Establishing the Straightness of a Line for Radial Distortion Correction through Conic Fitting

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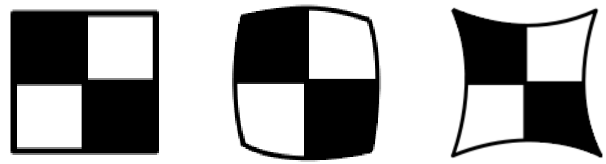
Summary
Establishing straight lines in plumb-line based radial distortion correction is very important as the straight-line will determine the success of distortion correction operation. Various researches have been made to describe the line using linear fitting or polynomial fitting assuming that these fittings will augur well for distortion correction operation. However linear or polynomial fitting may not work in certain situation such as in vertical line. This research proposes a conic fitting method for establishing a straight-line. Fitting to a conic is more robust in a sense it can accommodate lines of different orientation.

Key words:
Radial Distortion, Conic Fitting, Straight Line Fitting

1. Introduction

Distortion in an image occurs when a scene acquired using a camera is deformed due to the bending of light as the light goes through the camera lens. The distortion is most noticeable near the border of the image [6]. The distortion can be big or small depending on the quality and the type of camera lens. Low-cost camera is known to have lens that distort light considerably. This will cause the distortion to be apparent on the image. Therefore, distortion in an image acquired using camera need to be corrected before further processing can be done. This is to ensure that the image processed is more accurate and free from distortion noise. Distortion in image caused by the lens can be of two types namely tangential and radial. Researches have shown that of the two distortion attributes, radial distortion accounted for 90% of the distortion error [2], [14]. Since radial distortion causes the most error, this paper focuses on correcting the radial distortion as studies indicated that this is sufficient in handling standard lenses [14]. As such, for brevity the paper will be using the term distortion to refer to radial distortion unless stated otherwise.

Radial Distortion can be of two types namely barrel and pincushion. The barrel and pincushion effect can be best described using the following diagram:



(a) (b) (c)

Fig. 1. Types of Distortion. (a) Normal Image of Squares (b) Barrel Distortion (c) Pincushion Distortion.

Based on Fig. 1, image of squares can be radially distorted into a barrel-like image (a) or pincushion-like image (c) depending on the lens attributes.

There are various approaches employed to correct distortion correction. The approaches includes comparing the image and real world location, searching and correcting straight lines and constructing constraints based on different views that are able to undistort the image deformation.

The method of comparing image object with real world object is known as point correspondence problem. Using this method, an image object will be compared with real world representation. Often time, an image of a calibration pattern board consists of squares or dot with known value will be acquired and the locations of these dots or squares are compared with the real one. The approach can be seen implemented in [2] and [11]. The point correspondence approach main setback is the need for mapping between real world and image attributes. The unavailability of real world object information will render this approach useless.

The second approach of correcting distortion as demonstrated by [7] and [13] requires usage of multiple views, While the approach can increase the accuracy, requirement for multiple views will increase the amount of computation, complexities, input needed and not to be forgotten noise per-view that need to be handled.

The third approach is popularly known as plumb-line method as introduced by [3] and used by [1], [5] and [9]. Although it has been introduced 30 years ago, the fact that it is still being used up to this day shows that the technique

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remains relevant. Basically the technique will identify a distorted straight-line in an image. A transformation function coefficients will be established by the process of straightening the selected distorted line. Once established, the transformation function will be used to undistort the whole image. The advantage of this approach is point correspondences are not needed therefore the approach only relies on things that exist on the image. Moreover, the approach is simple in the sense that the main objective is to straighten a distorted straight line. No multi-views or specially arranged objects are needed.

As the plumb-line method depend on the straight line, detection and description of the straight-line becomes important. While existing techniques may rely on linear or polynomial fittings for establishing the straight line, polynomial and straight line fittings may require several fitting function to suit different orientations as will be explained later in this paper.

As such, this paper, while employing the plumb-line technique, will propose a solution of employing conical fitting to describe the straight-line. The usage of conical fitting is more practical as by using the proposed approach, lines of different orientation can be described in one function thus making the task of evaluating the straightness of a line easier.

Another issue with the distortion correction process is the distorted image pixel and its undistorted counterparts need to be of 1-to-1 relationship. In other word, cases where a pixel in distorted image cannot be mapped to the undistorted counterpart should not occur. As plumb-line technique evaluates selected line, the parameters found during the evaluation must be applicable to all pixels in the image without lost of data especially of rounding error. Assuming that image regeneration is done through transformation, the technique used cannot afford to have different values during line evaluation and overall transformation. Having said that, the paper proposed on employing backward mapping of the straight-line as a method to establish the transformation parameters. By having backward mapping of the straight-line, the mapping of the distorted and undistorted image can be assured. To alleviate the problem of using too much computer resources, sub-sampling will be employed before backward mapping is done.

This paper will review existing works in plumb-line method for radial distortion correction in section 2. The proposed method will be explained in detail in section 3, experiments will be detailed in section 4 followed by discussions and conclusions in section 5 and 6 respectively.

2. Literature Review

As plumb-line method relies on the availability of straight line, the quality of the straight line is very important. This is due to that the detected edges may belong to the line or it may as well belong to random noise abundantly available on the image. Various approaches have been proposed for the establishment of the straight line. [14] employs combination of polynomial and linear fitting in establishing straight line. Linear fitting can be tricky for a near vertical line as standard least square fitting approach cannot be conducted due to division by zero. [12] proposes for a piecewise function to handle this situation. In his paper, a check will be conducted to determine whether the line is vertical or horizontal. A function used to handled the vertical line is different from the one used for other lines. Our proposed conical fitting will not have to offer a piecewise function to handle this situation as the proposed fitting can handled both vertical and other line orientations. [14] also employs polynomial fitting. Like linear fitting, polynomial fitting perform poorly in almost vertical situation as illustrated in the following Figure 2:

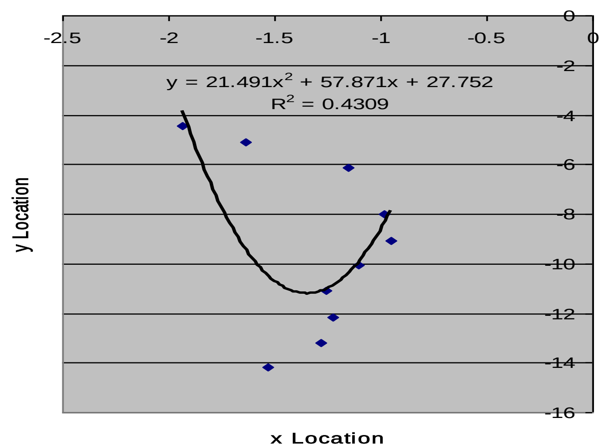

Fig. 2. Polynomial Fitting for a set of curve line using Least Square Approach. (Dots = Data point, Line=Predicted Polynomial Line)

As shown in Fig. 2, fitting for a line with this position will not give a good result if Polynomial Fitting is being used.

[9] on the other hand employs Hough Transform (HT) method to establish straight line. While HT method seems to be free from the axes problem, HT has its own setback. HT requires setting up of polling table that double up as an image by itself. Since the parameters for the polling table are predefined, certain values may be skipped which makes this method to be not comprehensive. Furthermore the computation of sine and cosine requires lots of computing power. Besides, usage of sine and cosine function will result in rounding errors.

Strand in [10] moves away from linear and polynomial fitting by introducing circle fitting. He claimed that distorted straight lines are circular arcs. A conic fitting seems to be a natural progression from Strand's approach as the image analyzed may not contained an ideal circle-based attributes as stated by Strand. In other word conic coefficients need to be introduced to handle situations where 4 coefficients (of Circle) are not enough to describe the line due to the existence of noise.

Clause in [4] handles straight lines as conic through rational function. In Clause, a set of lines are being used. The proposed solution differs from Clause by the fact that the proposed solution will used only a single line for distortion correction instead of multiple lines. The proposed solution tries to minimize on the dependency of the available line so that it can operate on the most minimum line possible.

Most of the researchers employ iterative method in establishing the distortion parameters. [5] utilizes Levernbeg-Marquart for distortion correction optimization, [12] uses RANSAC, [10] on the other-hand also utilize non-linear minimization through lsqnonlin function found in Matlab. [9] and [14] iteratively increment the parameter value and store the results in Hough Transform Image/Table. The proposed method will use iterative technique similar to [9] and [14] in our effort to avoid dealing too much with function manipulation that can result in rounding error. To compensate from the prospect of utilizing too much of computing power, the image will be sub-sampled before the iteration begin.

3. Methods

The proposed radial distortion correction uses the following model as implemented by Devernay in [5]:

$$x_u = x_d(1+\kappa_1 r_d^{\ 2}) \quad (1)$$

$$y_u = x_u(1+\kappa_1 r_d^{\ 2}) \quad (2)$$

$$r_d = \sqrt{x_d^{\ 2} + y_d^{\ 2}} \quad (3)$$

where:

x_u : post distortion correction value

x_d : pre distortion correction value

κ_1 : distortion parameter

While higher degree of parameters can be used $(k_2 r_d^{\ 4}, k_3 r_d^{\ 6} ..)$, usage of $\kappa_1 r_d^{\ 2}$ is enough to achieve the accuracy of 0.1 pixels [5], [9]. By setting the $\kappa_1 > 0$ barrel effect can be produced, setting $\kappa_1 < 0$ on the other hand pincushion effect can be produced.

To avoid 'holes in the picture' effect, the proposed method will be using backward mapping approach to recreate image after distortion correction. Backward mapping requires the creation of output file before backtracking to the original image to find the corresponding pixel value for each pixel location. The algorithm used for backward mapping is as follows, adapted from [8].

```
create g = subsampled(source,n) //resize by n
set κ1 to a preset value (κ1<0)
create an output image g'    // same size as g
for all pixel coordinate (xu,yu) in g' do
        ru = sqrt(xu^2 + yu^2)
        xd = xu(1+κ1 ru^2)
        yd = yu(1+κ1 ru^2)
        if (xd,yd) is inside g do
            g'(xu,yu) = g(xd,yd)
        else
            g'(xu,yu) = 0
```

Fig. 3. – Backward Mapping from Undistorted Image to Distorted Image

To reduce the amount of computation, sub sampling will be done prior to evaluation. Reduction of image size of equal proportion will not change the nature of a straight line which will still be straight. Therefore employing distortion correction function on sub-sampled image will suffice in establishing the κ_1 value while preserving the coordinate value to be the same as the one used for the final image.

In establishing the suitable κ_1 value, initial κ_1 will be set. A distorted straight line ℓ will be first extracted from the sub-sampled image. The line will be manually selected by the user. Distortion correction will be done in line ℓ and evaluation of the line quality will be conducted.

The quality of a line will be evaluated by fitting the line ℓ to a conic. Conic fitting aims to solve for conic function $f(x) = Ax^2 + Bxy + Cy^2 + Dx + Ey + F$ with constraints of $A^2 + B^2 + C^2 + D^2 + E^2 + F^2 = 1$. Conic coefficient can be represented in a matrix form C as follows:

$$C = \begin{bmatrix} F & \frac{D}{2} & \frac{E}{2} \\ \frac{D}{2} & A & \frac{B}{2} \\ \frac{E}{2} & \frac{B}{2} & c \end{bmatrix} \quad (4)$$

While C will be most useful in evaluating for circles, ellipses and hyperbolas, the degenerate condition where C suggests either a point, single line or two intersecting line can be useful in our effort to establish a straight line. The conic coefficients of C can be utilized to establish a straight line by calculating the determinant of C (det(C)) and the determinant of C_{23} (det(C_{23})) [15]. A det(C) equals 0 suggests a degenerate condition of either a point, single straight line or two intersecting line. A det(C_{23}) equals 0 on the other hand suggests that the given data will be a straight line thus factors out the point and two intersecting line possibilities. Given these preliminary facts, the paper proposed a summation of the determinants (det(C) and det(C_{23})) as a tool to establish the straight line. By summing the absolute value of these determinants, a low value will indicate a straight line. In other word, straightest line can be established if we solve the following:

$$\textit{arg min} \; |\det(C)| + |\det(C_{23})| \quad (5)$$

The overall flow of the proposed method will be as follows:

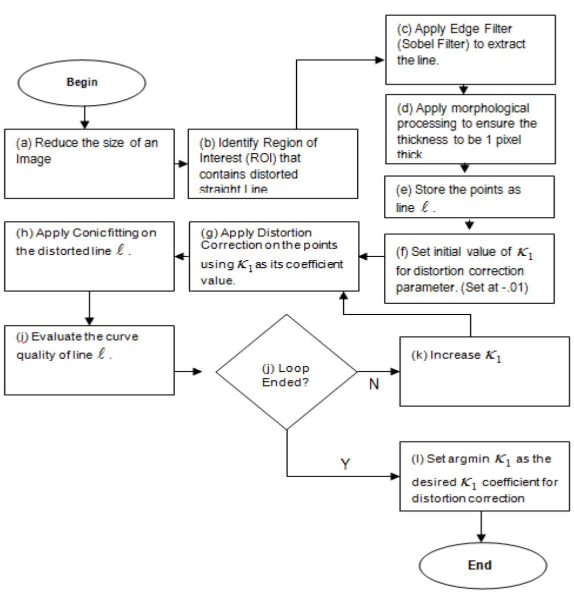

Fig. 4. Distortion Correction using Conic Fitting Method

4. Experiment

To test the proposed method, five pictures will be evaluated. Each picture consists of distorted straight lines with different orientation:

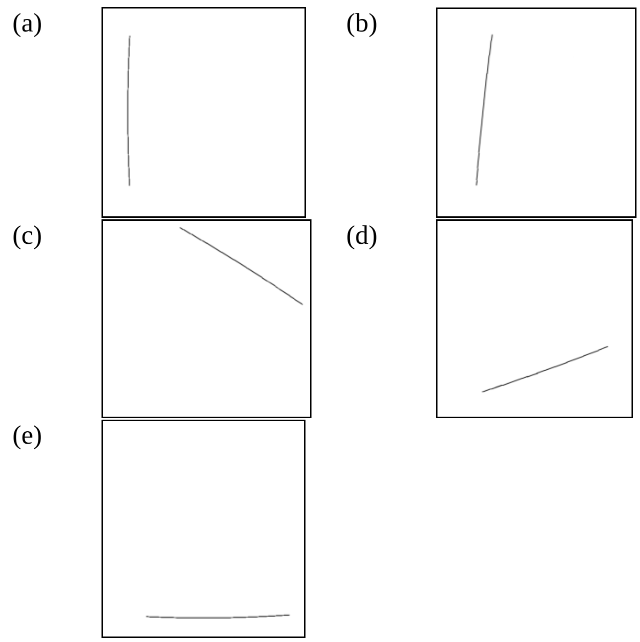

Fig. 5. Test Pictures with various line orientation (a) Pic-A (b) Pic-B (c) Pic-C (d) Pic-D (e) Pic E

The iteration will be initialized at κ_1 =0 and will be decremented by -.01. The iteration will stop at -0.1. The selection is based on observation that κ_1 =0 is the starting image where barrel distortion appears and κ_1 =-0.01 produces a distinct pincushion distortion. Iteration interval is selected to be at -.01 because based on observation, this interval results in more apparent changes. The interval value can be set at lower terms as need arises.

As a comparison, polynomial fitting will be conducted on these distorted lines. Coefficient of Correlation values (R^2) and the polynomial's full equation will be recorded in an attempt to find the straightest line. The Correlation of Coefficient (R^2) will be applied using the following formula:

$$R^2 = 1 - \frac{SSE}{SST} \quad (6)$$

Where:

$$SSE = \sum_{i=1}^{n} (y_i - f(x_i))^2 \qquad (7)$$

$$SST = \left(\sum_{i=1}^{n} {y_i}^2\right) - \frac{\left(\sum_{i=1}^{n} y_i\right)^2}{n} \qquad (8)$$

The high R^2 value indicates the quality of the line fitting. The coefficient value of near 0 for x^2 in the polynomial equation indicates that the fitting is a straight line.

The quality of conic fitting will be analyzed according to the approach as stated by section 3. The minimum value for *arg min* $|\det(C)| + |\det(C_{23})|$ will imply that the image contains the straightest line compared to other values.

To complete the test, a natural image is acquired, conic fitting will be done and the image will be corrected using κ_1 found from the conic fitting. The original distorted image will be displayed side by side with the undistorted image.

5. Results and Discussion

The results can be best described using the following tables.

Table I Fitting for Pic-A

Item	κ_1	Poly-fitting (R^2)	Conic-Fitting
1	0.00	y = -7.61 x^2 + 974.85 x – 30967 (0.020)	1.98E+28
2	-0.01	y = -7.68 x^2 + 968.63 x - 30283 (0.020)	3.18E+20
3	-0.02	y = -12.94 x^2 + 1592.5 x - 48763 (0.024)	1.49E+20
4	-0.03	y = 15.06 x^2 - 1841.8 x + 56637 (0.013)	4.40E+20
5	-.004	y = 20.29 x^2 - 2408.9 x + 71723 (0.015)	3.73E+20
6	-0.05	y = 14.66 x^2 - 1678.8 x + 48285 (0.009)	**1.53E+19**
7	-0.06	y = 0.09 x^2 - 9.16 x + 481.02 (0.000)	8.68E+19
8	-0.07	y = -0.28 x^2 + 31.191 x - 635.48 (0.001)	3.37E+20
9	-0.08	y = 1.1 x^2 - 110.49 x + 3032.9 (0.002)	2.97E+20
10	-0.09	y = 0.2 x^2 - 18.31 x + 657.85 (0.001)	7.22E+23
11	-0.10	y = -0.25 x^2 + 23.77 x - 322.57 (0.002)	6.27E+27

Bold indicates minimum value

Table I evaluates a vertical line. Looking at the polynomial fitting, all the iteration (itern) register a very weak R^2 value for the polynomial fitting. The *arg min* $|\det(C)| + |\det(C_{23})|$ evaluation yield a value of **1.53E+19** during the 6th iteration.

Table II Fitting for Pic-B

Item	κ_1	Poly-fitting (R^2)	Conic-Fitting
1	0.00	y = 0.067 x^2 - 24.82 x + 2228.2 (0.999)	1.24E-09
2	-0.01	y = 0.05 x^2 - 21.84 x + 2049.7 (0.999)	2.08E-09
3	-0.02	y = 0.02 x^2 - 14.68 x + 1621.9 (0.998)	1.27E-09
4	-0.03	y = 0.023 x^2 - 14.38 x + 1594.8 (0.999)	6.61E-10
5	-.004	y = -0.006 x^2 - 7.55 x + 1194.8 (0.998)	1.62E-10
6	-0.05	**y = -0.02 x^2 - 4.53 x + 1016.6** (0.998)	**1.33E-10**
7	-0.06	y = -0.038 x^2 - 0.34 x + 775.59 (0.999)	3.50E-10
8	-0.07	y = -0.058 x^2 + 4.15 x + 519.62 (0.998)	4.20E-09
9	-0.08	y = -0.07 x^2 + 6.75 x + 375.46 (0.999)	3.07E-09
10	-0.09	y = -0.091 x^2 + 11.36 x + 123.71 (0.998)	1.63E-08
11	-0.10	y = -0.10 x^2 + 13.3 x + 22.478 (0.997)	1.79E-08

Bold indicates minimum value

Table II evaluates a vertical line but slightly leaning to the right. This time, a better fitting of polynomial can be seen. Looking at the polynomial fitting, all the iteration (itern) register a very strong R^2 value for the polynomial fitting with values ranging from 0.997 to 0.999. The *arg min* $|\det(C)| + |\det(C_{23})|$ evaluation yields a value of **1.33E-10** during the 6th iteration.

Table III Fitting for Pic-C

Item	κ_1	Poly-fitting (R^2)	Conic-Fitting
1	0.00	y = 0.0002 x^2 + 0.5193 x - 85.616 (0.999)	6.23E-08
2	-0.01	y = 0.0002 x^2 + 0.5477 x - 90.92 (0.999)	3.64E-08

3	-0.02	y = 0.0001 x^2 + 0.5729 x - 96.393 (0.999)	2.54E-08
4	-0.03	y = 6E-05 x^2 + 0.6243 x - 105.38 (0.999)	5.71E-09
5	-.004	**y = 3E-05 x^2 + 0.644 x - 109.99** (0.999)	**9.92E-10**
6	-0.05	y = -5E-05 x^2 + 0.6932 x - 118.99 (0.999)	2.41E-09
7	-0.06	y = -9E-05 x^2 + 0.7219 x - 125.09 (0.999)	9.32E-09
8	-0.07	y = -0.0002 x^2 + 0.7738 x - 134.79 (0.999)	2.28E-08
9	-0.08	y = -0.0002 x^2 + 0.812 x - 142.48 (0.999)	3.59E-08
10	-0.09	y = -0.0003 x^2 + 0.864 x - 152.46 (0.999)	3.40E-08
11	-0.10	y = -0.0004 x^2 + 0.9129 x - 161.99 (0.999)	5.66E-08

Bold indicates minimum value

Table III evaluates a line which is placed diagonally at the top right corner of the image. Like Table II, Table III records a good R^2 value for the polynomial fitting. The value range however is too small that all the iteration record a value of 0.999 for R^2 . The *arg min* $|\det(C)|+|\det(C_{23})|$ evaluation on the other hand yields a better distinction between iteration with minimal value to be recorded at fifth iteration (9.92E-10).

Table IV Fitting for Pic-D

Item	κ_1	Poly-fitting (R^2)	Conic-Fitting
1	0.00	y = -0.0001 x^2 - 0.2918 x + 471.26 (0.999)	1.87E-10
2	-0.01	y = -9E-05 x^2 - 0.3067 x + 474.53 (0.999)	1.52E-10
3	-0.02	y = -7E-05 x^2 - 0.3231 x + 477.44 (0.999)	7.65E-11
4	-0.03	y = -5E-05 x^2 - 0.3326 x + 479.35 (0.999)	4.91E-11
5	-.004	**y = -1E-05 x^2 - 0.3554 x + 483.1** (0.999)	**3.42E-11**
6	-0.05	y = 3E-05 x^2 - 0.3799 x + 486.95 (0.999)	5.76E-11
7	-0.06	y = 5E-05 x^2 - 0.3916x + 489.32 (0.999)	4.02E-11
8	-0.07	y = 9E-05 x^2 - 0.4154 x + 493.17 (0.999)	1.10E-10
9	-0.08	y = 0.0001 x^2 - 0.4319x + 496.28 (0.999)	2.94E-10
10	-0.09	y = 0.0002 x^2 - 0.457 x + 500.34 (0.999)	3.48E-10
11	-0.10	y = 0.0002 x^2 - 0.4817 x + 504.51 (0.999)	4.93E-10

Bold indicates minimum value

Table IV is a variation of Table III where a straight line is placed at bottom right diagonal position. Like Table III, polynomial fitting yields a very high R^2 value. As R^2 is too high, three decimal points are not enough to differentiate between iterations. Conic fitting is capable of producing a very fine results. As such, the *arg min* $|\det(C)|+|\det(C_{23})|$ can be detected to yield a value at the fifth iteration (3.42E-11).

Table V Fitting for Pic-E

Item	κ_1	Poly-fitting (R^2)	Conic-Fitting
1	0.00	y = -0.0002 x^2 + 0.0802 x + 445.97 (0.975)	6.70E-10
2	-0.01	y = -0.0001 x^2 + 0.0667 x + 449.42 0.938)	1.96E-10
3	-0.02	y = -9E-05 x^2 + 0.0473 x + 453.56 (0.838)	2.45E-10
4	-0.03	y = -4E-05 x^2 + 0.0216 x + 458.22 (0.633)	1.70E-11
5	-.04	**y = -1E-05 x^2 + 0.0061 x + 461.9 (0.049)**	**8.13E-12**
6	-0.05	y = 4E-05 x^2 - 0.0188 x + 466.68 (0.589)	7.43E-11
7	-0.06	y = 1E-04 x^2 - 0.0475 x + 472.03 (0.889)	1.34E-10
8	-0.07	y = 0.0002 x^2 - 0.0769 x + 477.56 (0.952)	1.76E-10
9	-0.08	y = 0.0002 x^2 - 0.1098 x + 483.68 (0.972)	1.61E-09
10	-0.09	y = 0.0003 x^2 - 0.1436 x + 490.03 (0.980)	4.93E-09
11	-0.10	y = 0.0004 x^2 - 0.1841 x + 497.32 0.985)	1.40E-08

Bold indicates minimum value

Finally Table V is the result on fitting for almost horizontal line at the bottom of the image. As the polynomial fitting consider smaller data interval unit, the fitting results in some iterations to yield a weak R^2 . Distinction using conic fitting remains unperturbed where the straightest most line can be found at fifth iteration.

Polynomial fitting in general are not able to describe the line properly especially for the vertical line (Pic-A) and also for near-horizontal line (Pic-E). The R^2 value of vertical line (Pic-A) is a poor indicator for the fitting as many recorded R^2 value of under 0.01 for vertical fitting. In addition R^2 also is not indicative description for near-horizontal line (Pic-E). At κ_1 = -0.04 the R^2 recorded a mere value of 0.049 which make line identification process to be inconclusive. The problem with line orientation prompts paper such as [12] to propose piecewise function to check for line orientation where vertical or horizontal lines are handled differently.

Using conic fitting approach, the line orientation is no longer an issue. Using the rule of *arg min* $|\det(C)|+|\det(C_{23})|$, straight line can be consistently identified. This approach can accommodate various line orientations from vertical to diagonal to horizontal lines. Whilst the polynomial fitting cast doubts on vertical or horizontal line by reporting low R^2 value, the conic fitting approach is consistent with its report.

The proposed generalization seems to be at odd with vertical line as in Pic-A. Admittedly the *arg min* $|\det(C)|+|\det(C_{23})|$ of a vertical line is very big (Pic-A: 1.53E+19). A further evaluation shows that the fitting in Table I is either an ellipse or hyperbola depending on your thresholding of the coefficients. However, looking at the determinant value, the suggested ellipse or hyperbola will be very big given the value of the degree involved (10^{19}). Given the relative small range of our data (x:{60, ... , 70} and y:{150, ... , 250}), the fitting for that small range will yield an almost straight line. In addition, the idea of the line as part of a circle is already mooted by [10]. The proposed method looks at a more general perspective of conic equation instead of confining on circles.

Base on these facts, the rule of *arg min* $|\det(C)|+|\det(C_{23})|$ can still be used to establish a straight line as long as the range of the evaluated data is small. Moreover the proposed method emphasized on the minimum value of the evaluated data without putting the requisite that the sum must always equals to a value that is near to zero. In addition the situation of *arg min* $|\det(C)|+|\det(C_{23})|$ with big positive value can be interpreted as the possible existence of vertical line. Furthermore, the unique situation (a very big determinant values) only occurs at almost vertical situation, other line orientation from near-vertical (Pic-B), diagonals (Pic-C, Pic-D) and near-horizontal lines register *arg min* $|\det(C)|+|\det(C_{23})|$ value of almost zero which is in-line with original definition of straight line in conic equation.

The following Fig. 6 illustrates the usage of the proposed method applied on the manually selected near vertical line of the door.

(a)

(b)

Fig. 6 (a) Distorted Image (b) Corrected Image using Conic Fitting as Indicator for Straight Line (κ_1 value of -0.02)

Picture (a) is acquired using consumer digital camcorder (Panasonic NVS - 250). A conic fitting is done with κ_1 = -0.02 yields the straightest line possible. Distortion correction using backward mapping is applied using κ_1 of -0.02 with its corrected image is displayed at (b). Apparent change is visible on the outline of the door available in the image. The corrected version of the door no longer has the bulging effect caused by the camera lens which can be seen on the original image.

6. Conclusion

As a conclusion, evaluating a straight line by taking advantage of the degenerate cases that occurs during conic fitting is better than using the polynomial approach. Results as can be seen from the results in Table I – V shows that the polynomial fitting requires different indicators if straight line is to be evaluated in this way. In addition, usage of *arg min* $|\det(C)|+|\det(C_{23})|$ enables us to evaluate distorted straight lines of various orientations including the critical vertical and horizontal lines without having to introduce piecewise scheme. While sub-sampling the image promises faster computation, the impact of such technique with different settings is yet to be truly studied and can potentially be the future direction for the proposed method.

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