A Low-Complexity Algorithm for Improving Computing Power of Multiuser Receivers for Next Generation Wireless Systems

Syed S. Rizvi†, Aasia Riasat††, and Khaled M. Elleithy†††
srizvi@fuee.bridgeport.edu aasia.riasat@iobm.edu.pk elleithy@bridgeport.edu
†Computer Science and Engineering Department, University of Bridgeport, Bridgeport, CT, 06604 USA
††Computer Science Department, Institute of Business Management, Karachi, 78100, Pakistan
††† Computer Science and Engineering Department, University of Bridgeport, Bridgeport, CT, 06604 USA

Summary
A new transformation matrix (TM) algorithm for reducing the computational complexity of multiuser receivers for DS-CDMA wireless system is presented. Next generation multiuser wireless receivers will need to use low computational complexity algorithm in order to perform both fast signal detection and error estimation. Several multiuser signal detection algorithms have proposed for next generation wireless receivers, which are designed to give good performance in terms of signal to noise ratio (SNR) and bit error rate (BER), are discarded for a direct implementation as they have high computational complexity. In this paper, we propose a new low-complexity TM algorithm that can be used to perform fast signal detection for multiuser wireless receives. This reduction in computational complexity would likely to give us a considerable improvement in the performance of multiuser wireless receivers such as high computing power and low error rate. In addition, we also present a formal mathematical proof for computational complexities that verifies the low-complexity of the proposed algorithm
Key words:
Computational complexity, DS-CDMA systems, Multiuser communications, Wireless receivers

1. Introduction

Code division multiple access (CDMA) has been widely used and accepted for wireless access in terrestrial and satellite applications. CDMA cellular systems use state of the art digital communication techniques and build on some of the most sophisticated aspects of modern statistical communication theory. CDMA technique has significant advantages over the analog and conventional time-division-multiple access (TDMA) system. CDMA is a multiple access (MA) technique that uses spread spectrum modulation where each user has its own unique chip sequence. This technique enables multiple users to access a common channel simultaneously.

Multiuser direct-sequence code division multiple access (DS-CDMA) has received wide attention in the field of wireless communications [4, 8]. In CDMA communication systems, several users are active on the same fringe of the spectrum at the same time. Therefore, the received signal results from the sum of all the contributions from the active users [2]. Conventional spread spectrum mechanisms applied in DS-CDMA are severely limited in performance by MAI [3, 4], leading to both system capacity limitations and strict power control requirements. The traditional way to deal with such a situation would be to process the received signal through parallel devices.

Verdu's [1] proposed and analyzed the optimum multiuser detector and the maximum likelihood sequence detector, which, unfortunately, is too complex for practical implementation, since its complexity grows exponentially as the function of the number of users. Although the performance of multiuser detector is optimum, it is not a very practical system because the number of required computations increases as 2^k, where k is the number of users to be detected. Multiuser detectors suffer from their relatively higher computational complexity that prevents CDMA systems to adapt this technology for signal detection. However, if we could lower the complexity of multiuser detectors, most of the CDMA systems would likely to get advantage of this technique in terms of increased system capacity and a better data rate.

In this paper, we employ a new approach of TM technique that observes the coordinates of the constellation diagram to determine the location of the transformation points (TPs). Since most of the decisions are correct, we can reduce the number of required computations by using transformation matrixes only on those coordinates which are most likely to lead to an incorrect decision. By doing this, we can greatly reduce the unnecessary processing involves in making decisions about the correct region or the coordinate. Our mathematical results show that the proposed approach successfully reduces the computational complexity of the optimal ML receiver.

The rest of this paper is organized as follows. Section 2 describes the state of the art research that has already been done in this area. Section 3 presents both the original ML algorithm and the proposed TM algorithm along with a comprehensive discussion of their computational complexities. The numerical and simulation results are presented in Section 4. Finally, we conclude the paper in Section 5.

2. Related Work

Multiuser receivers can be categorized in the following two forms: optimal maximum likelihood sequence estimation (MLSE) receivers and suboptimal linear and nonlinear receivers. Suboptimal multiuser detection algorithms can be further classified into linear and interference cancellation type algorithms. The figurative representation of the research work that has been done so far in this area is shown in Fig. 1. Optimal multiuser wireless receiver consists of a matched filter followed by a maximum likelihood sequence detector implemented via a dynamic programming algorithm. In order to mitigate the problem of MAI, Verdu [6] proposed and analyzed the optimum multiuser detector for asynchronous Gaussian multiple access channels. The optimum detector searches all the possible demodulated bits in order to find the decision region that maximizes the correlation metric given by [1]. The practical application of this mechanism is limited by the complexity of the receiver [7]. This optimum detector outperforms the conventional detector, but unfortunately its complexity grows exponentially in the order of $\boldsymbol{O}\,(2)^{K}$, where K is the number of active users.

Much research has been done to reduce this receiver's computational complexity. Recently, Ottosson and Agrell [5] proposed a new ML receiver that uses the neighbor descent (ND) algorithm. They implemented an iterative approach using the ND algorithm to locate the region where the actual observations belong. In order to reduce the computational complexity of optimum receivers, the iterative approach uses the ND algorithm that performs MAI cancellation linearly. The linearity of their iterative approach increases noise components at the receiving end. Due to the enhancement in the noise components, the SNR and BER of ND algorithm is more affected by the MAI.

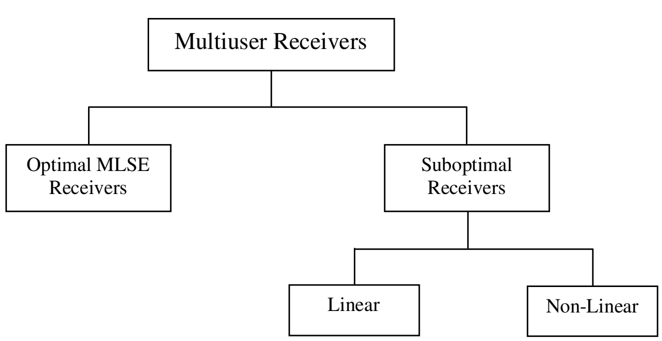

Fig.1.Multiuser optimal and suboptimal wireless receivers

Several tree-search detection receivers have been proposed in the literature [10, 11], in order to reduce the computational complexity of the original ML detection scheme proposed by Verdu. Specifically, [10] investigated a tree-search detection algorithm, where a recursive, additive metric was developed in order to reduce the search complexity. Reduced tree-search algorithms, such as the well known M-algorithms and T-algorithms [12], were used by [11] in order to reduce the complexity incurred by the optimum multiuser detectors.

In order to make an optimal wireless receiver that gives minimum mean square error (MMSE) performance, we need to provide some knowledge of interference such as phase, frequency, delays, and amplitude for all users. In addition, an optimal MMSE receiver requires the inversion of a large matrix. This computation takes relatively long time and makes the detection process slow and expensive [7, 8]. On the other hand, an adaptive MMSE receiver greatly reduces the entire computation process and gives an acceptable performance. Xie, Rushforth, Short and Moon [13] proposed an approximate MLSE solution known as the pre-survivor processing (PSP) type algorithm, which combined a tree search algorithm for data detection with the aid of the recursive least square (RLS) adaptive algorithm used for channel amplitude and phase estimation. The PSP algorithm was first proposed by Seshadri [14] for blind estimation in single user ISI-contaminated channels.

3. Proposed Low-Complexity TM Algorithm

We consider a synchronous DS-CDMA system as a linear time invariant (LTI) channel. In a LTI channel, the probability of variations in the interference parameters, such as the timing of all users, amplitude variation, phase shift, and frequency shift, is extremely low. This property makes it possible to reduce the overall computational complexity at the receiving end. Our TM technique utilizes the complex properties of the existing inverse matrix algorithms to construct the transformation matrices and to determine the location of the TPs that may occur in any coordinate of the constellation diagram. The individual TPs can be used to determine the average computational complexity.

The system may consist of K users. User k can transmit a signal at any given time with the power of W_k. With the binary phase shift keying (BPSK) modulation technique, the transmitted bits belong to either +1 or -1, (i.e., $b_k \in \{\pm 1\}$). The cross correlation can be reduced by neglecting the variable delay spreads, since these delays are relatively small as compared to the symbol transmission time. In order to detect signals from any user, the demodulated output of the low pass filter is multiplied by a unique signature waveform assigned by a pseudo random number generator. It should be noted that we extract the signal using the match filter followed by a Viterbi algorithm [15].

3.1 Original Optimum Multiuser Receiver

The optimum multiuser receiver exists and permits to relax the constraints of choosing the spreading sequences with good correlation properties at a cost of increased receiver complexity. Fig. 2 shows the block diagram of an optimum receiver that uses a bank of matched filters and a maximum likelihood Viterbi decision algorithm [15] for signal detection. It should be noted in Fig. 2 that the proposed TM algorithm is implemented in conjunction with the Viterbi decision algorithm [15] with the feedback mechanism. In order to detect signal from any user, the demodulated output of the low pass filter is multiplied by a unique signature waveform assigned by a pseudo random number generator.

When receiver wants to detect the signal from user-1, it first demodulates the received signal to obtain the base-band signal. The base-band signal multiplies with user-1's unique signature waveform, $C_1(t)$ .The resulting signal, $r_1(t)$, is applied to the input of the matched filter. The matched filter integrates the resulting signal $\{r_1(t)\}$ over each symbol period *T*, and the output is read into the decoder at the end of each integration cycle. The outputs of the matched filter and the Verdu's algorithm can be represented by $y_k(m)$ and $b_k(m)$, respectively where *m* is the sampling interval. We also assume that the first timing offset τ_1 is almost zero and $\tau_2 < T$. The same procedure applies to other users. The outputs of the matched filter for the first two users at the m^{th} sampling interval can be expressed as follows:

$$y_1(m) = \frac{1}{T}\left\{\int_{(m)T}^{(m+1)T} \{r_1(t)C_1(t)\}\,dt\right\} \quad (1)$$

$$y_2(m) = \frac{1}{T}\left\{\int_{2+(m)T}^{2+(m+1)T} \{r_2(t)C_2(t-\tau_2)\}\,dt\right\} \quad (2)$$

The received signal $r_1(t)$ and $r_2(t)$ can be expressed as follows:

$$r_1(t) = (E_{C1})^{0.5} \sum_{i=-M}^{M} \{b_1(i)C_1(t-iT_b)\} \quad (3)$$

$$r_2(t) = (E_{C_2})^{0.5} \sum_{i=-M}^{M} \{b_2(i)C_2(t-iT_b-\tau_2)\} \quad (4)$$

where E_{C_1} and E_{C_2} represent the original bit energy of the received signals with respect to their unique signature waveforms.

The received signals $r_1(t)$ and $r_2(t)$ can be treated as a single signal $r(t)$ that will be distinguished by the receiver with respect to its unique signature waveform. Based on the above analysis, we can combine equation (3) and (4).

$$r(t) = (E_{C_1})^{0.5} \sum_{i=-M}^{M} \{b_1(i)C_1(t-iT_b)\} + (E_{C_2})^{0.5} \sum_{i=-M}^{M} \{b_2(i)C_2(t-iT_b-\tau_2)\} \quad (5)$$

Substitute (5) as an individual equation into (1), we have

$$y_1(m) = \frac{1}{T}\left\{\int_{(m)T}^{(m+1)T} (E_{C_1})^{0.5} \left\{\sum_{i=-M}^{M} \{b_1(i)C_1(t-iT_b)\}\right\} C_1(t)\,dt\right\} \quad (6)$$

Substitute (5) as an individual equation into (2), we have

$$y_2(m) = \frac{1}{T}\left\{\int_{(m)T}^{(m+1)T} (E_{C_2})^{0.5} \left\{\sum_{i=-M}^{M} \{b_2(i)C_2(t-iT_b-\tau_2)\}\right\} \times C_2(t-\tau_2)\,dt\right\} \quad (7)$$

After performing integration over the given interval, we get the following results with the noise components as well as the cross correlation of signature waveforms.

$$y_1(m) = (E_{C_1})^{0.5} b_1(m) + (E_{C_2})^{0.5} b_2(m-1)\rho_1 + (E_{C_2})^{0.5} b_2(m)\rho_0 + (E_{C_2})^{0.5} b_2(m+1)\rho_{-1} + n_1(m) \quad (8)$$

$$y_2(m) = (E_{C_2})^{0.5} b_2(m) + (E_{C_1})^{0.5} b_1(m-1)\rho_1 + (E_{C_1})^{0.5} b_1(m)\rho_0 + (E_{C_1})^{0.5} b_1(m+1)\rho_{-1} + n_2(m) \quad (9)$$

where coefficients $b_1(m)$ and $b_2(m)$ represent MAI, $\rho_{-1/0/+1}$ are cross-correlations of signature waveforms, and $n_1(m)$ and $n_2(m)$ represent the minimum noise components. Since the channel is LTI, the probability of unwanted noise is minimum.

These symbols can now be decoded using a maximum likelihood Viterbi decision algorithm [15]. Viterbi algorithm can be used to detect these signals in much the same way as convolution codes. This algorithm makes decision over a finite window of sampling instants rather

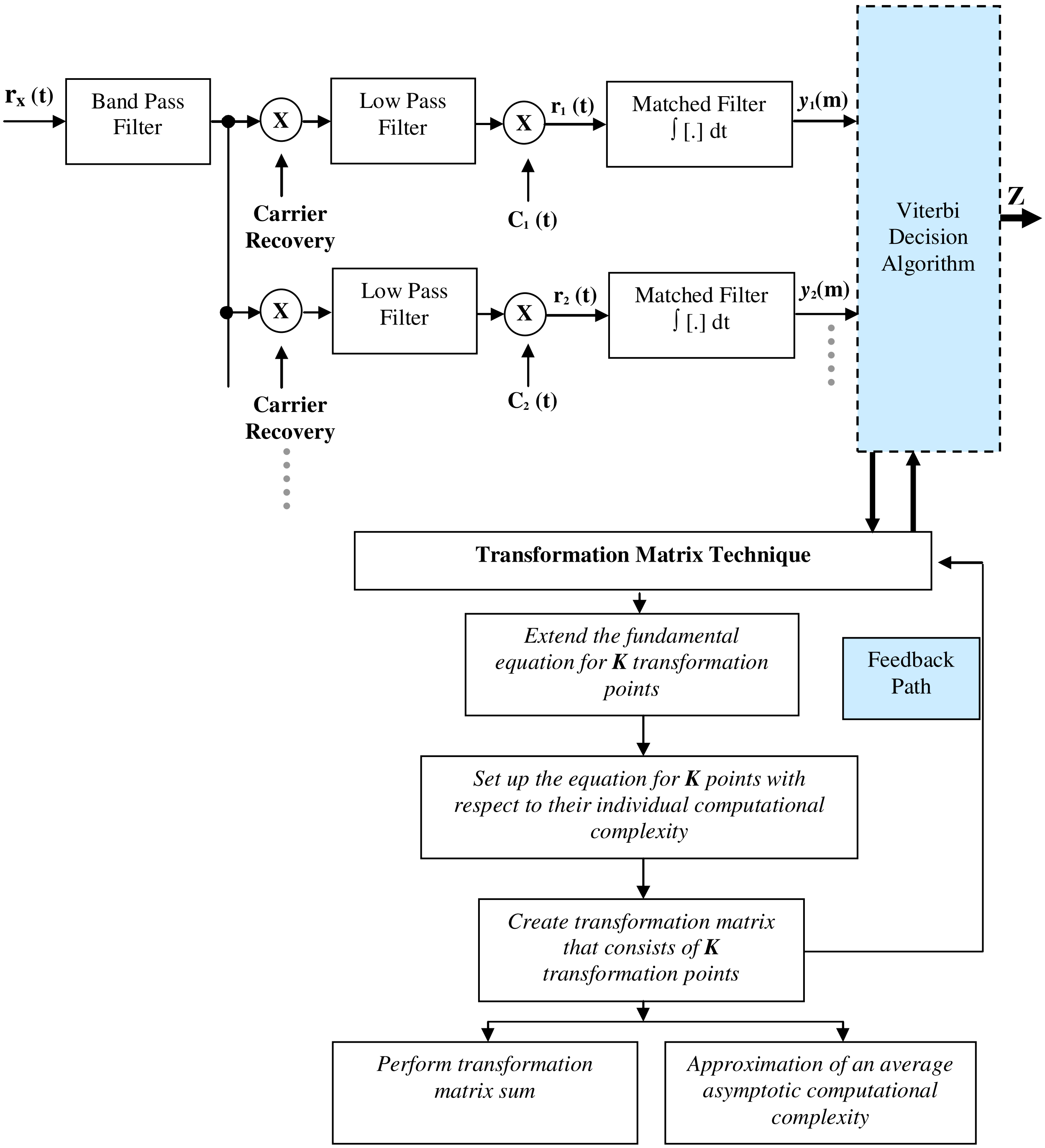

Fig.2 Implementation of proposed transformation matrix (TM) algorithm with the optimum multiuser receiver

than waiting for all the data to be received [4]. The above derivation can be extended from two users to *K* number of users. The number of operations performed in the Viterbi algorithm is proportional to the number of decision states, and the number of decision states is exponential with respect to the total number of users. The asymptotic computational complexity of this algorithm can be approximated as: $O(2)^K$.

3.2 Proposed Transformation Matrix (TM) Algorithm

According to original Verdu's algorithm, the outputs of the matched filter $y_1(m)$, *and* $y_2(m)$ can be considered as a single output $y(m)$.In order to minimize the noise components and to maximize the received demodulated bits, we can transform the output of the matched filter, and this transformation can be expressed as follows: $y(m) = \mathrm{T}\mathrm{b} + \eta$ where T represents the transformation matrix, $b_k \in \{\pm 1\}$ and η represents the noise components. In addition, if vectors are regarded as points in K-dimensional space, then the vectors constitute the constellation diagram that has K total points. This constellation diagram can be mathematically expressed as: $\aleph = \{\mathrm{T}b\}\ where\ b \in \{-1, +1\}$. We use this equation as a fundamental equation of the proposed algorithm. According to the detection rule, the constellation diagram can be partitioned into 2^K lines (where the total possible lines in the constellation diagram can be represented as $\int$) that can only intersect each other at the following points: $\aleph = \{\mathrm{Tb}\}_{\mathrm{b} \in \{-1, 1\}^K} ¥ \int$

Fig. 3 shows the constellation diagram that consists of three different vectors (lines) with the original vector '**X**' that represents the collective complexity of the receiver. *Q*, *R*, and *S* represent vectors or TPs within the coverage area of a cellular network as shown in Fig. 3. In addition, $Q^\neg$, $R^\neg$, and $S^\neg$ represent the computational complexity of each individual TP. In order to compute the collective computational complexity of the optimum wireless receiver, it is essential to determine the complexity of each individual TP. The computational complexity of each individual TP represents by $X^\neg$ of the TP which is equal to the collective complexity of $Q^\neg$, $R^\neg$, and $S^\neg$. A TM defines how to map points from one coordinate space into another. A transformation does not change the original vector, instead it alters the components. In order to derive the value of the original vector **X**, we need to perform the following derivations. We consider the original vector with respect to each transmitted symbol or bit. The following system can be derived from the above equations:

$$\begin{pmatrix} X^\neg Q \\ X^\neg R \\ X^\neg S \end{pmatrix} = \begin{pmatrix} ii^\neg & ji^\neg & ki^\neg \\ ij^\neg & jj^\neg & kj^\neg \\ ik^\neg & jk^\neg & kk^\neg \end{pmatrix} \begin{pmatrix} XQ \\ XR \\ XS \end{pmatrix} \qquad (10)$$

Equation (10) represents the following: *QRS* with the unit vectors $i, j,$ and k ; $X^\neg Q$, $X^\neg R$, and $X^\neg S$ with the inverse of the unit vectors $i^\neg$, $j^\neg$, and $k^\neg$. The second matrix on the right hand side of (10) represents **b**, where as the first matrix on the right hand side of (10) represents the actual TM. Therefore, the TM from the global reference points (which could be *Q*, *R*, or *S*) to a particular local reference point can now be derived from (10):

$$\begin{pmatrix} X^\neg Q \\ X^\neg R \\ X^\neg S \end{pmatrix} = T_{L/G} \begin{pmatrix} XQ \\ XR \\ XS \end{pmatrix} \qquad (11)$$

Equation (11) can also be written as:

$$T_{L/G} = \begin{pmatrix} ii^\neg & ji^\neg & ki^\neg \\ ij^\neg & jj^\neg & kj^\neg \\ ik^\neg & jk^\neg & kk^\neg \end{pmatrix} \qquad (12)$$

In equation (12), the dot products of the unit vectors of the two reference points are in fact the same as the unit vectors of the inverse TM of (11). We need to compute the locations of the actual TPs described in equations (11) and (12). Let the unit vectors for the local reference point be:

$$\begin{aligned} i^\neg &= \left[t_{11}(i), t_{12}(j), t_{13}(k)\right] \\ j^\neg &= \left[t_{21}(i), t_{22}(j), t_{23}(k)\right] \\ k^\neg &= \left[t_{31}(i), t_{32}(j), t_{33}(k)\right] \end{aligned} \qquad (13)$$

Since, $i^\neg (i + j + k) = i^\neg$, where $(i + j + k) = 1$. The same is true for the rest of the unit vectors (i.e., $i^\neg = i^\neg$). Therefore, (13) can be rewritten as:

$$\begin{aligned} i^\neg &= \left[t_{11}, t_{12}, t_{13}\right] \\ j^\neg &= \left[t_{21}, t_{22}, t_{23}\right] \\ k^\neg &= \left[t_{31}, t_{32}, t_{33}\right] \end{aligned} \qquad (14)$$

By substituting the values of $i^\neg$, $j^\neg$, and $k^\neg$ from (14) into (12), we obtain

$$T_{L/G} = \begin{pmatrix} i(t_{11}i + t_{12}j + t_{13}k) & j(t_{11}i + t_{12}j + t_{13}k) & k(t_{11}i + t_{12}j + t_{13}k) \\ i(t_{21}i + t_{22}j + t_{23}k) & j(t_{21}i + t_{22}j + t_{23}k) & k(t_{21}i + t_{22}j + t_{23}k) \\ i(t_{31}i + t_{32}j + t_{33}k) & j(t_{31}i + t_{32}j + t_{33}k) & k(t_{31}i + t_{32}j + t_{33}k) \end{pmatrix} \triangleq \qquad (15)$$

$$T_{L/G} = \begin{pmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ t_{31} & t_{32} & t_{33} \end{pmatrix}$$

Substituting $\boldsymbol{T_{L/G}}$ from (15) into (11), yields

$$\begin{pmatrix} X^\neg Q \\ X^\neg R \\ X^\neg S \end{pmatrix} = \begin{pmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ t_{31} & t_{32} & t_{33} \end{pmatrix} \begin{pmatrix} XQ \\ XR \\ XS \end{pmatrix} \qquad (16)$$

Equation (16) corresponds to the following standard equation used for computing the computational complexity at the receiving end: $\aleph = \{\mathrm{Tb}\}_{\,\mathrm{b} \in \{-1,\,+1\}^k}$ If the target of one transformation $(U : Q \rightarrow R)$ is the same as the source of other transformation $(T : R \rightarrow S)$, then we can combine two or more transformations and form the following composition: $TU : Q \rightarrow S,\ TU(Q) = T(U(Q))$.

This composition can be used to derive the collective computational complexity at the receiving end using (16). Since the channel is assumed to be LTI, the TPs may occur in any coordinate of the constellation diagram. The positive and negative coordinates of the constellation diagram do not make any difference for a LTI propagation channel. In addition, the TPs should lie within the specified range of the system. Since we assumed that the transmitted signals are modulated using BPSK which can at most use 1 bit out of 2 bits (that is, $b_k \in \{\pm 1\}$), consider the following set of TPs to approximate the number of demodulated received bits that need to search out by decision algorithm:

$$\aleph = \left(\begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} -1 & 1 & 1 \\ -1 & 1 & 1 \\ -1 & 1 & 1 \end{pmatrix} \right)^K$$

$$+ \left(\begin{pmatrix} 1 & -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{pmatrix} + \begin{pmatrix} 1 & -1 & 1 \end{pmatrix} \begin{pmatrix} -1 & -1 & -1 \\ -1 & -1 & -1 \\ -1 & -1 & -1 \end{pmatrix} \right)^K$$

Using (16), a simple matrix addition of the received demodulated bits can be used to approximate the number of most correlated TPs. The set of TPs correspond the actual location with in the TM as shown in (16). The entire procedure for computing the number of demodulated bits that need to search out by the decision algorithm can be used to approximate the number of most correlated signals for any given set of TPs. This is because, we need to check weather or not the TPs are closest to either (+1, +1) or (-1, -1). The decision regions or the coordinates where the TPs lie for (+1, +1) and (-1, -1) are simply the corresponding transformation matrixes that store the patterns of their occurrences. If the TPs do not exist in the region (coordinate) of either (+1, +1) or (-1, -1), then it just a matter of checking weather the TPs are closest to (+1, -1) or to (-1, +1). In other words, the second matrix on the right hand side of (16) requires a comprehensive search of at most 5^K demodulated bits that indirectly correspond to one or more users. The minimum search performed by the decision algorithm is conducted if the TPs exist within the incorrect region. Since the minimum search saves computation by one degree, the decision algorithm has to search at least 4^k demodulated bits. The average number of computations required by a system on any given set always exists between the maximum and the minimum number of computations performed in each operational cycle [9]. This implies that the total number of demodulated bits that need to search out by the decision algorithm can not exceed by5^K -4^K. In other words, the total numbers of most correlated pairs are upper-bounded by5^K -4^K.

Since most of the decisions are correct, we can reduce the number of computations by using the transformation matrixes only on those coordinates that are most likely lead to an incorrect decision. In other words, TM does not process those coordinates which are most likely lead to a correct decision. By doing this, we greatly reduce the

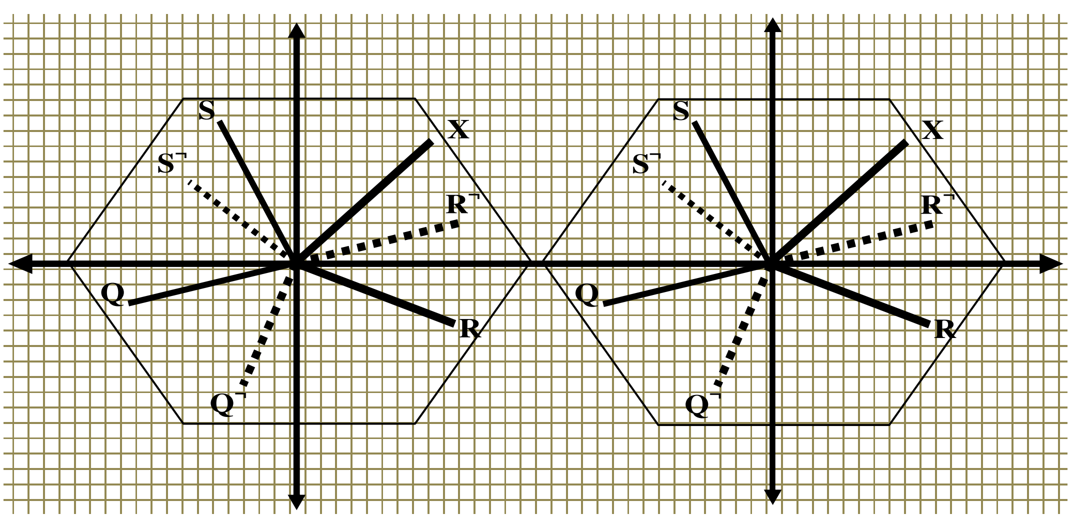

Fig. 3 A constellation diagram consisting of three different vectors

unnecessary processing that requires to make a decision about the correct region or the coordinate. Thus, the number of received demodulated bits that need to search out can be approximated as: 5^K -4^K. The total number of pair in the upper-bound describes the computational complexity at the receiving end.

The computational complexity of any multiuser receiver can be quantified by its time complexity per bit [9]. The collective computational complexity of the proposed algorithm is achieved after performing the TM sum using the complex properties of the existing inverse matrix algorithms. In other words, the computational complexity can be computed by determining the number of operations required by the receiver to detect and demodulate the transmitted information divided by the total number of demodulated bits. Therefore, both quantities T and b from our fundamental equation can be computed together and the generation for all the values of demodulated received bits b can be done through the sum of the actual T that approximately takes $\boldsymbol{O}$ $(5/4)^K$ operations with an asymptotic constant. We determine the collective complexity of optimum multiuser receiver by performing the TM sum.

After selecting the BPSK modulated bits ($b_k \in \{\pm 1\}$) and the TPs that may occur in any coordinate of the constellation diagram, the collective asymptotic computational complexity of the optimal ML receiver can be approximated after performing the TM sum. The resultant approximation has no concern with the decision algorithm, since the approximate result can only be used to analyze the number of operations performed by the receiver. The computational complexity of the proposed algorithm for multiuser detection is not polynomial in the number of users, instead the number of operations required to maximize the demodulation of the transmitted bits and to choose an optimal value of b is $\boldsymbol{O}$ $(5/4)^K$, and therefore the time complexity per bit is $\boldsymbol{O}$ $(5/4)^K$. Even though, the computational complexity of the proposed algorithm is not polynomial in terms of the total number of users, but it still gives significantly reduced computational complexity.

3.2 Proofs for Computational Complexity

This section provides the formal mathematical proof of the above discussion that proves the efficiency of the proposed algorithm with given input sizes. We provide a mathematical proof for both the upper bound and the lower bound of the proposed algorithm over the ND and the ML algorithms.

Proof (1): $f(x)$ is upper bound of $\mathbf{g_1}(\mathbf{x})$ and $\mathbf{g_2}(\mathbf{x})$

For the sake of this proof, we consider each algorithm represents by the growth of a function as follows: Let $f(x) = \left(5/4\right)^K$ for the proposed algorithm, $g_1(x) = (2)^K$ for the ML algorithm, and $g_2(x) = \left(3/2\right)^K$ for the ND algorithm. Equation (17) shows that the proposed algorithm $f(x)$ is in the lower bound of both $g_1(x)$ and $g_2(x)$. Therefore, the values of the function $f(x)$, with different input sizes, always exist as a lower limit of both $g_1(x)$ and $g_2(x)$. In order to prove this hypothesis mathematically, we need to consider the following equations:

$$f(x) = \boldsymbol{O}\left(g_1(x)\right) \text{ and } f(x) = \boldsymbol{O}\left(g_2(x)\right) \quad (17)$$

$$f(x) = \left(5/4\right)^K < c_1\left(g_1(x)\right),$$

$$f(x) = \left(5/4\right)^K < c_2\left(g_2(x)\right)$$

Solving for $g(x)$, we get the following two equations:

$$f(x) = \left(5/4\right)^K < c_1(2.0)^K \quad (18)$$

$$f(x) = \left(5/4\right)^K < c_2\left(3/2\right)^K \quad (19)$$

Solving for $g_1(x)$, we can write an argument using (18), such as: *$f(x)$ is said to be $\boldsymbol{O}\left(c_1 \times g_1(x)\right)$, if and only if there exists a constant c_1 and the threshold n_o such that:*

$$f(x) < c_1\left|g(x)\right| \text{ whenever } x > n_o.$$

$$f(x) = \boldsymbol{O}\left(c_1 \times g\ \ (x)\right)$$

Thus this is proved using (18). It should be noted that the n_0 is the threshold value at which both functions approximately approaches each other. Solving for $g_2(x)$, we can write a similar argument using (19), such as:

$$f(x) \text{ is said to be } \boldsymbol{O}\left(c_2 \times g_2(x)\right),$$

If and only if there exists a constant c_2 and the threshold n_o such that:

$$f(x) < c_2 |g(x)| \text{ Whenever } x > n_o.$$
$$f(x) = \boldsymbol{O}(c_2 \times g(x))$$

Thus this is proved using (19).

Proof (2): $\boldsymbol{f(x)}$ ***is lower bound of*** $\mathbf{g_1(x)}$ ***and*** $\mathbf{g_2(x)}$

In order to analyze the lower bound, we provide a proof in the reverse order to define a lower bound for the function $f(x)$. Equation (20) demonstrates that both functions $g_1(x)$ and $g_2(x)$ is the upper bounds for the function $f(x)$. The corresponding values of $g_1(x)$ and $g_2(x)$ with different input sizes always lie as a maximum upper limit of $f(x)$, and hence both functions $g_1(x)$ and $g_2(x)$ always yield a greater complexity. In order to prove this hypothesis mathematically, we need to consider the following equations:

$$g_1(x) = \Omega(f(x)) \text{ and } g_2(x) = \Omega(f(x)) \quad (20)$$

$$g_1(x) = (2.0)^K > c_1(f(x))$$
$$g_2(x) = \left(3/2\right)^K > c_1(f(x))$$

Solving for $f(x)$, we get the following two equations:

$$g_1(x) = (2.0)^K > c_1(5/4)^K \quad (21)$$
$$g_2(x) = (3/2)^K > c_2(5/4)^K \quad (22)$$

Solving for $g_1(x)$, we can make the following argument using (21), such as

$$g_1(x) \text{ is said to be } \Omega(c_1 \times f(x))$$

If and only if there exists a constant c_1 and the threshold n_o such that:

$$g_1(x) > c_1 |f(x)| \text{ whenever } x > n_o.$$
$$g_1(x) = \Omega(c_1 \times f(x))$$

Thus this is proved using (21). Solving for $g_2(x)$, we can claim a similar argument using (22), such as

$$g_2(x) \text{ is said to be } \Omega(c_2 \times f(x)),$$

If and only if there exists a constant c_2 and the threshold n_o such that:

$$g_2(x) > c_2 |f(x)|$$
$$g_2(x) = \Omega(c_2 \times f(x))$$

Thus this is proved using (22). As we have proved here (referring (17) and (20)) that:

$$f(x) = \boldsymbol{O}(c \times g(x)) \text{ and } g(x) = \Omega(c \times f(x))$$

4. Performance Analysis and Experimental Verifications

The order of growth of a function is an important criterion for proving the complexity and efficiency of an algorithm. It gives simple characterization of the algorithm's efficiency and also allows us to compare the relative performance of algorithms with given input sizes. In this section, we present a comparative analysis of the asymptotic computational complexity of the proposed algorithm over the ML and the ND algorithms. The original asymptotic computational complexity of the ML optimal receiver is $(2)^k$ [1]. Another research paper [5] has reduced the complexity from $(2)^k$ to $(3/2)^k$. This paper [5], also known as ND algorithm, has reduced the computational complexity after considering a synchronous DS-CDMA system.

According to our numerical results, we successfully reduced the computational complexity at an acceptable BER after considering the DS-CDMA synchronous LTI system. The numerical results show the asymptotic computational complexities with respect to the number of users as shown in Fig. 4 and 5 for 10 and 100 users,

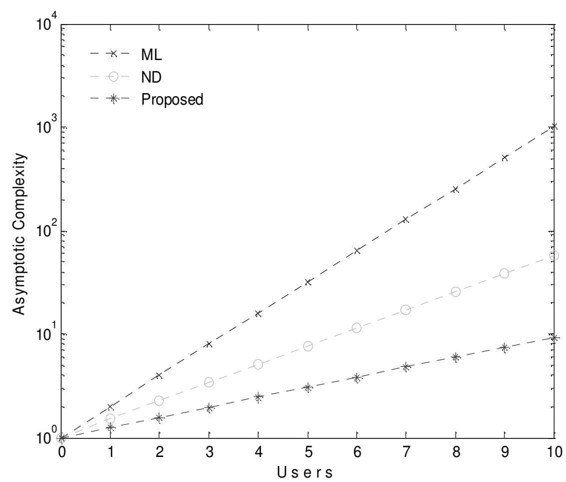

Fig. 4. The asymptotic computational complexities versus small number of users

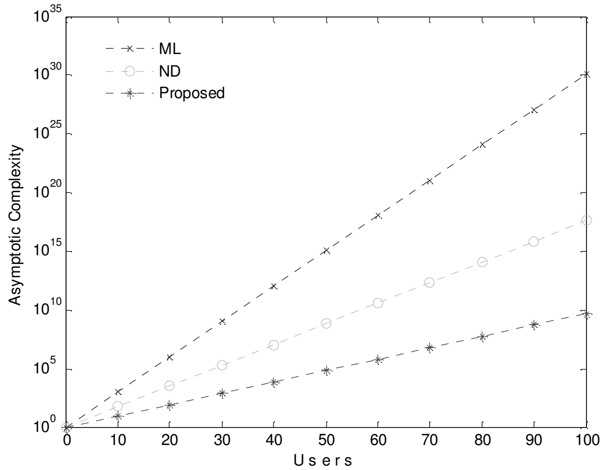

Fig. 5. The asymptotic computational complexities versus intermediate number of users

respectively. As the number of users increases in the system, the computational complexity differences among the three approaches will be obvious.

Fig. 4 shows the computational complexities for a network that consists of 10 users. As we can see that the proposed algorithm for a small network of 10 users requires fewer computations as compare to the ML and the ND algorithms. In addition, the proposed algorithm greatly reduces the unnecessary computations involve in signal detection by storing the pattern of occurrence of the demodulated bits in the TM and uses it only on those coordinates or decision regions which are most likely lead to an incorrect decision. The computational complexity for a network that consists of 100 users is shown in Fig. 5. It should be noted that the computational complexity curve for the proposed algorithm is growing in a linear order rather than in an exponential order. The computational linearity of the proposed algorithm comes by employing the TM technique that avoids considering all the decision variables and thus provides much better performance over the ND and the ML algorithms. In other words, this can be considered as an extension of the former results that demonstrates the consistency in the linear growth for the required computations of the proposed algorithm. As we increase the number of users in the system, more transformation matrixes will be used to determine that which coordinate(s) or decision region(s) within the constellation diagram is most likely to produce errors.

5. Conclusion

In this paper, a novel approach for reducing the computational complexity of multiuser receivers was proposed that utilizes the TM technique to improve the performance of multiuser receiver. In addition to the low-complexity algorithm, we provided a complete implementation of the proposed algorithm with the support of a well driven mathematical model. In order to prove the low-complexity and the correctness of the proposed algorithm, we provided the formal mathematical proofs for both the upper and the lower bounds of the proposed complexity. The mathematical proofs for both bounds demonstrated that the computational complexity of the TM algorithm with any input size always be less than the ML and the ND algorithms. The reduction in computational complexity increases the computing power of a multiuser receiver. Consequently, the increase in computing power would likely to result fast signal detection and error estimation which do not come at the expense of performance. For the future work, it will be interesting to implement the proposed approach for asynchronous systems with non-linear time variant properties of the channel.

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Syed S. Rizvi is a Ph.D. student of Computer Science and Engineering at University of Bridgeport. He received a B.S. in Computer Engineering from Sir Syed University of Engineering and Technology and an M.S. in Computer Engineering from Old Dominion University in 2001 and 2005, respectively. In the past, he has done research on bioinformatics projects where he investigated the use of Linux based cluster search engines for finding the desired proteins in input and outputs sequences from multiple databases. For last three year, his research focused primarily on the modeling and simulation of wide range parallel/distributed systems and the web based training applications. Syed Rizvi is the author of 68 scholarly publications in various areas. His current research focuses on the design, implementation and comparisons of algorithms in the areas of multiuser communications, multipath signals detection, multi-access interference estimation, computational complexity and combinatorial optimization of multiuser receivers, peer-to-peer networking, network security, and reconfigurable coprocessor and FPGA based architectures.

Aasia Riasat is an Associate Professor of Computer Science at Collage of Business Management (CBM) since May 2006. She received an M.S.C. in Computer Science from the University of Sindh, and an M.S in Computer Science from Old Dominion University in 2005. For last one year, she is working as one of the active members of the wireless and mobile communications (WMC) lab research group of University of Bridgeport, Bridgeport CT. In WMC research group, she is mainly responsible for simulation design for all the research work. Aasia Riasat is the author or co-author of more than 40 scholarly publications in various areas. Her research interests include modeling and simulation, web-based visualization, virtual reality, data compression, and algorithms optimization.

Khaled Elleithy received the B.Sc. degree in computer science and automatic control from Alexandria University in 1983, the MS Degree in computer networks from the same university in 1986, and the MS and Ph.D. degrees in computer science from The Center for Advanced Computer Studies at the University of Louisiana at Lafayette in 1988 and 1990, respectively. From 1983 to 1986, he was with the Computer Science Department, Alexandria University, Egypt, as a lecturer. From September 1990 to May 1995 he worked as an assistant professor at the Department of Computer Engineering, King Fahd University of Petroleum and Minerals, Dhahran, Saudi Arabia. From May 1995 to December 2000, he has worked as an Associate Professor in the same department. In January 2000, Dr. Elleithy has joined the Department of Computer Science and Engineering in University of Bridgeport as an associate professor. Dr. Elleithy published more than seventy research papers in international journals and conferences. He has research interests are in the areas of computer networks, network security, mobile communications, and formal approaches for design and verification.