Application of affine invariant Fourier descriptor to shape-based image retrieval

P. Lionel Evina Ekombo, Noureddine Ennahnahi, Mohammed Oumsis, Mohammed Meknassi

Informatics Statistics & Quality Laboratory (LISQ), Dhar Mehraz Sciences Faculty, Sidi Mohammed Ben Abdellah University – Fez-MOROCCO

Summary
Contour-based descriptors are among the main approaches in content based image retrieval. Most of these descriptors are based on Fourier transform and use various shape signatures and retrieval methods. There retrieval rates are good, and they perform generally in speeded ways. This paper presents a new approach which looks able to upgrade these results. Prior to feature extraction, the shape undergoes moment-based preprocessing in order to ensure affine transformations robustness. A double signature is computed from shape radius and specific angles. Then, we compute the coefficients of Fourier descriptors, and with a specific similarity measure we get an efficient shape retrieval performance. Our approach is compared to a classical Fourier descriptor and to another variant using PCA. We also design a comparison with other shape contour-based descriptors.

Key words:
Closed curve, Fourier Descriptor, similarity measure, affine transformation

1. Introduction

Recent years show a tremendous increase in the using of digital images, due to the high development of technologies and Internet. Because it is easier to capture and store an image, we now possess huge databases accumulated in various fields. So, content based image retrieval (CBIR) can be used as a powerful tool intended for good exploitation of these data. The retrieval process tends to be more complex and critical. In most of cases, images in database are retrieved based on their content's information, like color, texture and shape. In this work, we focus our attention on methods related to the shape. In this context, we distinguish two main categories: region-based and contour-based.

Region-based descriptors take into account all the pixels inside the shape. These descriptors require some statistical methods and support grayscale images. The known approaches usually cited in the literature are: the “shape matrix” [1], the moments (Zernike, pseudo-Zernike or Legendre) [2]-[4], the histograms [5] and the generic Fourier descriptor [6].

In the contour-based approach, we notice some natural and elementary methods like polygonal approximation, chain code and smoothed curvatures [7]-[9]. We can also mention some advanced approaches based on spectral descriptors such as the Hough transformation and the CSS (curvature scale space) descriptor. Introduced by Mokhtarian [10], the CSS consists in building a map containing inflexion points of the contour after several smoothing steps performed by a Gaussian function. This descriptor seems reliable against similarities and noised contours. This CSS method is commonly used for the indexation [11] and it has been adopted like the MPEG-7 descriptor. The Hough transform, originally used only for detecting lines, was adapted under the denomination of the Generalized Hough Transformation (GHT) [12] to allow detection of any form and was used like a descriptor. Other spectral descriptors have been developed such as the Fourier descriptors (FD) [13], declinations as Fourier-Mellin [14] and the wavelet descriptors [15]. All these global descriptors, except wavelets, are invariant to similarities. The Fourier Descriptor (FD) is more promising, computationally efficient and easier to implement. It supports a variety of shape signatures (curvatures, tangents' angles, complex coordinates, centroid distance, etc.) and can be used for the multi-scale approach. The main information on contour is present in the low frequencies (first harmonics) and it is particularly robust to the noise.

In this article, we suggest a new approach in using the Fourier Descriptor to contour-based shape matching. We propose to use moments to get the canonical shape before applying the FD. This step allows removing all affine transformations. A new angle signature and the centroid distance are combined to have a strong shape fingerprint. The normalization of both FDs is expected before the similarity measure step. Then, with the measure of similarity defined and adapted for this double feature vector, we obtain a powerful and robust descriptor. This descriptor gives us efficient performance retrieval assessed by several tests and analysis. A tool that extracts the proposed shape features from images, compares these features with query image features and retrieves the images based on similarity between these two descriptor sets was implemented. The tests were performed on the database containing affine transformations of contours [1].

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The paper is organized in four parts. The section 2 presents the Fourier descriptor. In the section 3, the description of our shape signature and the normalization scheme of the FD are exposed. The section 4 explains the use of moments to get an affine invariant shape. The section 5 analyses the experimental results and the conclusion closes this paper.

2. The Fourier Descriptor

For a given shape defined by a closed curve *C*, let *s(t) with t=0,1,…,T*, be a signal or shape signature such as (radius, abscissa, curvature…) extracted from the curve *C*. if *s(t)* is periodic of T period, it can be decomposed into Fourier series in the following way:

$$s(t) = \sum_{-\infty}^{\infty} a_n \exp(j2\pi nt / T) \quad (1)$$

The Fourier coefficients a_n are given by the Fourier transform of *s(t)*:

$$a_n = \frac{1}{T}\int_0^T s(t)\exp(-j2\pi nt / T)dt \quad n \in \mathrm{Z} \quad (2)$$

The discrete Fourier transform of *s(t)* is given by:

$$a_n = \frac{1}{N}\sum_{t=0}^{N-1} s(t)\exp(-j2\pi nt / N) \quad n = 0,1,\ldots,\mathrm{N-1} \quad (3)$$

Since *s(t)* is a 1D function representing the shape, it is crucial for it to have an effective capability of description. All coefficients a_n, *n=0,1,…,N-1*, are belonged to a feature vector called Fourier descriptor (FD). Coefficients a_n are usually noted FD_n, *n=0,1,…,N-1.* The value *N* represents the order of descriptor FD. This Fourier descriptor represents the shape in spectral domain and the coefficients a_n are the frequency components. The lower frequencies, the coefficients of lower index, contain information about general features of the shape. And the higher frequencies, the coefficients of high index, contain information about finer details. Thus the larger the number of coefficients is, the more the curve is well described. But, the great interest of the Fourier descriptor is to describe a shape by only few values.

The regular sampling of the data signal is important for the signal reconstruction. Thus, the non-uniformity of the data signal leads to distortions in the rebuild, and results in slower convergence to the original signal and even a possible divergence of the reconstruction. Uniformity in sampling is a key property in the signal modeling Fourier transform-based process.

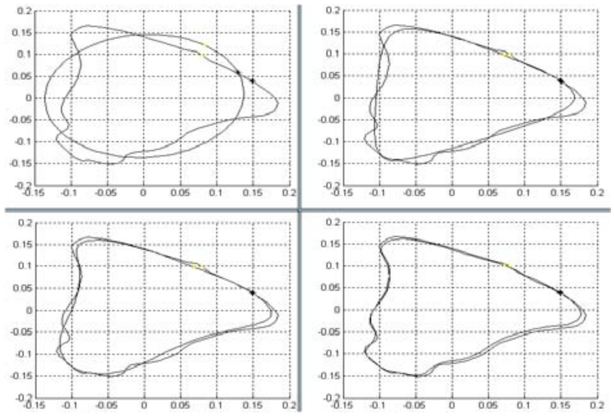

Fig. 1 Reconstruction of a shape with the 2D Fourier descriptor (X and Y) with a regular sampling. The order of the descriptor is set respectively from left to right and from top to bottom 2, 3, 7 and 15.

In this paper, the centroid signature is adopted and considered like the classic FD signature since recent work has proven that it outperforms other signatures in shape retrieval [16]. Zhang shows in [17] that Manhattan distance (city-block) applied to the FD provides the best performance compared to the others distances. Zhang has also underlined the fact that just the first 15 harmonics of FD are sufficient to get efficient results, and beyond this limit, no significant increase of the retrieval rate is gained.

3. Shape signatures and the Fourier descriptor normalization

3.1 Shape signatures

As stated in the previous section, FD needs a shape signature to be modeled. To ensure efficient shape retrieval, we must choose a signature able to keep a significant description of the original shape. Thus, our interest has been oriented on a dual signature which combines both radius and angles. Existing works had already exploited both signatures. However, in our case we pick out a very special signature. It is significant and makes it possible to reconstruct the shape faithfully.

Distance between the center of gravity and each points of the contour provides us the radius signature.

$$r_i = \sqrt{(x_i - x_c)^2 + (y_i - y_c)^2} \quad \text{with } i=1,\ldots,N \quad (4)$$

$$x_c = \sum_{i=1}^{N} x_i / N \textit{ and } y_c = \sum_{i=1}^{N} y_i / N$$

where *N* denotes the number of boundary points.

The second signature is constructed by the measure of the angle between successive points. The angle is positive in the counterclockwise and negative in clockwise.

$$\theta_i = \cos^{-1}\left((c^2 + a^2 - b^2)/2ca\right) \quad (5)$$

$$if\ i < N\ c = r_{i+1}\ else\ c = r_1;\ a = r_i;\ b = \sqrt{c^2 + a^2}$$

θ_i changes its sign following the position of point $i+1$ and the reference point i.

In the section intended for experimental results, we will compare the results of our angle signature and the classic angle signature below:

$$\delta_i = \tan^{-1}\left(y_i - y_c / x_i - x_c\right) \quad (6)$$

3.2 Normalization of Fourier Descriptor

In sub-section *A.*, we extract signature which runs our retrieval process. The normalization of the Fourier coefficients within [0 1] is required to get efficient retrieval and it is done following the expression:

$$FD = \left(\left|\frac{FD_1}{FD_0}\right|, \left|\frac{FD_2}{FD_0}\right|, \ldots, \left|\frac{FD_n}{FD_0}\right|\right) \quad (7)$$

Where n is the number of the FD harmonic and FD_i is the i^{th} harmonic.

With the signature (radius and angle) defined in this section, we have invariance to the translation. The rotation of the shape and the starting point, involve only a rotation of the periodic signatures, and has no influence on the FD since it is a global descriptor. So, our approach is now invariant to translation, rotation and starting point and reflection. And what about the rest of affine transformations? Normalization contours provide a solution to this problem. However, it is performed before the extraction of the signature.

4. Curve normalization

Curve Normalization is designed to get a canonical curve pose and to eliminate the effect of any possible rotation. The Moment and the Principal Component Analysis (PCA) are the main techniques used for this purpose. The steps of these two methods are described in this section. We present firstly curve normalization by the moments. Then, in the second part, the curve normalization by PCA is carried out.

4.1 Curve normalization by moments

To do this kind of normalization, we use moments which seem be able to eliminate the main translation, the scaling and the skewing.

For a curve C, the (p,q)-order moments is written as :

$$m_{pq}(C) = \frac{1}{N}\sum_{i=1}^{N} x_i^p y_i^q \quad (8)$$

with N is the number of boundary points.

The normalization of a curve C implies specific values for the moments with order 1 and 2. So, we have:

$$\begin{aligned} m_{10}(C) = m_{01}(C) = m_{11}(C) = 0 \\ m_{20}(C) = m_{02}(C) = 1 \end{aligned} \quad (9)$$

The normalization stages of the curve C in C' are:

$$x_1 = x - m_{10}(C);\ y_1 = y - m_{01}(C) \quad (10)$$

$$x_2 = x_1 / \sqrt{m_{20}(C)};\ y_2 = y_1 / \sqrt{m_{02}(C)} \quad (11)$$

$$\begin{bmatrix} x_3 \\ y_3 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} x_2 & -y_2 \\ x_2 & y_2 \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} \quad (12)$$

$$x' = x_3 / \sqrt{m_{20}(C)};\ y' = y_3 / \sqrt{m_{02}(C)} \quad (13)$$

In chronological order, the steps for normalization reduce the curve in the center of gravity and eliminate the vertical and horizontal stretch. After that, we make a rotation of 45° (π/4) in order to remove again the vertical and horizontal stretch. Then, we get a standard shape ready to undergo the signature extraction process.

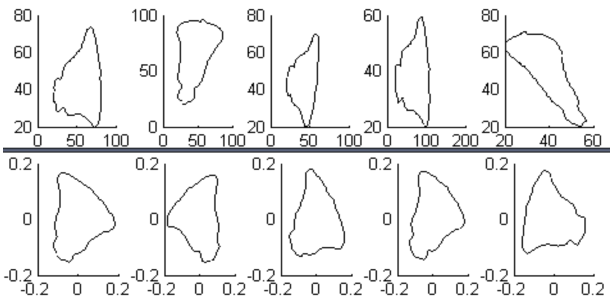

Fig. 2 Curves before (top line) and after (bottom line) normalization

4.2 Curve normalization by PCA

PCA allows normalization along the principle component directions. To apply it, we follow the steps below that are better explained by Sener [18].

Let $P_i = [x_i, y_i]^T$ for $i=1,2,\ ...,N$, which represents the range data of a curve $C = [P_1, P_2, \cdots, P_N]$ and $\hat{C}$ his normalized form.

The center μ and the covariance matrix ∑ of C are respectively defined by

$$\mu \overset{def}{=} \frac{1}{N}\sum_{i=1}^{N} P_i \quad \text{and} \quad \Sigma \overset{def}{=} \frac{1}{N}\sum_{i=1}^{N} (P_i - \mu)(P_i - \mu)^T \quad (14)$$

The covariance matrix ∑ is symmetric and positive definite. Therefore, it is diagonalizable by an orthogonal matrix E composed by the eigenvectors of ∑, so that:

$$D = E^T \Sigma E \Rightarrow \Sigma = EDE^T \quad (15)$$

where D is a positive definite and diagonal matrix. D is composed of $\sum$ eigenvalues.

Now, we consider the normalized data set $\hat{C}$, obtained from C via the transformation:

$$\hat{C} = D^{-1/2} E^T (C - \mu) \quad (16)$$

By this PCA normalization, the curve signature is invariant to translation, scale and shear. The PCA is often combined with Independent Component Analysis (ICA) [19] and high order moments to ensure rotation and reflection invariance.

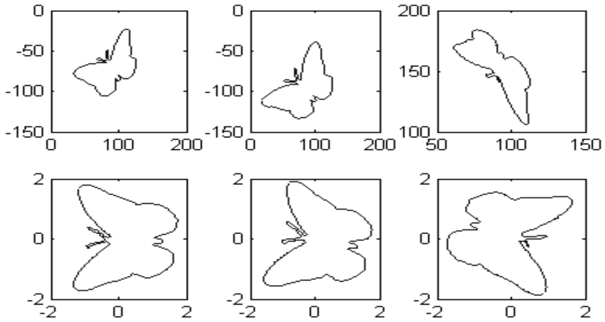

Fig. 3 Curves before (top line) and after (bottom line) PCA normalization

With this both curve normalization, we get a template curve. With the properties of this new curve and those of signature, we have a complete invariance to affine transformations. The simulation stages led us to pick up Moment normalization against PCA. The Moment method gives better retrieval rate than PCA method.

5. Experimental results

To evaluate the retrieval performance of our approach described in section 3 and 4, we apply our tests on Multiview Curve Dataset [1]. A suitable similarity measure is defined for the descriptor. The performance measures use here are described. At the end, comparative studies are conducted to show the insight of our approach.

5.1 Curve database

The evaluation and tests will be done on Multiview Data Curve (MCD). This dataset contains contours extracted from shapes found in the MPEG-7 set B database. It regroups 40 shape categories from different angles of views. These views correspond clearly to different affine transformations (perspective distortions, rotation, scaling and reflections) of each original shape. There are 14 views for each object, which means 560 curves in the whole dataset. So, this database is suitable for evaluating affine-invariant descriptors.

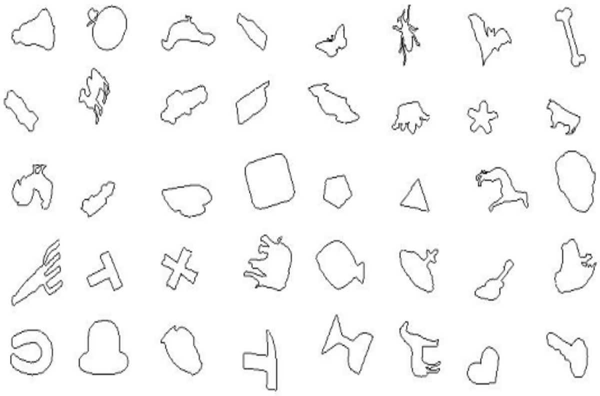

Fig. 4 Each shape represents a class of database MCD

5.2 Similarity measure

The measure of similarity to our descriptor, noted *Sim*, provides a classification of objects following the curve shape criteria. It is a measure based on the Manhattan distance. When a request is done, we measure the distance between radius vectors and angle vectors separately. Then, the merger of both distance values is made to obtain the similarity between the request object and each dataset shape. After preliminary tests, in merge process, we give more weight to radius than angle.

$$Sim(C_1, C_2) = D_{manhatt\tan}(FD_{r1}, FD_{r2}) + 0.25 * D_{manhatt\tan}(FD_{\theta 1}, FD_{\theta 2}) \quad (17)$$

In simulation, we also use *Sim* measure on Fourier descriptor which uses signature radius and classic angle δ.

5.3 Performance measures

We present briefly the performance tools used to evaluate our approach. These tools are particularly useful for studying the quality of ranked matches on descriptors having specific properties. a complete description of these quantitative statistics should be found in [18]. To complete this list of performance tools, we built the performance per classes (PP) and mean average precision (MAP).

Precision-recall: is a chart describing the relationship between precision and recall in a ranked list of matches. Recall and precision are easy to compare across queries and engines. Precision is a measure of the usefulness or exactness of top ranked matches. Precision also measures the how well the engine performs in not returning non-relevant documents. Recall is a measure of completeness. Also, it measures the engine performs in finding relevant documents.

$$\Pr ecision = \frac{|\{revelant\ documents\} \cap \{documents\ retrieved\}|}{|\{documents\ retrieved\}|}$$
$$\operatorname{Re} call = \frac{|\{relevant\ documents\} \cap \{documents\ retrieved\}|}{|\{relevant\ documents\}|} \quad (18)$$

Nearest neighbor: the percentage of the closest matches that belong to the same class as the query. This statistic provides an indication of how well a nearest neighbor classifier would perform. Obviously, an ideal score is 100%, and higher scores represent better results.

First-tier and Second-tier: the percentage of models in the query's class that appear within the top *K* matches, where *K* depends on the size of the query's class. Specifically, for a class with $|C|$ members, $K = |C| - 1$ for the first tier, and $K = 2*(|C| - 1)$ for the second tier. The first tier statistic indicates the recall for the smallest *K* that could possibly include 100% of the models in the query class, while the second tier is a little less stringent. Always, an ideal matching result gives a score of 100%, and higher values indicate better matches.

E-Measure: a composite measure of the precision and recall for a fixed number of retrieved results [21]. The intuition is that a user of a search engine is more interested in the first page of query results than in later pages. So, this measure considers only the first 32 retrieved models for every query and calculates the precision and recall over those results. The maximum score of e-measure is 1.0, and higher values indicate better results. The E-Measure is defined as [21], [22]:

$$E = \frac{2}{1/P + 1/R} \quad (19)$$

Discounted Cumulative Gain (DCG): is a statistic that weights correct results near the front of the list more than correct results later in the ranked list under the assumption that a user is less likely to consider elements near the end of the list. Specifically, the ranked list *R* is converted to a list *G*, where element G_i has value 1 if element R_i is in the correct class and value 0 otherwise. Discounted cumulative gain is then defined as follows [23]:

$$DCG_i = \begin{Bmatrix} G_1, & i = 1 \\ DCG_{i-1} + \dfrac{G_i}{\lg_2(i)}, & otherwise \end{Bmatrix} \quad (20)$$

This result is then divided by the maximum possible DCG (i.e., that would be achieved if the first *C* elements were in the correct class, where *C* is the size of the class) to give the final score:

$$DCG = \frac{DCG_k}{1 + \sum_{j=2}^{|C|} 1/\lg_2(j)} \quad (21)$$

where *k* is the number of models in the database.

The performance per classes (PP): is the mean retrieval rate of all items belong to the same class. In this case, we check top 28 matches to determine the most similar shapes for every query shape. The retrieval rate for a method is equal to the mean of all performances collected per classes.

Mean average precision (MAP): is the average of Average Precision (AP) values over several topics. It is also an overall summary measure quality of retrieval engines. AP is average of precision value obtained for set of top k documents existing after each relevant document is retrieved. The measure AP that combines precision, relevance ranking, and overall recall is defined as follows:

$$AP = \frac{\sum_{r=1}^{k} \left(P(r) * rel(r)\right)}{number\ of\ relevant\ documents} \quad (22)$$

where r is the rank, k the number retrieved, rel a binary function on the relevance of a given rank, and P precision at a given cut-off rank.

5.4 Evaluation of normalization and signature

This section is intended for evaluating the effect of the normalization technique used, and the shape signature chosen for the Fourier Descriptor.

To reveal the strength of normalization chosen, we compare the Fourier descriptor in three different contexts: - the classic centroid distance FD (FDs); - the FD after a normalization by Moments (FDn); - the FD after a normalization by PCA (FDpca).

Table 1: Global performance per classes (PP) and MAP on MCD database

Descriptor / Discrimination	FDs	FDpca	FDn
Nearest Neighbor	100%	100%	100%
First Tier	31.8%	75.0%	79.5%
Second Tier	41.0%	86.5%	88.88%
E-Measure	25.5%	51.2%	53.0%
DCG	65.9%	93.0%	94.6%
PP	48.5%	89.7%	92.52%
MAP	44.1%	85.46%	88.51%

s = no normalized curve, pca = pca normalized curve and n = moment normalized curve

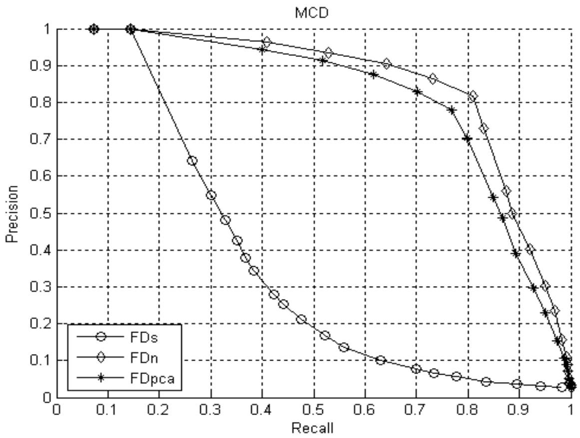

Fig. 5 Precision and recall for FD on classic form and both normalizations, by Moment and PCA.

Fig. 5 and Table 1 gather good views of performance produced by the normalization on Fourier descriptors. The retrieval accuracy carried out by descriptor FDn is higher than the others.

Now, it is time to evaluate the descriptor signature. For this goal, we measure the performance of our signature and two other ones. The experiments are done with and without moment normalization. We investigate the impact produced while combining a good signature with an efficient normalization technique.

Table 2: Statistics on MCD database for descriptors

Descriptor / Discrimination	FDs	FDn	FDs (R+δ)
Nearest Neighbor	100%	100%	100%
First Tier	31.8%	79.5%	27.6%
Second Tier	41.0%	88.8%	35.8%
E-Measure	25.5%	53.0%	22.6%
DCG	65.9%	94.6%	62.2%
PP	48.5%	92.5%	43.87%
MAP	44.1%	88.5%	40.15%

Descriptor / Discrimination	FDn (R+δ)	FDs (R+θ)	FDn (R+θ)
Nearest Neighbor	93.0%	100%	100%
First Tier	56.5%	37.3%	89.0%
Second Tier	69.3%	45.5%	94.8%
E-Measure	42.3%	28.2%	55.2%
DCG	83.0%	70.2%	97.5%
PP	75.0%	52.7%	96.4%
MAP	68.33%	49.0%	94.43%

s = no normalized curve and n = moment normalized curve

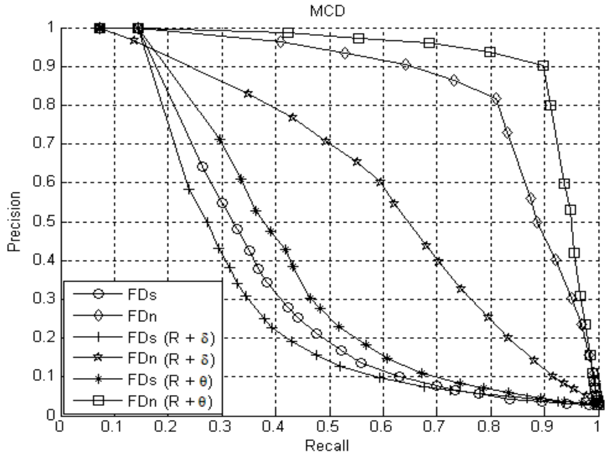

Fig. 6 Precision and recall for three descriptors before and after normalizations of MCD curves.

5.5 Comparative study with some affine invariants

In this section, we compare the descriptor with some affine invariant methods. We choose three methods based on Fourier descriptors: - the affine invariant Fourier descriptor (AIFD) presented by Chaker in [24, 25]; - the affine curvature using a B-spline [24] which is an upgrade of previous work proposed by Weiss [26]; - and the affine invariant Fourier descriptors defined by Arbter [27].

In this evaluation, each shape from the MCD was used as a query. And the number of similar shapes was counted in the top 16 matches. In Table 3, we show the retrieval rates for the same 10 random query shapes presented by Chaker in [24]. Our invariant affine descriptor significantly outperforms the other three descriptors.

Table 3: Retrieval rates for 10 random query shapes on MCD database

Shape	**AIFD**	**Arbter' Invariant**	**Affine Curvature**	**FDn (R+θ)**
Bat	100%	28%	14%	100%
Bell	86%	57%	0%	92
Butterfly	86%	43%	14%	91%
Insect	86%	43%	14%	82%
Bone	72%	72%	28%	88%
Camel	72%	43%	28%	100%
Bird	72%	14%	28%	66%
Apple	72%	72%	14%	64%
Bottle	57%	28%	14%	90%
Brick	57%	14%	14%	97%
Mean	**76%**	**41.4%**	**16.8%**	**84.2%**

5.6 Comparative study with other well known methods

In this new analysis, we have confronted our approach to three well known descriptors in the research field. These methods, although contour-based, used other approaches. Thus, we gather the recall-precision chart of our descriptor and the CSS descriptor one [10]. The inverse generalized Hough transform descriptor (IGHTD) of Bourgeois [28], and the Helmholtz curve descriptor (HCD) of Zuliani [29] complete this comparison. We underline the fact that these descriptors are also affine transformations invariant.

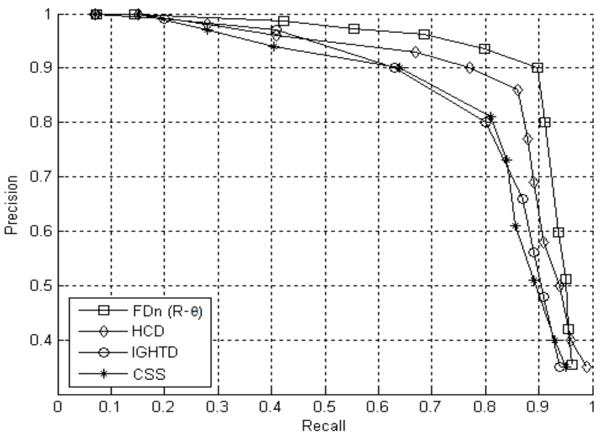

Fig. 7 Precision and recall for four descriptors on MCD database.

We dare to declare after the analysis of the various performance measures, the best quality of our approach. The outperformance of our descriptor is clear on other approaches independently of their origins: -similar to our approach, -invariant to affine transformations or -having various sources approaches.

6. Conclusion

This paper presents a new image retrieval approach based on Fourier descriptor. This approach has two points which make its strength: the double signature based on the radius and specific computed angles, and the curve normalization made by a combination of moments. These two requirements allow our descriptor to be invariant according to many affine transformations. With an adapted similarity measure, our descriptor gives noteworthy performance in shape retrieval. Experiments and tests are made on 560 shapes database. Performance comparisons are made between our descriptor and many others descriptors, and it displays the best results always. The good retrieval rate and others good statistics show clearly that our descriptor is very robust under distortions and affine variations.

In a near future work, we will apply evaluation of descriptor on a bigger database and will compare it with other descriptors.

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