Fault-Tolerant Guaranteed Cost Control of Uncertain Networked Control Systems with Time-varying Delay

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Summary
In practice, there inevitably exists the offset or disturbances induced by actuator failures. This is an ignored problem in integrity fault-tolerant control theory. In order to satisfy the robustness of system, the closed-loop model of uncertain networked control system with time-varying delay and actuator failures is established aiming at a class of controlled objective with uncertain parameters. The attenuation performance index of system for fault is defined. Combing with fault-tolerant control and guaranteed cost control, a fault-tolerant guaranteed cost controller is designed adopting Lyapunov stability analysis method. Simulation results indicate the controller can not only guarantee the asymptotic stability, but also ensure the robustness and anti-disturbance performance.

Key words:
Networked control systems, Fault-tolerant, Guaranteed cost, Ttime-varying delay, Uncertain

1. Introduction

Networked control system (NCS) is a hot research area. At present,many workes have been done about system modeling,stability analysis,guaranteed cost controller design and so on. Researches on robust guaranteed cost controller ensure the stability and robustness of networked control system. This meets the practical needs and has the important meaning.

Guaranteed cost control is to design a controller which not only makes uncertain closed-loop system stable, but also limits the bound of certain performance index. In Ref[1], guaranteed cost control of discrete networked control system is studied aiming at time-varying delay. Guaranteed cost control of networked control system is given based on discrete jump system aiming at random delay[2]. Yue[3] proposes guaranteed cost control based on model given in Ref[4] aiming at quadratic performances. In Ref[5], H∞ guaranteed cost control of networked control system adopting proportion-integral output feedback controller aiming at quadratic cost function. For networked control systems with time-varying delay less than one sample period and data-packet dropout, a compensator is introduced to compensate the effect of data dropout[6]. And NCS is modeled as a discrete switched system with parametrical uncertainties. Based on this model,a cooperative design approach of controller and the compensator are given in terms of a group of linear matrix inequality. Guaranteed cost control of networked control system with uncertain time delay adopting output feedback controller is studied in Ref[7].

In existing achievements,guaranteed cost control of networked control system with faults is not taken into account. Otherwise, most of cotrol methods are given based on discrete networked control system with constant network-induced time delay. However, the robustness of continuous networked control system with uncertainty and time-varying delay is seldom considered.

In the paper, guaranteed cost controller of uncertain networked control system with time-varying delay and actuator failures is designed. Firstly, networked control system with time-varying delay and actuator failures is modeled. Secondly, the stability of closed-loop fault system is analyzed considering zero disturbance caused by actuator faults. When disturbance caused by actuator faults is not zero, performance index reflecting disturbance degradation is defined. And guaranteed cost controller is designed in terms of Lyapunov stability analysis method. At last, the validity of proposed method is validated by two examples.

2. Modeling of The Closed-loop Fault Systems

Consider the continuous-time linear plant described by state-space equations of the form

$$\begin{cases} \dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t) \\ \boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{x}(t) \end{cases} \tag{1}$$

where $\boldsymbol{x}(t) \in \boldsymbol{R}^n$ is the state and $\boldsymbol{u}(t) \in \boldsymbol{R}^m$ is the control input. $\boldsymbol{y}(t) \in \boldsymbol{R}^p$ is the output of plant. $\boldsymbol{A} \in \boldsymbol{R}^{n\times n}$ is the state matrix and $\boldsymbol{B} \in \boldsymbol{R}^{n\times m}$ is the input matrix. $\boldsymbol{C} \in \boldsymbol{R}^{p\times n}$ is the output matrix. And $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ are constants matrices.

In this paper, it is assumed that:

(1)Continuous plant without network and state feedback are stability or meet certain needs of control.

(2)Controller is time-varying and continuous.

(3)Noise of system is not taken into account. And no error exist in communication.

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(4)Suppose $d_{sc}(t)$ is time delays caused by network from sensor to controller. $d_{sc}(t)$ denotes time delays caused by network from controller to actuator. Above time delays are regarded as $d(t) = d_{sc}(t) + d_{ca}(t)$.

Under above assumption, system model considering time-varying network-induced delay is obtained:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t - d(t)) \tag{2}$$

where $d(t)=d_{sc}(t)+d_{ca}(t)$ is time-varying delay satisfied $0 \le d(t) \le \tau$. Here, τ is upper limit of time-varying delay. Consider actuator failure, fault model of networked control system is formed:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{L}\boldsymbol{u}(t - d(t)) + \boldsymbol{B}(\boldsymbol{I} - \boldsymbol{L})\boldsymbol{f}_a(t) \tag{3}$$

where $\boldsymbol{L} = diag\{l_1, l_2, \cdots, l_m\}$ is actuator failure matrix satisfied $\boldsymbol{L} \neq 0$ and $\boldsymbol{L} \in \psi$. Here, ψ indicates an aggregate containing all possible actuator failure matrixes.

$$l_i = \begin{cases} 1, & i\text{-th actuator is normal} \\ 0 & i\text{-th actuator is failure} \end{cases} \quad i = 1, 2, \cdots, m \tag{4}$$

It is obvious that system is normal when $L=I$. If $L \neq I$, there exists actuator faults and $f_a(t)$ is the disturbance or offset caused by actuator faults.

Consindering uncertainty of system, $\Delta\boldsymbol{A}(t)$ and $\Delta\boldsymbol{B}(t)$ are unknown limited coefficient matrixes. That is, $\Delta\boldsymbol{A}(t)\Delta\boldsymbol{A}^T(t) \le \overline{\boldsymbol{A}}^2$, $\Delta\boldsymbol{B}(t)\Delta\boldsymbol{B}^T(t) \le \overline{\boldsymbol{B}}^2$. Here, $\overline{\boldsymbol{A}} \in \boldsymbol{R}^{n\times n}$ and $\overline{\boldsymbol{B}} \in \boldsymbol{R}^{n\times n}$ are known constant matrixes. If state feedback controller is adopted shown as $\boldsymbol{u}(t) = \boldsymbol{K}\boldsymbol{x}(t)$, above model of system shown in formula(3) is transformed as follows.

$$\dot{\boldsymbol{x}}(t) = (\boldsymbol{A}+\Delta\boldsymbol{A}(t))\boldsymbol{x}(t) + (\boldsymbol{B}+\Delta\boldsymbol{B}(t))\boldsymbol{L}\boldsymbol{K}\boldsymbol{x}(t-d(t)) + \boldsymbol{B}(\boldsymbol{I}-\boldsymbol{L})\boldsymbol{f}_a(t) \tag{5}$$

Beasd on above-mentioned model, researches are done aiming at different disturbance:

(1) $\boldsymbol{f}_a(t) \equiv 0$ means that when actuator faults happen, output of controller is zero. The goal of the control system is to obtain sufficiency conditions ensuring the asymptotic stability of closed-loop fault system and determine proportion constant matrix of controller adopting Lyapunov stability analysis method.

(2) If $\boldsymbol{f}_a(t) \neq 0$, the offset of faults is regarded as unknown disturbance of system. In order to decrease the influence caused by disturbance, degradation performance shall satisfy $\|\boldsymbol{y}(t)\| < \rho \|\boldsymbol{f}_a(t)\|$. Here, $0<\rho<1$. Corresponding performance index is defined as $J = \int_0^\infty \left[\boldsymbol{y}^T(t)\boldsymbol{y}(t) - \rho^2 \boldsymbol{f}_a^T(t)\boldsymbol{f}_a(t)\right]dt$. The goal of the control system is to determine proportion constant matrix of controller which ensures the asymptotic stability of system whether actuator fault happens and make $J < 0$.

3 Design of Fault-tolerant Guaranteed Cost Controller

3.1 Stability analysis of closed-loop fault systems

Theorem 1: Considering the system shown in formula(5), for given positive constane κ and τ, if there exist positive definite symmetric matrix X, symmetry matrix Y and positive scalars α, β, ζ, η, δ1, δ2, the following linear matrix inequality is satisfied:

$$\boldsymbol{M} = \begin{bmatrix} \boldsymbol{M}_{11} & \boldsymbol{M}_{12} \\ \boldsymbol{M}^T_{12} & \boldsymbol{M}_{22} \end{bmatrix} < 0 \tag{6}$$

$$s.t. \alpha + \beta + 1 - \delta_1 > 0, \alpha\boldsymbol{I} - \delta_2\overline{\boldsymbol{A}}^2 > 0$$

Then the system is exponentially asymptotically stable. From a group of feasible solution (X,Y), controller with $\boldsymbol{K} = \boldsymbol{Y}\boldsymbol{X}^{-1}$ is obtained, where

$$\boldsymbol{M}_{11} = \boldsymbol{X}\boldsymbol{A}^T + \boldsymbol{A}\boldsymbol{X} + \boldsymbol{Y}^T\boldsymbol{L}^T\boldsymbol{B}^T + \boldsymbol{B}\boldsymbol{L}\boldsymbol{Y} + \zeta\overline{\boldsymbol{A}}^2 + \eta\overline{\boldsymbol{B}}^2$$

$$\boldsymbol{M}_{12} = \Big[\boldsymbol{X} \quad \boldsymbol{Y} \quad \tau\kappa\lambda\boldsymbol{B}\boldsymbol{L} \quad \tau\kappa\lambda\overline{\boldsymbol{B}} \quad \tau\boldsymbol{X}\boldsymbol{A}^T \quad \tau\boldsymbol{X}$$
$$\tau\boldsymbol{Y}^T\boldsymbol{L}^T\boldsymbol{B}^T \quad \tau\boldsymbol{Y}^T\boldsymbol{L}^T\boldsymbol{B}^T\overline{\boldsymbol{B}} \quad \tau\boldsymbol{Y}^T \quad \tau\sigma_{\max}(\overline{\boldsymbol{B}})\boldsymbol{Y}^T\Big]$$

$$\boldsymbol{M}_{22} = diag\{-\zeta\boldsymbol{I}, -\eta\boldsymbol{I}, -\tau\kappa(\lambda-\delta_1)\boldsymbol{I}, -\tau(\alpha\boldsymbol{I}-\delta_2\overline{\boldsymbol{A}}^2)\boldsymbol{I},$$
$$-\tau\delta_2\boldsymbol{I}, -\tau\beta\boldsymbol{I}, -\tau\beta\boldsymbol{I}, -\tau\beta\boldsymbol{I}, -\tau\beta\boldsymbol{I}\}$$

Proof: Based on Newton-Leibniz theorem,

$$Q_1(t) = \int_{-d(t)}^0 \{[\boldsymbol{A}+\Delta\boldsymbol{A}(t+\theta)]x(t+\theta) + \boldsymbol{B}(\boldsymbol{I}-\boldsymbol{L})\boldsymbol{f}_a(t+\theta))\}d\theta$$
$$+[\boldsymbol{B}+\Delta\boldsymbol{B}(t+\theta)]\boldsymbol{L}\boldsymbol{K}\boldsymbol{x}(t-d(t+\theta)+\theta) \tag{7}$$

The closed-loop model of fault system is rewritten:

$$\dot{\boldsymbol{x}}(t) = [(\boldsymbol{A}+\Delta\boldsymbol{A}(t)) + (\boldsymbol{B}+\Delta\boldsymbol{B}(t))\boldsymbol{L}\boldsymbol{K}]\boldsymbol{x}(t)$$
$$+\boldsymbol{B}(\boldsymbol{I}-\boldsymbol{L})\boldsymbol{f}_a(t) - (\boldsymbol{B}+\Delta\boldsymbol{B}(t))\boldsymbol{L}\boldsymbol{K}Q_1(t) \tag{8}$$

Construct a Lyapunov functiona as $V(\boldsymbol{x},t) = V_1(\boldsymbol{x},t) + V_2(\boldsymbol{x},t)$.

$$V_1(\boldsymbol{x}, t) = \boldsymbol{x}^T(t)\boldsymbol{P}\boldsymbol{x}(t)$$
$$V_2(\boldsymbol{x},t) = V_{21}(\boldsymbol{x},t) + V_{22}(\boldsymbol{x},t) + V_{23}(\boldsymbol{x},t)$$

$$=\frac{1}{\alpha}\int_{-\tau}^{0}\int_{t+\theta}^{t}\boldsymbol{x}^T(\vartheta)[\boldsymbol{A}+\Delta\boldsymbol{A}(\vartheta)]^T[\boldsymbol{A}+\Delta\boldsymbol{A}(\vartheta)]\boldsymbol{x}(\vartheta)d\vartheta d\theta$$
$$+\frac{1}{\beta}\int_{-\tau}^{0}\int_{t-d(t+\theta)+\theta}^{t}\boldsymbol{x}^T(\vartheta)\boldsymbol{K}^T[\boldsymbol{BL}+\Delta\boldsymbol{B}(\vartheta+d(t+\theta))\boldsymbol{L}]^T$$
$$\times[\boldsymbol{B}+\Delta\boldsymbol{B}(\vartheta+d(t+\theta))]\boldsymbol{LKx}(\vartheta)d\vartheta d\theta$$
$$+\int_{-\tau}^{0}\int_{t+\theta}^{t}\boldsymbol{f}_a^T(\vartheta)(\boldsymbol{I}-\boldsymbol{L})^T\boldsymbol{B}^T\boldsymbol{B}(\boldsymbol{I}-\boldsymbol{L})\boldsymbol{f}_a(\vartheta)d\vartheta d\theta$$

Along with random trajectory of system, the derivative of $V(\boldsymbol{x},t)$ is $\dot V(\boldsymbol{x},t)=\dot V_1(\boldsymbol{x},t)+\dot V_2(\boldsymbol{x},t)$.

$$\dot V_1(\boldsymbol{x},t)=\dot{\boldsymbol{x}}^T(t)\boldsymbol{Px}(t)+\boldsymbol{x}^T(t)\boldsymbol{P}\dot{\boldsymbol{x}}(t) \tag{9}$$
$$=\boldsymbol{x}^T(t)\{[(\boldsymbol{A}+\Delta\boldsymbol{A}(t)+(\boldsymbol{B}+\Delta\boldsymbol{B}(t))\boldsymbol{LK}]^T\boldsymbol{P}$$
$$+2\boldsymbol{x}^T(t)\boldsymbol{PB}(\boldsymbol{I}-\boldsymbol{L})\boldsymbol{f}_a(t)-2\boldsymbol{x}^T(t)\boldsymbol{P}(\boldsymbol{B}+\Delta\boldsymbol{B}(t))\boldsymbol{LK}Q_1(t)$$

$$-2\boldsymbol{x}^T(t)\boldsymbol{P}(\boldsymbol{B}+\Delta\boldsymbol{B}(t))\boldsymbol{LK}Q_1(t)$$
$$=-2\int_{-d(t)}^{0}\boldsymbol{x}^T(t)\boldsymbol{P}(\boldsymbol{B}+\Delta\boldsymbol{B}(t))\boldsymbol{LK}(\boldsymbol{A}+\Delta\boldsymbol{A}(t+\theta))\boldsymbol{x}(t+\theta)d\theta$$
$$-2\int_{-d(t)}^{0}\boldsymbol{x}^T(t)\boldsymbol{P}(\boldsymbol{B}+\Delta\boldsymbol{B}(t))\boldsymbol{LK}(\boldsymbol{B}+\Delta\boldsymbol{B}(t+\theta))\boldsymbol{x}(t-d(t+\theta)+\theta)d\theta$$
$$-2\int_{-d(t)}^{0}\boldsymbol{x}^T(t)\boldsymbol{P}(\boldsymbol{B}+\Delta\boldsymbol{B}(t))\boldsymbol{LKB}(\boldsymbol{I}-\boldsymbol{L})\boldsymbol{f}_a(t+\theta)d\theta \tag{10}$$

Lemma 1[8]: For any vectors or matrices X,Y,Z and any positive constants $\alpha>0$, $\beta>0$, the following inequalities are satisfied:

$$\boldsymbol{X}^T\boldsymbol{Y}+\boldsymbol{Y}^T\boldsymbol{X}\le\alpha\boldsymbol{X}^T\boldsymbol{X}+\frac{1}{\alpha}\boldsymbol{Y}^T\boldsymbol{Y}$$
$$\pm 2\boldsymbol{Z}^T\boldsymbol{Y}\le\beta\boldsymbol{Z}^T\boldsymbol{Z}+\frac{1}{\beta}\boldsymbol{Y}^T\boldsymbol{Y}$$

Based on Lemma 1, fomula (10) is rewritten as:

$$-2\int_{-d(t)}^{0}\boldsymbol{x}^T(t)\boldsymbol{P}(\boldsymbol{B}+\Delta\boldsymbol{B}(t))\boldsymbol{LKB}(\boldsymbol{I}-\boldsymbol{L})\boldsymbol{f}_a(t+\theta)d\theta$$
$$\le\tau\boldsymbol{x}^T(t)\boldsymbol{P}(\boldsymbol{B}+\Delta\boldsymbol{B}(t))\boldsymbol{LKK}^T(\boldsymbol{BL}+\Delta\boldsymbol{B}(t)\boldsymbol{L})^T\boldsymbol{Px}(t)$$
$$+\int_{-d(t)}^{0}\boldsymbol{f}_a^T(t+\theta)(\boldsymbol{I}-\boldsymbol{L})\boldsymbol{B}^T\boldsymbol{B}(\boldsymbol{I}-\boldsymbol{L})\boldsymbol{f}_a(t+\theta)d\theta \tag{11}$$

Now consider $\dot V_2(\boldsymbol{x},t)=\dot V_{21}(\boldsymbol{x},t)+\dot V_{22}(\boldsymbol{x},t)+\dot V_{23}(\boldsymbol{x},t)$.

$$\dot V_{21}(\boldsymbol{x},t)=\frac{\tau}{\alpha}\boldsymbol{x}^T(t)(\boldsymbol{A}+\Delta\boldsymbol{A}(t))^T(\boldsymbol{A}+\Delta\boldsymbol{A}(t))\boldsymbol{x}(t)$$
$$-\frac{1}{\alpha}\int_{-\tau}^{0}\boldsymbol{x}^T(t+\theta)(\boldsymbol{A}+\Delta\boldsymbol{A}(t+\theta))^T(\boldsymbol{A}+\Delta\boldsymbol{A}(t+\theta))\boldsymbol{x}(t+\theta)d\theta$$
$$\le\frac{\tau}{\alpha}\boldsymbol{x}^T(t)(\boldsymbol{A}+\Delta\boldsymbol{A}(t))^T(\boldsymbol{A}+\Delta\boldsymbol{A}(t))\boldsymbol{x}(t)$$
$$-\frac{1}{\alpha}\int_{-d(t)}^{0}\boldsymbol{x}^T(t+\theta)(\boldsymbol{A}+\Delta\boldsymbol{A}(t+\theta))^T(\boldsymbol{A}+\Delta\boldsymbol{A}(t+\theta))\boldsymbol{x}(t+\theta)d\theta \tag{12}$$

$$\dot V_{22}(\boldsymbol{x},t)=\frac{\tau}{\beta}\boldsymbol{x}^T(t)\boldsymbol{K}^T(\boldsymbol{BL}+\Delta\boldsymbol{B}(t+d(t+\theta)\boldsymbol{L}))^T(\boldsymbol{BL}+\Delta\boldsymbol{B}(t+d(t+\theta)\boldsymbol{L})\boldsymbol{Kx}(t)$$
$$-\frac{1}{\beta}\int_{-\tau}^{0}\boldsymbol{x}^T(t-d(t+\theta)+\theta)\boldsymbol{K}^T(\boldsymbol{BL}+\Delta\boldsymbol{B}(t+\theta)\boldsymbol{L})^T$$
$$(\boldsymbol{BL}+\Delta\boldsymbol{B}(t+\theta)\boldsymbol{L})\boldsymbol{Kx}(t-d(t+\theta)+\theta)d\theta$$
$$\le\frac{\tau}{\beta}\boldsymbol{x}^T(t)\boldsymbol{K}^T(\boldsymbol{BL}+\Delta\boldsymbol{B}(t+d(t+\theta)\boldsymbol{L}))^T(\boldsymbol{BL}+\Delta\boldsymbol{B}(t+d(t+\theta)\boldsymbol{L})\boldsymbol{Kx}(t)$$
$$-\frac{1}{\beta}\int_{-d(t)}^{0}\boldsymbol{x}^T(t-d(t+\theta)+\theta)\boldsymbol{K}^T(\boldsymbol{BL}+\Delta\boldsymbol{B}(t+\theta)\boldsymbol{L})^T$$
$$(\boldsymbol{BL}+\Delta\boldsymbol{B}(t+\theta)\boldsymbol{L})\boldsymbol{Kx}(t-d(t+\theta)+\theta)d\theta \tag{13}$$

$$\dot V_{23}(\boldsymbol{x},t)=\tau\boldsymbol{f}_a^T(t)(\boldsymbol{I}-\boldsymbol{L})\boldsymbol{B}^T\boldsymbol{B}(\boldsymbol{I}-\boldsymbol{L})\boldsymbol{f}_a(t)$$
$$-\int_{-\tau}^{0}\boldsymbol{f}_a^T(t+\theta)(\boldsymbol{I}-\boldsymbol{L})\boldsymbol{B}^T\boldsymbol{B}(\boldsymbol{I}-\boldsymbol{L})\boldsymbol{f}_a(t+\theta)d\theta$$
$$\le\tau\boldsymbol{f}_a^T(t)(\boldsymbol{I}-\boldsymbol{L})\boldsymbol{B}^T\boldsymbol{B}(\boldsymbol{I}-\boldsymbol{L})\boldsymbol{f}_a(t)$$
$$-\int_{-d(t)}^{0}\boldsymbol{f}_a^T(t+\theta)(\boldsymbol{I}-\boldsymbol{L})\boldsymbol{B}^T\boldsymbol{B}(\boldsymbol{I}-\boldsymbol{L})\boldsymbol{f}_a(t+\theta)d\theta \tag{14}$$

With (9)-(14), the following inequalities are obtained:

$$\dot V(\boldsymbol{x},t)\le\boldsymbol{x}^T(t)\{[(\boldsymbol{A}+\Delta\boldsymbol{A}(t)+(\boldsymbol{B}+\Delta\boldsymbol{B}(t))\boldsymbol{LK}]^T\boldsymbol{P}$$
$$+\boldsymbol{P}[(\boldsymbol{A}+\Delta\boldsymbol{A}(t)+(\boldsymbol{B}+\Delta\boldsymbol{B}(t))\boldsymbol{LK}]\}\boldsymbol{x}(t)+2\boldsymbol{x}^T(t)\boldsymbol{PB}(\boldsymbol{I}-\boldsymbol{L})\boldsymbol{f}_a(t)$$
$$+\tau(\alpha+\beta+1)\boldsymbol{x}^T(t)\boldsymbol{P}(\boldsymbol{B}+\Delta\boldsymbol{B}(t))\boldsymbol{LKK}^T(\boldsymbol{BL}+\Delta\boldsymbol{B}(t)\boldsymbol{L})^T\boldsymbol{Px}(t)$$
$$+\frac{\tau}{\alpha}\boldsymbol{x}^T(t)(\boldsymbol{A}+\Delta\boldsymbol{A}(t))^T(\boldsymbol{A}+\Delta\boldsymbol{A}(t))\boldsymbol{x}(t)+\tau\boldsymbol{f}_a^T(t)(\boldsymbol{I}-\boldsymbol{L})\boldsymbol{B}^T\boldsymbol{B}(\boldsymbol{I}-\boldsymbol{L})\boldsymbol{f}_a(t)$$
$$+\frac{\tau}{\beta}\boldsymbol{x}^T(t)\boldsymbol{K}^T(\boldsymbol{BL}+\Delta\boldsymbol{B}(t+d(t+\theta)\boldsymbol{L}))^T(\boldsymbol{BL}+\Delta\boldsymbol{B}(t+d(t+\theta)\boldsymbol{L})\boldsymbol{Kx}(t) \tag{15}$$

According to Lemma 1[8], the following inequalities are obtained:

$$\boldsymbol{P}\Delta\boldsymbol{A}(t)+\Delta\boldsymbol{A}(t)^T\boldsymbol{P}\le\zeta\boldsymbol{P}\overline{\boldsymbol{A}}^2\boldsymbol{P}+\frac{1}{\zeta}\boldsymbol{I}$$
$$\boldsymbol{P}\Delta\boldsymbol{B}(t)\boldsymbol{LK}+\boldsymbol{K}^T\boldsymbol{L}^T\Delta\boldsymbol{B}(t)^T\boldsymbol{P}\le\eta\boldsymbol{P}\overline{\boldsymbol{B}}^2\boldsymbol{P}+\frac{1}{\eta}\boldsymbol{KK}^T$$
$$\frac{\tau}{\beta}\boldsymbol{K}^T(\boldsymbol{BL}+\Delta\boldsymbol{B}(t+d(t+\theta)\boldsymbol{L}))^T(\boldsymbol{BL}+\Delta\boldsymbol{B}(t+d(t+\theta)\boldsymbol{L})\boldsymbol{K}$$
$$=\frac{\tau}{\beta}\boldsymbol{K}^T[\boldsymbol{L}^T\boldsymbol{B}^T\boldsymbol{BL}+2\boldsymbol{L}^T\boldsymbol{B}^T\Delta\boldsymbol{B}(t+d(t+\theta)\boldsymbol{L})$$
$$+\boldsymbol{L}^T\Delta\boldsymbol{B}^T(t+d(t+\theta))\Delta\boldsymbol{B}(t+d(t+\theta)\boldsymbol{L})]\boldsymbol{K}$$
$$\le\frac{\tau}{\beta}\boldsymbol{K}^T[\boldsymbol{L}^T\boldsymbol{B}^T\boldsymbol{BL}+\boldsymbol{L}^T\boldsymbol{B}^T\overline{\boldsymbol{B}}^2\boldsymbol{BL}+(1+\sigma_{\max}^2(\overline{\boldsymbol{B}}))\boldsymbol{I}]\boldsymbol{K} \tag{16}$$

Lemma 2[9]: Let $\boldsymbol{A}$ and $\Delta\boldsymbol{A}$ be $n\times n$ real matrices and assume inequality $\Delta\boldsymbol{A}(t)\Delta\boldsymbol{A}^T(t)\le\overline{\boldsymbol{A}}^2$ is satisfied, where $\overline{\boldsymbol{A}}$ is a symmetric matrix. Then for any $0<\varepsilon<1$, we have

$$(\boldsymbol{A}+\Delta\boldsymbol{A})(\boldsymbol{A}+\Delta\boldsymbol{A})^T\le\frac{1}{1-\varepsilon}\boldsymbol{AA}^T+\frac{1}{\varepsilon}\overline{\boldsymbol{A}}^2$$

Let $\delta(\alpha+\beta+1)=\delta_1$, then

$$\tau(\alpha+\beta+1)\boldsymbol{P}(\boldsymbol{B}+\Delta\boldsymbol{B}(t))\boldsymbol{LKK}^T(\boldsymbol{BL}+\Delta\boldsymbol{B}(t)\boldsymbol{L})^T\boldsymbol{P}$$
$$\le\tau\kappa\boldsymbol{P}\left(\frac{(\alpha+\beta+1)^2}{\alpha+\beta+1-\delta_1}\boldsymbol{BLL}^T\boldsymbol{B}+\frac{(\alpha+\beta+1)^2}{\delta_1}\overline{\boldsymbol{B}}^2\right)\boldsymbol{P} \tag{17}$$

where $\alpha+\beta+1-\delta_1>0$. κ is a given constant satisfying $\kappa>0$ and $KK^T \le \kappa I$.

Lemma 3[9]: Let *A* and Δ*A* be $n\times n$ real matrices and assume inequality $\Delta A(t)\Delta A^T(t)\le \bar{A}^2$ is satisfied, where $\bar{A}$ is a symmetric matrix. Then for any $0<\varepsilon<1$ and $1-\varepsilon\bar{A}^2>0$, we have

$$(A+\Delta A)^T(A+\Delta A)\le A^T(1-\varepsilon\bar{A}^2)^{-1}A+\frac{1}{\varepsilon}\sigma I$$

where $\sigma=\begin{cases}0 & \Delta A\equiv 0\\ 1 & otherwise\end{cases}$. Let $\alpha\delta'=\delta_2$, the following inequalities are satisfied:

$$\frac{\tau}{\alpha}x^T(t)(A+\Delta A(t))^T(A+\Delta A(t))x(t)\le \tau[A^T(\alpha I-\delta_2\bar{A}^2)^{-1}A+\frac{1}{\delta_2}I]$$

Based on above inequalities, formula (15) becomes:

$$\begin{aligned}\dot{V}(x,t)\le{}& x^T(t)(A^TP+AP+(BLK)^TP+PBLK)x(t)\\ &+x^T(t)\{(\zeta P\bar{A}^2P+\frac{1}{\zeta}I+\eta P\bar{B}^2P+\frac{1}{\eta}KK^T)\\ &+\tau\kappa P(\frac{(\alpha+\beta+1)^2}{\alpha+\beta+1-\delta_1}BLL^TB+\frac{(\alpha+\beta+1)^2}{\delta_1}\bar{B}^2)P\\ &+\tau[A^T(\alpha I-\delta_2\bar{A}^2)^{-1}A+\frac{1}{\delta_2}I]\\ &+\frac{\tau}{\beta}K^T[L^TB^TBL+L^TB^T\bar{B}^2BL+(1+\sigma^2_{\max}(\bar{B}))I]K\}x(t)\\ &+2x^T(t)PB(I-L)f_a(t)+\tau f_a^T(t)(I-L)B^TB(I-L)f_a(t)\end{aligned}\quad(18)$$

If $f_a(t)\equiv 0$, the stability of system is only considered, then

$$x^T(t)PB(I-L)f_a(t)+\tau f_a^T(t)(I-L)B^TB(I-L)f_a(t)=0$$

So formula(18) is simplified as $\dot{V}(x,t)\le x^T(t)\hat{M}x(t)$, where

$$\begin{aligned}\hat{M}={}&A^TP+AP+(BLK)^TP+PBLK+\zeta P\bar{A}^2P+\frac{1}{\zeta}I\\ &+\tau\kappa P(\frac{(\alpha+\beta+1)^2}{\alpha+\beta+1-\delta_1}BLL^TB+\frac{(\alpha+\beta+1)^2}{\delta_1}\bar{B}^2)P\\ &+\frac{\tau}{\beta}K^T(L^TB^TBL+L^TB^T\bar{B}^2BL+(1+\sigma^2_{\max}(\bar{B}))I)K\\ &+\frac{1}{\eta}KK^T+\tau(A^T(\alpha I-\delta_2\bar{A}^2)^{-1}A+\frac{1}{\delta_2}I)+\eta P\bar{B}^2P\end{aligned}\quad(19)$$

Therefore, if $\hat{M}<0$, $\dot{V}(x,t)<0$. That means the closed-loop system express by (5) is stable.

According to Shur Lemma, $\hat{M}<0$ is equal to matrix inequality:

$$\hat{M}=\begin{bmatrix}\hat{M}_{11} & \hat{M}_{12}\\ \hat{M}_{12}^T & \hat{M}_{22}\end{bmatrix}<0\quad(20)$$

Where

$$\hat{M}_{11}=A^TP+AP+(BLK)^TP+PBLK+\zeta P\bar{A}^2P+\eta P\bar{B}^2P,$$

$$\begin{aligned}\hat{M}_{12}=[&I\quad K\quad \tau\kappa\lambda PBL\quad \tau\kappa\lambda P\bar{B}\quad \tau A^T\quad \tau I\\ &\tau K^TL^TB^T\quad \tau K^TL^TB^T\bar{B}\quad \tau K^T\quad \tau\sigma_{\max}(\bar{B})K^T]\end{aligned}$$

$$\begin{aligned}\hat{M}_{22}=diag\{&-\zeta I,-\eta I,-\tau\kappa(\lambda-\delta_1)I,-\tau(\alpha I-\delta_2\bar{A}^2)I,\\ &-\tau\delta_2 I,-\tau\beta I,-\tau\beta I,-\tau\beta I,-\tau\beta I\}\end{aligned}$$

Because matrix inequalities(20) is a nonliear matrix inequality about P and K, it can not compute using Linear Matrix Inequality toolbox. So $diag\{P^{-1},I,I,I,I,I,I,I,I,I\}$ is multiplied with both sides of inequality(19). Let $X=P^{-1}, Y=KX$, then above matrix inequality becomes inequality (6). From a group of feasible solution as (X,Y), controller with $K=YX^{-1}$ is obtained.

3.2 Design of fault-tolerant guaranteed cost controller

Theorem 2:Consider the system (5), for given positive constane k, τ, ρ, if there exist positive definite symmetry matrix X, symmetry matrix Y and positive scalar α, β, ζ, η, δ1, δ2，the following linear matrix inequality is satisfied:

$$\begin{bmatrix}M_0 & 0\\ 0 & M_f\end{bmatrix}<0\quad(21)$$

s.t. $\alpha+\beta+1-\delta_1>0$, $\alpha I-\delta_2\bar{A}^2>0$

Then the system shown in formula(5) with $K=YX^{-1}$ is stable and $J<0$. When system is normal , *M* is rewritten as M_0.

$$M_f=\begin{bmatrix}M & XC^T & B(I-L)\\ * & -I & 0\\ * & * & -(\rho I-\tau(I-L)^TB^TB(I-L)\end{bmatrix}$$

Proof: Considering the performance index:

$$\begin{aligned}J&=\int_0^\infty\left[y^T(t)y(t)-\rho^2f_a^T(t)f_a(t)\right]dt\\ &\le\int_0^\infty\begin{bmatrix}x(t)\\ f_a(t)\end{bmatrix}^T\begin{bmatrix}\hat{M}+C^TC & PB(I-L)\\ (I-L)^TB^TP & -(\rho^2I-\tau(I-L)^TB^TB(I-L))\end{bmatrix}\begin{bmatrix}x(t)\\ f_a(t)\end{bmatrix}dt\\ &\quad-V(\infty)\end{aligned}$$

If $J<0$, $\begin{bmatrix}\hat{M}+C^TC & PB(I-L)\\ (I-L)^TB^TP & -(\rho^2I-\tau(I-L)^TB^TB(I-L))\end{bmatrix}<0$ where $(\rho I-\tau(I-L)^TB^TB(I-L)>0$.

According to Shur Lemma, formula(21) is equal to matrix inequality:

$$\begin{bmatrix}\hat{M} & C^T & PB(I-L)\\ * & -I & 0\\ * & * & -(\rho^2I-\tau(I-L)^TB^TB(I-L))\end{bmatrix}<0\quad(22)$$

Because fomula (22) is a nonliear matrix inequality, it can not be computed using LMI toolbox. So

$diag\{\boldsymbol{P}^{-1}, \boldsymbol{I}, \boldsymbol{I}, \boldsymbol{I}, \boldsymbol{I}, \boldsymbol{I}, \boldsymbol{I}, \boldsymbol{I}, \boldsymbol{I}\}$ is multiplied with both sides of inequality (22), we have

$$\begin{bmatrix} \boldsymbol{P}^{-1}\hat{\boldsymbol{M}}\boldsymbol{P}^{-1} & \boldsymbol{P}^{-1}\boldsymbol{C}^T & \boldsymbol{B}(\boldsymbol{I}-\boldsymbol{L}) \\ * & -\boldsymbol{I} & 0 \\ * & * & -(\rho\boldsymbol{I}-\tau(\boldsymbol{I}-\boldsymbol{L})^T\boldsymbol{B}^T\boldsymbol{B}(\boldsymbol{I}-\boldsymbol{L}) \end{bmatrix} < 0 \tag{23}$$

Let $\boldsymbol{X}=\boldsymbol{P}^{-1}$, $\boldsymbol{Y}=\boldsymbol{K}\boldsymbol{X}$, then above matrix inequality becomes:

$$\boldsymbol{M}_f := \begin{bmatrix} \boldsymbol{M} & \boldsymbol{X}\boldsymbol{C}^T & \boldsymbol{B}(\boldsymbol{I}-\boldsymbol{L}) \\ * & -\boldsymbol{I} & 0 \\ * & * & -(\rho\boldsymbol{I}-\tau(\boldsymbol{I}-\boldsymbol{L})^T\boldsymbol{B}^T\boldsymbol{B}(\boldsymbol{I}-\boldsymbol{L}) \end{bmatrix} < 0 \tag{24}$$

If inequality(2) has solution, closed-loop system shown in formula (5) can guarantee the asymptotic stability and satisfy $J<0$.

4 Simulations Samples and Analysis

Consider a network control system with uncertain parameters, the closed-loop model is

$$\dot{\boldsymbol{x}}(t) = (\boldsymbol{A}+\Delta\boldsymbol{A}(t))\boldsymbol{x}(t) + (\boldsymbol{B}+\Delta\boldsymbol{B}(t))\boldsymbol{L}\boldsymbol{K}\boldsymbol{x}(t-d(t)) + \boldsymbol{B}(\boldsymbol{I}-\boldsymbol{L})\boldsymbol{f}_a(t)$$

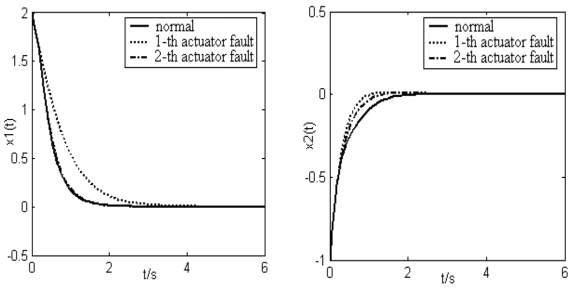
where $\boldsymbol{A}=\begin{bmatrix} -1.3 & -0.5 \\ 0.7 & -1.8 \end{bmatrix}$, $\Delta\boldsymbol{A}=\begin{bmatrix} 0.2\sin t & 0 \\ 0 & 0.2\cos t \end{bmatrix}$, $\boldsymbol{B}=\begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix}$, $\Delta\boldsymbol{B}=\begin{bmatrix} 0.2\sin t & 0 \\ 0 & 0.2\cos t \end{bmatrix}$, $\boldsymbol{C}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\bar{\boldsymbol{A}}=\bar{\boldsymbol{B}}=\begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}$.

In general, initial states are randomly chosen as $[x_1(0) \quad x_2(0)]^T = [2 \quad -1]^T$. Suppose $d_{sc}(t)=d_{ca}(t)=0.1+0.1\sin t$ is network-induced time-delay. So $d(t)=0.2+0.2\sin t$. Let $\tau=0.4$. Impulse signal is adopted as disturbance caused by faults. If no actuator fault happens in control system, $\boldsymbol{L}_0=diag\{1,1\}$. $\boldsymbol{L}_1=diag\{0,1\}$ or $\boldsymbol{L}_2=diag\{1,0\}$ respectively indicate 1-th actuator fault or 2-th actuator fault.

A. Example1: Fault-tolerant controller

Suppose $k=1.5$. Global optimal solution of inequality(6) is obtained as $t\min=-0.0505$ adopting LMI toolbox. Because of $t\min<0$, LMI is feasible. And a group of feasible solutions are obtained. That is, $\zeta=87.6505$, $\eta=94.9209$, $\lambda=108.2793$, $\delta_1=108.2793$, $\delta_2=157.6753$, $\alpha=110.2793$, $\beta=159.1502$. Variable matrixes of the system is $\boldsymbol{X}=\begin{bmatrix} 20.4171 & -0.6198 \\ -0.6198 & 19.0000 \end{bmatrix}$, $\boldsymbol{Y}=\begin{bmatrix} -19.0230 & 2.4639 \\ -8.6409 & 15.2920 \end{bmatrix}$. Proportion constant of fault-tolerant guaranteed cost controller is $\boldsymbol{K}=\begin{bmatrix} -0.9287 & 0.0994 \\ -0.4481 & -0.8195 \end{bmatrix}$.

Beasd on above parameters, response curves of states are shown in Fig.1.



$x_1(t)$ $x_2(t)$

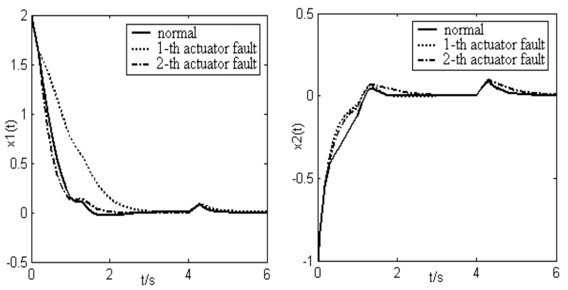
Fig.1 Response curves of states

It is obvious that whether actuator fault happens, the controller can guarantee the asymptotic stability.

B. Example2:Fault-tolerant guaranteed cost controller

Suppose $k=1.5$. $\rho=0.9$ is degradation degree of disturbance. Global optimal solution of inequality (24) is obtained as $t\min=-0.0031$ adopting LMI toolbox. Because of $t\min<0$, LMI is feasible which ensure $J<0$. And a group of feasible solutions are obtained. That is, $\zeta=0.7415$, $\eta=0.7297$, $\lambda=2.4106$, $\delta_1=0.4106$, $\delta_2=0.4999$, $\alpha=0.3508$, $\beta=1.3213$. Variable matrixes of the system is $\boldsymbol{X}=\begin{bmatrix} 0.1290 & -0.0188 \\ -0.0188 & 0.1300 \end{bmatrix}$, $\boldsymbol{Y}=\begin{bmatrix} -0.1554 & 0.0175 \\ 0.0601 & -0.1148 \end{bmatrix}$. Proportion constant of fault-tolerant guaranteed cost controller is $\boldsymbol{K}=\begin{bmatrix} -1.2109 & -0.0403 \\ -0.6076 & -0.9705 \end{bmatrix}$.

Beasd on above parameters, response curves of states are shown in Fig.2.



$x_1(t)$ $x_2(t)$

Fig.2 Response curves of states

It is obvious that whether actuator fault happens, the controller can not only guarantee the asymptotic stability, but also ensure the robustness and anti-disturbance performance.

4. Conclusion

Aiming at a class of controlled plant with uncertain parameters, the closed-loop model of uncertain networked control system with time-varying delay is established considering disturbance caused by actuator fault. The degradation performance index of systems for fault is defined. Combing with fault-tolerant control and guaranteed cost control, a fault-tolerant guaranteed cost controller is designed adopting Lyapunov stability analysis method. Simulation results indicate the controller can not only guarantee the asymptotic stability, but also ensure the robustness and anti-disturbance performance.

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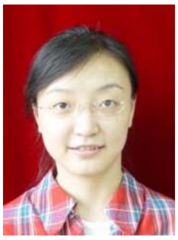

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