 1$, i.e. no phase shift, and all signal paths for negative coefficients contain two cross couplings resulting in $-j\times-j = -1$, i.e. 180° phase shift. Shown in *Figure 8-4(b)* is the proposed transversal filter structure that does not require phase modulators to achieve negative coefficients. The phase of the signal at various points of the filter is shown in gray scale.

The proposed structure is particularly suited to the transfer functions generated by matrix decomposition as it can implement a 1-D FIR transfer function with very little modifications(needed for the coupling attenuations) to its filter coefficients. How the additional sections can be accommodated to form a 2-D filter is shown later in this section.

The advantages of the proposed filter structure over the simple transversal filter structure in [7] are threefold. First, as mentioned previously, is the fact that no phase modulators are required to achieve negative coefficient. The second advantage is that no signal biasing or differential detection is required. The implication of the second advantage is that the filter structure is modular and can therefore be connected in parallel or in series without any modification to the filter module. Another advantage of the proposed structure is that the signal goes through less number of couplers than the implementation of [7]. This property can be observed if one considers the fact that the signal only goes through in average *N/2* couplers for the proposed structure (since the coefficients are split into two groups) whereas for the traditional transversal filter the number of couplers the signal must travel through is always *N*, where *N* is the number of coefficients of the filter transfer function which may or may not be the same as the order of the filter.

The proposed structure can also be used for adaptive filtering as only the coefficients of the optical amplifier/attenuator need to be changed to modify the filtering operation. However for adaptive operation, the filter coefficients that are zero and therefore not included in the original structure may have to be included in case the adapting algorithm changes them to non-zero values. The situation is slightly disadvantageous for the proposed structure compared to the simple transversal structure of [7], because the number of positive and negative coefficients (N_{-ve} and N_{+ve}) of an N^{th} order filter can range from 0 to *N+1* and both sets of coefficients need to be fully implemented. Fortunately the situation is not as bad as one might expect. At first, it appears that the number of optical attenuators (OAs) needed is *2N* since both negative and positive sections should implement the full order of the filter transfer function. For the proposed structure, it can be shown that the number of OAs required is approximately *(N+1)×1.5 (N+1)* components are required for the simple transversal filter structure of [7] for adaptive operation). The reason for requiring only *(N+1)×1.5* instead of *2N* can be explained as follows.

Assume that we can exchange the input connections to two parallel sections using optical switches so that the signs of the two parallel sections can be reversed. First consider the case where the number of positive coefficients and negative coefficients of the filter transform function are exactly the same, i.e., $N_{+ve}=N_{-ve}=(N+1)/2$. Clearly *(N+1)/2* OAs are needed for each parallel subsection and therefore total of *N+1* OAs are required. Now consider the case where one sign is completely dominant over the other, i.e. $N_{+ve}=N+1$ and $N_{-ve}=0$, or $N_{-ve}=N+1$ and $N_{+ve}=0$. In this case, *N+1* OAs are necessary in at least one of the parallel subsections even though the other subsection does not need any. To accommodate for both situations with one filter structure, it is clear that *(N+1) + (N+1)/2 = 1.5(N+1)* OAs are necessary. The result for optical attenuators can be extended to phase modulators and couplers with a few simple modifications.

In the case where adaptive filtering is not required, total number of components required is minimized by the use of the proposed structure. For 1-D digital filtering purposes also, this structure is economical, modular and easy to implement. *Example 8-1* is a simple exemplar of a fiber-optic filter realized from a transfer function using the parallel structure.

Photonic filter implementation using the proposed transversal structure

Design Aim: A sample implementation of a simple transfer function.

Method: The proposed transversal structure. Fiber losses and other non-linear effects are ignored.

Results:

$$H(z^{-1}) = 1 + 0.2z^{-1} - 0.3z^{-2} - 0.2z^{-3} + 0.4z^{-4} + 0.6z^{-5} - 0.5z^{-6} \quad (5)$$

The first step is to divide the transfer function into two parts, namely the positive coefficients and the negative coefficients, denoted by k_+ and k_-. It is assumed that all the coefficients are realized using optical attenuators and not amplifiers (optical attenuator is known to produce less noise than does the optical amplifier).

An observation of the filter structure shown previously in *Figure 8-4* will reveal that all streams of lightwaves in each parallel section pass through the same number of couplers and therefore suffers the same attenuation from coupling. In this example, the positive coefficients suffer $(\sqrt{2})^7$ attenuation whereas the negative coefficients suffer a $(\sqrt{2})^6$ attenuation. Optical preamplifier gain is therefore set at $(\sqrt{2})^7$ and the negative coefficient attenuators are set at $h_{-n}\div\sqrt{2}$ where h_{-n} denotes the negative coefficient values. This factor of $\sqrt{2}$ compensates for the effect of the extra coupler positive coefficient signals go through. Optical preamplifier is set at $(\sqrt{2})^k$ where k is the greater of k_+ and k_- to compensate for the coupling losses.

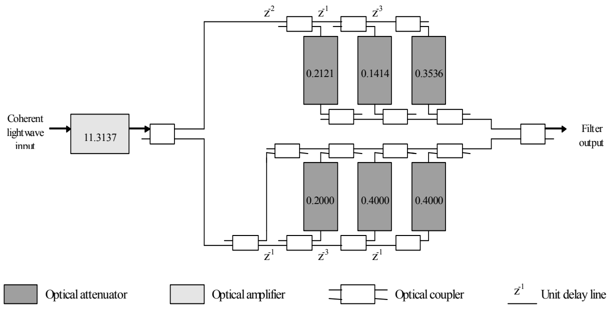

Figure 5: Fiber optic implementation of (5) using the proposed transversal filter structure

Because the binary tree splitting stages of Figure 3 add further attenuation to the lightwave when the transversal structure is incorporated into a 2-D system, the filter amplifier and the attenuator settings must be adjusted accordingly. The number of optical elements required for the transversal structure is given by

$$\begin{aligned} C_{TF} &= 2N + 2 \\ N_{PMTF} &= 0 \\ N_{AtTF} &= N \\ N_{ATF} &= 1 \end{aligned} \quad (6)$$

where C_{TF} = number of optical couplers required; N_{PMTF} = number of optical phase modulators required; N_{AtTF} = number of optical attenuators required; N_{ATF} = number of OAs required(minimum) and N = order of the 1-D filter sections

2.7 1-D Direct Structure Photonic Filter

1-D direct structure provides the classic alternative to the binary tree structure and transversal filter structure for implementation of 1-D filters. Unlike the structures mentioned, direct structure can also implement IIR filters very easily. The direct structure signal flow diagrams can be found in most signal processing books and is quite straightforward. However, photonic implementation of the direct structure has not been developed as yet and this section shows an implementation of the structure using fiber-optic elements.

$$H(z) = \frac{\sum_{k=0}^{L} b_k z^{-k}}{1 - \sum_{k=1}^{N} a_k z^{-k}} \tag{7}$$

This transfer function can be represented in signal flow diagram format as shown in *Figure 6* and the photonic implementation is shown in *Figure 7*.

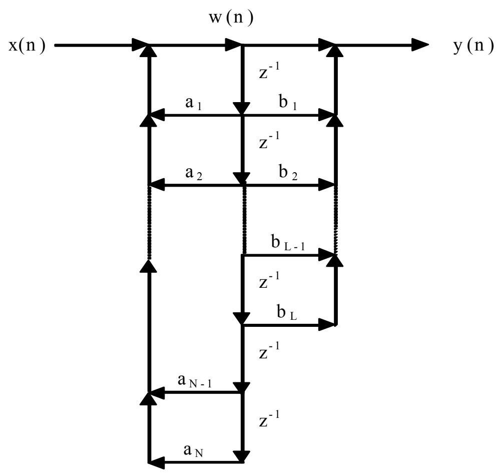

Figure 6: Signal flow diagram of 1-D direct structure.

As with the 2-D direct structure, each signal path contains four cross-couplings and therefore suffers no overall phase shift ($-j\times-j\times-j\times-j=1$). The boxes in the horizontal branches represent coefficient module identical to that in 2-D direct structure and the boxes in the vertical branches represent the delay elements. The structure is optimal in the sense that the number of delay elements is minimal and can be used to implement 1-D IIR functions in the photonic domain. The couplers used in the middle section of the implementation represent either 3×3 couplers or a cascade arrangement of two 2×2 couplers shown in Figure 8.
The number of photonic elements required for the direct structure is given by

$$\begin{gathered} C_{DF} = 3N + N_{num} + 3 \\ N_{PMDF} = N_{AtDF} = N_{num} + N_{den} \\ N_{ADF} = 1 \end{gathered} \tag{8}$$

where C_{DF} = number of optical couplers required; N_{PMDF} = number of optical phase modulators required; N_{AtDF} = number of optical attenuators required; N_{ADF} = number of OAs required; N_{num} = order of the 1-D filter numerator and N_{den} = order of the 1-D filter section denominator

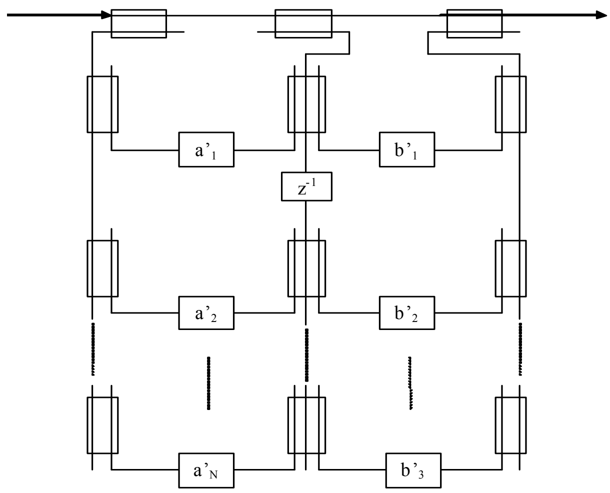

Figure 7: Photonic implementation of direct 1-D structure

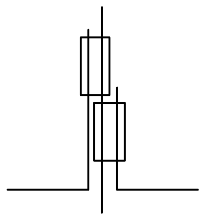

Figure 8: An arrangement of two 2×2 couplers to form a 3×3 coupler

2.8 Parallel Structure Filters

Parallel structure is a simple variation on the theme on the direct structure. This structure is slightly different from the direct structure in that signals pass through less number of couplers and therefore the signals are not attenuated as much.
Basically the structure can be thought of as a parallel arrangement of 2nd order direct structures. The signals are split into several branches at the beginning and after the signals have traveled through the parallel sub-structures, they are merged back into one signal path. To implement the parallel structure, the filter transfer function must be factorized into a sum of several sub-transfer functions. If each sub-transfer function is in the form of 2nd order substructure, it is then possible to turn the sub-transfer function into signal flow diagram form which in turn can be realized in the optical domain.

A Filter design using parallel structure filter

Design Aim: A 1-D low-pass filter of 7th order with normalized cut-off frequency at 0.3

Method: Parallel factorization

Program used: PARALLEL.m

Results: The transfer function designed using Chebychev approximation routine in MATLAB simulation package is decomposed as shown below in signal flow diagram format. It is found that the performances of the two implementations are nearly identical (see Figure 9).

For photonic implementations of parallel structures, the splitting of signal at the beginning of the structure can be performed using a binary tree of couplers in which case the amplitude and the phase change caused by cross-coupling must be considered. The task is very similar to that performed for binary filter structure. Essentially, the phase changes can be corrected by including a 180°shift every second parallel branch, and amplitude compensation is equal to 2 raised to $\log_2 P$ where P is the number of parallel stages (which must be a power of 2).

The performance of the parallel structure is similar to that of the direct structure as can be seen in *Figure 8-10*, and the number of delays as well as the number of amplifiers/attenuators required are also the same. However, the number of summer/splitters in parallel structure is significantly increased, it will be shown that the number of optical couplers needed for the implementation of parallel structure is also very much larger than that required for other 1-D structures.

The number of photonic elements required for the parallel structure is given by

$$
\begin{aligned}
C_{PF} &= \frac{N_{den}}{2} \times 7 + \sum_{i=0}^{\log_2 \frac{N_{den}}{2}} 2^{i+1} \\
N_{PMPF} &= 2N_{den} + \frac{\log_2 \frac{N_{den}}{2}}{2} \\
N_{AtPF} &= 2N_{den} \\
N_{APF} &= 1
\end{aligned}
\qquad (9)
$$

where C_{PF} = number of optical couplers required; N_{PMPF} = number of optical phase modulators required; N_{AtPF} = number of optical attenuators required; N_{APF} = number of OAs required (minimum); N_{num} = order of the 1-D filter numerator and N_{den} = order of the 1-D filter section denominator.

2.9 Other 1-D Filter Structures

Several other filter structures exist for filter realization in optical domain. Lattice form shown in *Figure 11* is a structure particularly suited to incoherent systems since each section of the signal flow diagram is identical to the optical coupler signal flow diagram shown in Figure 10*(a).* The coefficients k can be implemented using optical amplifier/attenuators. The lattice structure is not suitable for coherent signal processing as coherent systems must take phase into account and since the lattice structure assumes that the cross coupling does not cause any phase shift, it will be inefficient and difficult to implement coherent systems using the lattice structure.

2.10 Realization of Poles

In discussing feed forward structures such as the binary tree structure or the direct structure, the realization of the denominator part of filter transfer functions are ignored. This can be justified in the case of finite impulse response filters (FIR) where the denominator is simply 1. In cases of IIR filters, the denominator must be incorporated into the system as in 1-D direct structure. An alternative is to form a pure pole filter by using a feed forward structure in a feedback loop as shown in Figure 12.

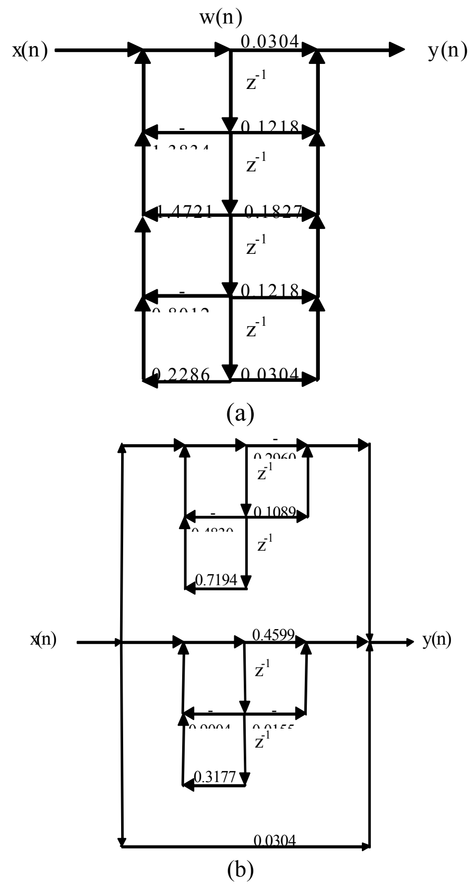

Figure 9: Parallel structure example signal flow diagram (a) Direct structure (b) Parallel structure

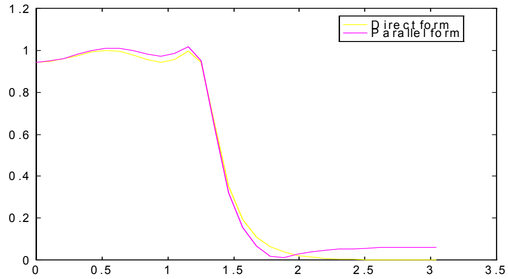

Figure 10: Frequency response of direct and parallel structures

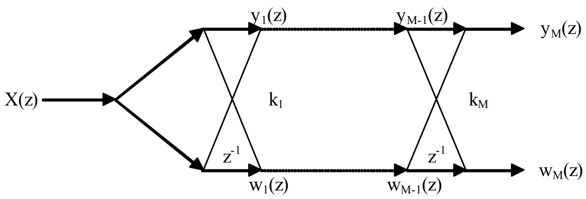

Figure 11: Lattice form signal flow diagram

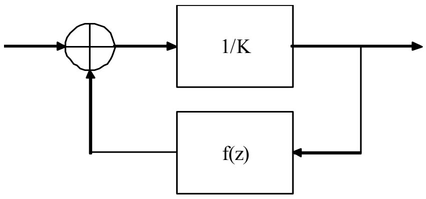

Figure 12: Denominator realization using feed forward structure.

Assuming that the denominator of transfer function can be written as

$$p(z) = K + f(z) \tag{10}$$

where *f(z)* has no constant terms, applying the feedback structure results in the following transfer function:

$$H_p(z) = \frac{\frac{1}{K}}{1 + \frac{1}{K} f(z)} \tag{11}$$

It is therefore possible to use the feed forward structure such as transversal structure without modifications in feedback branch to form the denominator of an IIR filter transfer function. The overall filter structure incorporating numerator q(z) and denominato*r K+f(z)* is shown in Figure 13.

Note that q(z) and f(z) can be replaced by $q(z_1, z_2)$ and $f(z_1, z_2)$. The structure in Figure 13 can therefore be used to implement a subsection of separable filters of Part II [2} or a complete 2-D direct filter implementation. We implement a case of the former as follows.

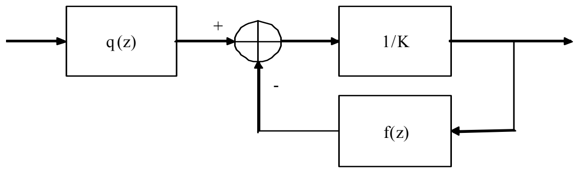

Figure 13: IIR filter structure using FIR subsections.

A 2-D recursive filter subsection realization

Design Aim: The recursive transfer function given in *Eq. 8-3.*

$$H(z^{-1}) = \frac{1 + 0.2z^{-1} - 0.3z^{-2}}{1 - 0.2z^{-3} + 0.4z^{-4} + 0.6z^{-5} - 0.5z^{-6}} \tag{12}$$

The filter subsection can be implemented in photonic domain as shown in Figure 14.

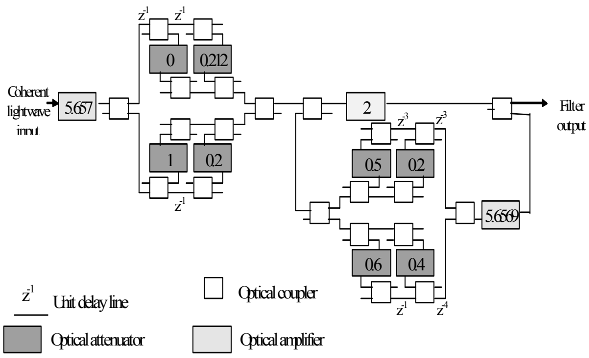

Figure 14: A 1-D IIR filter subsection example

Finally, when the SFG is ready, the graphical technique introduced in Part II [2} can be used to check the validity of the design. However, applying the technique by hand to photonic circuits shown in this section would be time-consuming and error prone as there is too many photonic elements to consider. In fact, applying any graphical technique by hand to the photonic circuits in this section would be impractical. Due to the impracticality of performing a graphical analysis by hand, the graphical analysis of the photonic circuits is omitted here.

2.11 Remarks

In this section (i) The significance of using coherent systems for PSP is stated ; (ii) 2-D direct structure illustrated ; (iii) Various matrix decomposition realizations using 1-D filter structures shown; and (iv) Realizations of IIR filters using FIR filter sections shown.

3. Design Chart and Discussions

The design of 2-D photonic filter can be summarized in this section. Another example of such design is also illustrated and thence some concluding remarks are given.

3.1 2-D Photonic Filter Design Flowchart

In designing and implementing a 2-D filter, several design decisions must be made. The following flowchart illustrates the possible paths that may be taken when designing and implementing a 2-D filter in photonic domain. Tables in the chart show various design methods any one of which may be substituted for another within the same table. For example for separable design methods path (the branch on the right side of the flowchart), any one of single stage SVD, multistage SVD, iterative SVD, or OD may be used to calculate the appropriate 2-D transfer function. The numbers inside brackets show the relevant section which to be referred to. Obviously, if a design result is not satisfactory, some of the procedures must be repeated to find a better solution. The iteration paths are not shown in the flowchart as they are reasonably obvious.

3.2 Examples of Photonic 2-D PSP Implementation

In this section, 2-D filter implementations of the same frequency specification are performed from scratch first using a matrix decomposition method, and then a direct method. The design methods used in the example are by no means 'ideal'. As shown in the flowchart show in the previous page, there are many possible paths and ultimately, the choices lie with the filter designer.

3.2.1 Specification

The filter to be designed is a low-pass filter with normalized spatial cut off frequency of 0.7 in x and y directions (see Figure 16). The overall filter magnitude response error is to be less than 10%. The 2-D signal is transmitted coherently through an optical fiber medium using linear sequencing of a 256×256 pixel frame starting from *x=0 and y=0.* The frequency response of the filter is to be circularly symmetric. The order of the filter should be sufficiently low to enable it to be implemented in photonic domain. The phase response of the filter should be linear. The filter is intended to be a noise remover as most noise components are in the high frequency band.

3.2.2 Choice of a Design Methodology

As the first decision to be made, a choice between direct design methods and separable design methods must be made. The following table shows the differences between separable and direct design methods. Although the two methodologies result in very different designs, there are no apparent advantages to be gained from preferring one structure over another. There are multitude of factors contributing to the actual performance and the economy of the designed filter and the best design will be obtained by using both methodologies and comparing various statistics to find out which design method gives better results for the particular application.

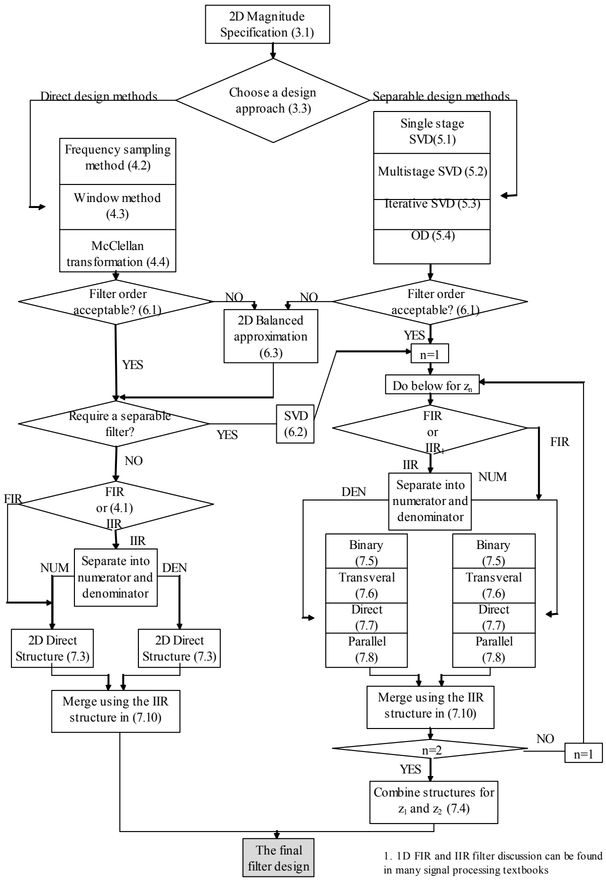

Figure 15 Flow Chart of the design of Multi-dimension photonic filters.

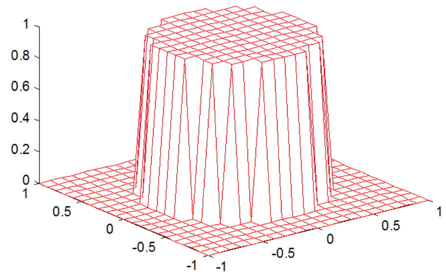

Figure 16: Magnitude specification

Property	Direct design methods	Separable design methods
Number of parallel stages	1	≥ 1
Filter structure	FIR or IIR	FIR or IIR
Filter design procedure	One 2-D filter design	Many 1-D filter designs
Coherent processing	Yes	Yes
Performance	Depends on the order of the filter and the 2-D filter design method	Depends on number of additional stages and the 1-D filter design method
Fiber-optic implementation	Direct structure only	Binary, parallel, transversal, direct

Table 1: Comparison of direct structure and separable structure

Separable Implementation Using Matrix Decomposition Methods

3.2.2.1 Choice of a Decomposition Method

There are four decomposition methods to consider. All methods are tried to compare their performances. The comparison is made on the basis of the least number of parallel stages that will satisfy the specified 10% overall error.

	Single stage SVD	Multiple stage SVD	Iterative SVD	Optimal decompositi on
Number of parallel stages	1	2	2	2
Resulting design	FIR (or IIR)[1]	FIR (or IIR)	FIR (or IIR)	FIR (or IIR)
Computational requirements	Minimal	Intermediate	Intermediat e	Very heavy
Major claim for advantage	Simple and quick	Accurate	Easy 1-D filter design	Accurate and easy 1-D filter design
Major disadvantage	Too rough (large error)	Phase must be considered when designing 1-D filter sections	Not accurate enough	Not accurate enough & heavy computational load
Filter Error(%)	10.7262	8.2740	9.9603	9.8521
Order	16	16	16	16
Total no of multiplications required	32	64	64	64

Table 2: Comparison of various decomposition methods.

Also, the 1-D filter design methods used are identical for the four different decomposition methods (note that perhaps this is not quite fair on methods such as ISVD algorithm or OD algorithm which put their strengths on making filter design task easier by keeping the 1-D magnitudes all positive).As can be seen in the table, the best performance can be obtained by multiple stage SVD algorithm that, with two parallel stages, results in a filter error of 8.274%.

As multiple stage SVD algorithm produces four sets of 1-D magnitude responses, there are four 1-D filters to be implemented and their coefficients are shown in Table 3. The 1-D filter design method used is least-squares algorithm.

Order	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
z_1 stage 1	0.0004	-0.0004	-0.0049	0.0114	0.0083	-0.0704	0.0819	0.5015	0.5015	0.0819	-0.0704	0.0083	0.0114	-0.0049	-0.0004	0.0004
z_1 stage 2	0.0022	0.0055	-0.0074	0.0037	0.0406	-0.0841	-0.1752	0.0082	0.0082	-0.1752	-0.0841	0.0406	0.0037	-0.0074	0.0055	0.0022
z_2 stage 1	-0.0004	0.0028	-0.0167	-0.0246	0.0600	0.0849	0.0071	0.0139	0.0139	0.0071	0.0849	0.0600	-0.0246	-0.0167	0.0028	-0.0004
z_2 stage 2	-0.0025	0.0080	0.0130	-0.0165	-0.0338	-0.0109	0.0028	0.0050	0.0050	0.0028	-0.0109	-0.0338	-0.0165	0.0130	0.0080	-0.0025

Table 3: Coefficients of the designed filter

The filters shown above are symmetric about the centre (which is formed by the coefficients for 7th order and 8th order), therefore the phase response is guaranteed to be linear. The repeated coefficients also raise the possibility of saving of components through exploitation of symmetry. A filter order of 15, although slightly too high for photonic implementation, will be retained for the example purpose. If the filter order is to be reduced at this stage, then one would merge the separable expressions into a single expression, then use the balanced approximation method illustrated in Part II [2}. Such a procedure would result in an IIR filter design of reduced order.

3.2.2.2 Photonic Implementation of the Separable 2-D Filter

Because we are dealing with 2-D separable structure, we need to decide which 1-D photonic filter structure should be used to implement the four 1-D filter stages. *Table 4* is a comparison of the structures on the number of components required.

In Table 4, the number of splitting stages per OA is used to indicate how many signal splitting stages the optical pre-amplifier must compensate for. If 30 stages must be compensated for as in the direct structure, the amplification factor would need to be $(\sqrt{2})^{30}=32768$ which is clearly unacceptable for OAs. A solution such as including several OAs in cascade would be required in such situations provided the accumulated ASE noises do not surpass the signal level. For the example, transversal filter structure is chosen as the implementation as it appears to be the most efficient structure. Below diagram

[1] ( ) contains the possible alternatives.

shows the schematic diagram for the photonic implementation.

	Binary	Transversal	Direct	Parallel
No of splitting stages per amplifier	10	19	30	10
Total no. of optical phase modulators	34	0	64	96
Total no. of optical attenuators	64	64	64	96
Total no. of optical couplers	126	142	136	184
Total no. of OAs	4	4	4	4
Total total no. of components	228	210	268	380
Remarks	Low attenuation	No phase modulators required	Simple, but inefficient	Inefficient, but low attenuation

Table 4: Comparison of different fiber-optic filter structures

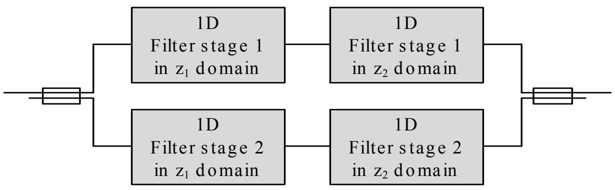

Figure 17: Schematic diagram of the separable 2-D filter.

Because the filter is circularly symmetric, the filter structure and attenuator/amplifier coefficients are the same for the second dimension except for the delay which must be replaced by z_2^{-1}. Physically, this delay would correspond to $(z_1^{-1})^{255}$ as one line delay is same as the entire row of pixels. For the bottom parallel section of Figure 18, the positive subsections and the negative subsections of the two 1-D subsections will need to be exchanged. This is because the binary splitting stages at the start and end of the structure shown in Figure 17 introduce another phase shift of 180° for the lower stage.
Exchanging of subsections can compensate for the phase shift. The filter coefficients are related to the attenuator settings by

$$a_{+i} = \frac{h_i}{\sqrt{2}^{k+2}}, a_{-i} = -h_i \quad \text{where} \qquad (13)$$

$$k = |\text{No of +ve terms - No of -ve terms}|$$

where the number of positive terms is greater than the number of negative terms in the transfer function. a_{+i} and a_{-i} can be exchanged and multiplied by -1 to obtain the settings if the reverse is true. The magnitude response of the filter is plotted in Figure 20.

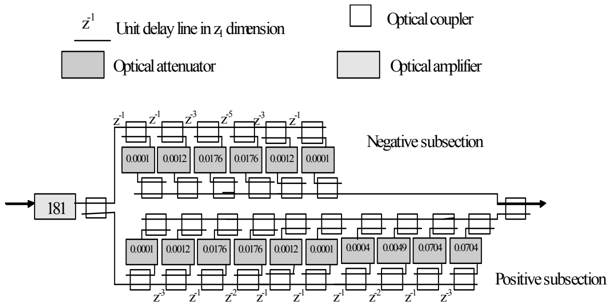

Figure 18: 1-D filter stage 1

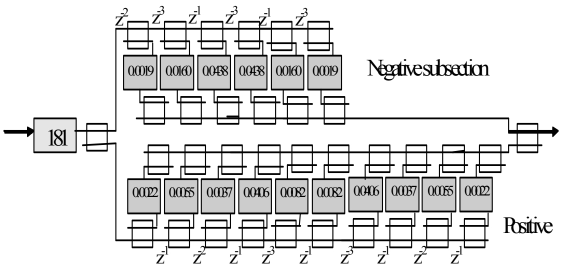

Figure 19: 1-D filter stage 2

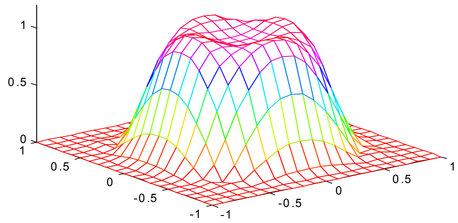

Figure 20: Magnitude response of the designed 2-D filter

3.2.3 Non-Separable Implementation Using Direct Methods

3.2.3.1 Choice of a Design Method

There are two methods to choose from: frequency sampling method and McClellan transformation method. The designs results are shown in Table 5.
The poor performance of McClellan transformation method can be attributed to its low attenuation in the stopband. It is envisaged that with a better transformation function, McClellan transformation method will perform better. For the current design task however, frequency sampling method is chosen. The resultant filter coefficients

are not shown due to the number of coefficients being too large to show here.

Property	Frequency sampling method	McClellan Transformation Method
Resulting design	FIR	FIR or IIR
Requirements	Magnitude specification	1-D Magnitude specifications and a transformation function
Remarks	The filter accuracy depends heavily on the accuracy of frequency sampling	Better transformation functions may result in much improved error
Filter order	20×20	12×12
Magnitude spec. error(%)	7.61	22.68%

Table 5: Comparison of direct design methods.

Reduction of Filter Order Using Balanced Approximation Method

The filter design in the current form requires a 20×20 order filter. This implies that for some signal paths, the signal must go through 20+20 = 40 power splitting stages. A huge pre-amplification factor will therefore be required. A solution to this problem is to use the filter order reduction method of Part II. In this case, the reduced filter is 10×10 resulting in 11+11 = 22 splitting stages which is still large, but again for example purpose, is retained.

Order	0	1	2	3	4	5	6	7	8	9	10
0	0.0015	-0.0046	0.0120	-0.0239	0.0290	-0.0258	0.0246	-0.0250	0.0262	-0.0197	0.0092
1	-0.0045	0.0098	-0.0251	0.0628	-0.0895	0.0991	-0.1155	0.1212	-0.1174	0.0816	-0.0373
2	0.0128	-0.0325	0.0702	-0.1456	0.1945	-0.2091	0.2341	-0.2318	0.2181	-0.1479	0.0706
3	-0.0273	0.0852	-0.1757	0.2946	-0.3387	0.3073	-0.2844	0.2461	-0.2319	0.1602	-0.0844
4	0.0370	-0.1279	0.2614	-0.3985	0.4165	-0.3249	0.2366	-0.1628	0.1567	-0.1155	0.0711
5	-0.0437	0.1660	-0.3410	0.4815	-0.4550	0.2844	-0.1124	0.0031	-0.0112	0.0317	-0.0394
6	0.0520	-0.2103	0.4309	-0.5751	0.5001	-0.2461	-0.0139	0.1646	-0.1466	0.0590	0.0059
7	-0.0531	0.2156	-0.4349	0.5640	-0.4755	0.2151	0.0571	-0.2204	0.2045	-0.1033	0.0123
8	0.0469	-0.1903	0.3841	-0.4995	0.4244	-0.1944	-0.0508	0.1957	-0.1731	0.0941	-0.0124
9	-0.0291	0.1171	-0.2344	0.3020	-0.2527	0.1154	0.0245	-0.1134	0.1145	-0.0534	0.0051
10	0.0117	-0.0487	0.1017	-0.1362	0.1171	-0.0529	-0.0127	0.0468	-0.0388	0.0156	0.0003

Table 6: Denominator coefficients

The filter error is found to be 6.57% which is actually less than the original design! It has therefore been shown that the application of filter order reduction did not result in any notable degradation in the performance. As can be deduced from the fact that there are two sets of filter coefficients, the resulting design is an infinite impulse response design. Another factor which must be taken into account is the fact that neither numerator nor the denominator is symmetrical. No components are duplicated therefore removing the possibility for saving of components by exploitation of symmetry.

Order	0	1	2	3	4	5	6	7	8	9	10
0	1.0000	-3.3531	6.6740	-9.7374	11.1831	-10.3924	7.8531	-4.7539	2.2164	-0.7249	0.1283
1	-2.9261	9.8116	-19.5290	28.4926	-32.7230	30.4093	-22.9790	13.9103	-6.4854	2.1211	-0.3753
2	5.4318	-18.2134	36.2521	-52.8915	60.7444	-56.4494	42.6564	-25.8220	12.0390	-3.9375	0.6967
3	-7.5706	25.3851	-50.5266	73.7179	-84.6629	78.6767	-59.4526	35.9896	-16.7795	5.4880	-0.9710
4	8.4280	-28.2599	56.2486	-82.0662	94.2508	-87.5866	66.1854	-40.0654	18.6797	-6.1095	1.0809
5	-7.6733	25.7295	-51.2120	74.7179	-85.8115	79.7440	-60.2591	36.4779	-17.0071	5.5624	-0.9841
6	5.7326	-19.2220	38.2595	-55.8202	64.1080	-59.5752	45.0184	-27.2519	12.7057	-4.1556	0.7352
7	-3.4579	11.5948	-23.0783	33.6710	-38.6703	35.9360	-27.1553	16.4385	-7.6641	2.5067	-0.4435
8	1.6190	-5.4287	10.8053	-15.7649	18.1056	-16.8254	12.7142	-7.6965	3.5884	-1.1736	0.2076
9	-0.5366	1.7992	-3.5812	5.2250	-6.0008	5.5765	-4.2139	2.5509	-1.1893	0.3890	-0.0688
10	0.0965	-0.3236	0.6441	-0.9398	1.0793	-1.0030	0.7579	-0.4588	0.2139	-0.0700	0.0124

Table 7: Numerator coefficients

3.2.3.3 Photonic Implementation of Non-Separable Filters

As the filter structure is in form of IIR filter, the structure proposed in *Section 2* must be employed in realizing the fiber-optic filter. The filter shown in Figure 1 will form the subsections of the IIR structure shown in Figure 14. The actual filter schematic diagram is omitted as the diagram will be too cluttered to make any clear statement of the structure of the filter. The attenuator settings can be calculated by considering how many splitting stages the signal being attenuated by the coefficient module must go through. The relationship between the attenuator settings a_{ij} and the filter coefficients h_{ij} is given by

$$a_{ij} = \frac{h_{ij}}{\sqrt{2}^{k_i+2j+2}} \tag{14}$$

where k_i is the order of the filter in n_1 dimension. The filter statistics are given in *Table 8*. The filter is shown to be very inefficient compared to the separable implementations as the number of components required is very much higher.

Clearly, the trade off for the good error response is the large number of components required to implement the design in practice.

Factor	2-D Direct Form
No of splitting stages per amplifier	33 (worst case)
No. of OPMs	242
No. of optical attenuators	242
No. of optical couplers	528
No. of OAs	2
Total no. of components	1014
Remarks	Very small error

Table 8: Direct structure filter statistics

The filter magnitude response is shown below in *Figure 21*. Although the passband contains some irregularities, the phase response of IIR filters designed using balanced approximations remains approximately linear (refer to Section 6) therefore satisfying the phase requirement of the specification.

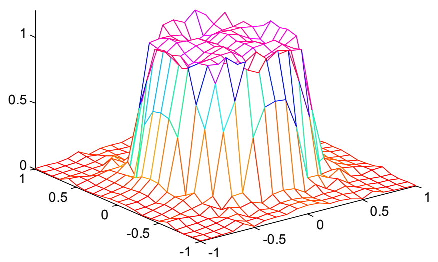

Figure 21: Normalized filter magnitude response

3.2.4 Comparison of Matrix Decomposition Method Design and Direct Method Design

Although we have started out with the exactly the same specification, the two design results are strikingly different. Direct method results in a filter with over 1000 elements whereas matrix decomposition method results in a filter with around 200 elements. The error is also different. Various factors such as performance and economy must be taken into account when deciding which structure to implement. The table below shows the comparison of the two methods using the results obtained for this example. For the example, the better implementation technique would be matrix decomposition method as it offers a filter of reasonable performance with smaller number of components.

3.3 Possible Areas of Applications

The fiber-optic and integrated photonic signal processing has found applications in areas such as optical matrix multiplications, convolutions and broadband signal processing, time division multiplexing in the femtosecond time resolution, high order optical correlation, higher order spectrum analysis for amplitude and phase, spatial and time photonic Fourier transformation. Because of the wideband properties of optical fibers and integrated photonic waveguides, PSP technique makes a natural processing architecture for fiber transmission systems carrying wide bandwidth data. An application of 2-D fiber-optic signal processing system would be the signal processing of ultra-fast time division multiplexing (TDM) information that is the photonic packet switching technique. Novel time division coding used as photonic headers is also another potential application of M-D PSP. 2-D fiber-optic signal processing architecture can be therefore a natural candidate for the processing of ultra-fast signals transmitted over optical guided media.

Property	Direct design method	Matrix decomposition method
No of splitting stages per OA	33(worst case)	19
No of OAs	2	4
No of attenuators	242	64
No of couplers	528	142
No of phase modulators	242	64
Total no of components	1014	210
Error(%)	6.57	8.27
Remark	Very good transition band and low error	Economical

Table 9: Comparison of direct method and matrix decomposition method

Further capabilities of 2-D fiber-optic signal processing can be explored by considering multiplexing of signals by some form of wavelength division multiplexing. By transmitting the entire frame of the ultra-fast signals at once, it becomes possible to process the signal without using delays and by just using optical attenuators and OAs. However there are many practicalities to be overcome in realizing such as system and further works is required.

4. Concluding Remarks

The objective of the research presented in this three part series is to explore possible ways of realizing a 2-D signal processing system using fiber-optic signal processing architecture. A general technique for designing a 2-D filter is illustrated and numerous examples of utilization of the

technique are given. Although the discussion is focused on fiber-optic systems, the design procedure for 2-D filters are just as applicable to any other signal processing architectures. For example, the 2-D filter order reduction method given in Part I [1] and Part II [2] can be used to simplify 2-D lightwaves systems which may or may not be fiber-optic systems.
The design of 2-D filters is classified into two different classes. One class used matrix decomposition to reduce the design of 2-D filters into a set of 1-D filter design procedures. The other class used direct extensions of 1-D filter design methods. It is found that neither has a distinctive superiority over another and that the designer has to choose what is the best for the particular application, most likely by designing both and comparing the performances. All of the design procedures are implemented using the MATLAB™ programming language.
Among the matrix decomposition methods, the multiple stage singular value decomposition method of Part II [2] performed the best whereas for direct methods, frequency sampling method produced filters with smallest errors. However, the result should be taken with caution as there are many factors to be considered before declaring one method superior over another. The differences between the various methods are outlined in *Table 2* and *Table 5*. A 2-D Filter order reduction method is applied to make fiber- and integrated optic signal processing more feasible. The technology allows the filter designer to produce filters of orders that are implementable in practice without sacrifices in performance.
Different possible filter structures are proposed and illustrated for photonic implementation of 2-D filters. Most of the filter structures discussed can be used in 1-D coherent fiber-optic signal processing and are not limited to 2-D coherent fiber-optic signal processing. Some of the proposed structures such as transversal structure are extremely efficient in the number of components used to achieve a certain performance requirement. To make the efficient structures possible, the phase shifting property of optical couplers when the incoming lightwaves is cross-coupled is utilized. Filter structures of FIR and IIR types are also shown and examples are given. It is evident that the fiber-optic signal processing technology presents a new direction in the usage of optical fiber, lasers, and photonics technologies which are evolving very fast. In [6, 10] an incoherent signal processing system operating at 100 MHz is demonstrated. The authors note that the raising this capability to over 10 GHz is a relatively straightforward procedure involving shorter fiber lengths and lasers and detectors with faster rise and fall time. They also note that although conventional digital signal processing and analog signal processing techniques are limited in their usefulness for signal bandwidths exceeding one or two GHz. Current research efforts on fiber-optic signal processing on lightwaves of millimeter wavelength region will allow signal processing at bandwidths of up to 100GHz even to THz region if parametric amplification is employed. The field of 2-D signal processing which requires ultra-fast processing capability has a great deal to gain from the usage of the high speed processing capability of fiber-optic architectures. In particular especially with the fast pace of research and inventions of photonic circuits reaching the nano-scale employing photonic crystal wave guiding techniques will allow multi-dimensional processing in the photonic domain flourishing in the near future.
Furthermore advanced modulation formats for 100 Gb/s such as quadrature amplitude modulation (QAM), quadrature differential phase shift keying (QPSK) schemes are two dimensional phase coding on information signals. The proposed multi-dimension optical signal processing may offer significant advantages for demodulation at the front of the optical receivers.

References

[1] Le Nguyen Binh, **“**Multi-dimensional Photonic Processing: A Discrete-Domain Approach: Part I”, Int. J. Comp Sc. And Network Security, Vol. , No.5, May 2009.

[2] Le Nguyen Binh, **“**Multi-dimensional Photonic Processing: A Discrete-Domain Approach: Part II”, Int. J. Comp Sc. And Network Security, Vol. , No.5, May 2009.

[3] Ngo N.Q. and L.N. Binh, *“Programmable incoherent Newton-Cotes Optical Integrator”*, *Optics Communications*, vol. 119, 1995, pp. 390-402.

[4] H. F. Taylor, “*Application of Guided-Wave Optics in Signal Processing and Sensing,*” Proc. IEEE, vol. 75, No 11, pp1524-1535, November 1987

[5] See for example B. Moslehi, J. W. Goodman, M. Tur, and H. J. Shaw, “Fiber-Optic Lattice Signal Processing,” Proc. IEEE, Vol. 72, No 7, July 1984;

[6] C.K. Marsden and J.H. Zhao “*Optical filter analysis and design: A digital signal processing approach*”, J Wiley, NY 1999.

[7] J. Capmany, and J. Cascon, “Optical programmable transversal filters using fiber amplifiers,” Electronic Letters, vol. 28, pp 1245-1246, 1992

[8] W. Lu, H. Wang, and A. Antoniou, “*Design of two-dimensional digital filters using singular-value decomposition and balanced approximation method*,” IEEE Trans. Sig. Proc., VOL. 39, No. 10, Oct 1991, pp 2253-2262.

[9] LN Binh and D. Trowser *“OPTMASON”: A program for Automatic Derivation of the optical transfer functions of photonic circuits from their connection graphs, Report No.*MECSE-34-2004, 2004, http://www.ds.eng.monash.edu.au/techrep/reports/post2003index.html.

[10] Le Nguyen Binh “Photonic Signal Processing”, CRC Press, Boca Raton, USA, 2009.

Le Nguyen Binh received the degrees of B.E. (Hons) and PhD in Electronic Engineering and Integrated Photonics in 1975 and 1980 respectively, both from the University of Western Australia, Nedlands, Western Australia. In 1981 he joined in the Department of Electrical Engineering of Monash University after a three year period with CSIRO Australia as a research scientist developing parallel micro-computing systems. He was appointed to reader of Monash University in 1995. He has worked for and lead major research and development programs in the Department of Optical Communications of Siemens AG Central Research Laboratories in Munich and the Advanced Technology Centre of Nortel Networks in Harlow, U.K. He was a visiting professor in 2007-2008 and is currently an adjunct professor of the Faculty of Engineering of Christian Albretchs University of Kiel, Germany.
Dr. Binh has published more than 200 papers in leading journals and international conferences and three books, one on Photonic Signal Processing (2007) and the other on Digital Optical Communications (2008) both by the publishing house CRC Press (Florida, USA). His third book scheduled for publication in 2009 is “Optical Fiber Communications Systems: Principles, Practices and MATLAB Simulink Models” also by the same publisher. He is an associate editor of the International Journal of Optics and the Research Letters in Optics, Series Editor of the Series “Optics and Photonics” for CRC Press Pub.
His current research interests are Tb/s optical transmission systems and networks, especially advanced modulation formats and associate electronic equalization methods, photonic signal processing, integrated photonics and multi-bound soliton fiber lasers.