# A Novel Model for Multilayer GSM Network Design

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**Summary**
GSM Network Designs usually offers big challenges for achieving an efficient cost while respecting the complex combinatorial technical constraints. This networks have hundreds or thousands BTS (Base Transceiver Station). They have their traffic grouped in hubs, then in BSC (Base Station Controller) nodes to reach the MSC. Hubs must be elected within the BTS set and BSC nodes have to be geographically allocated in the available sites. Also, the number and model of these BSC impact in the overall cost while the distances affect the transmission costs. This paper presents a mathematical model for designing a GSM network from the BTS lower layer until the MSC layer.

***Key words:***
*Operations Research, GSM Network, Optimization.*

## 1. Introduction

A GSM mobile network is a very complex mix of equipments working on specific functions in an integrated manner. These network elements are organized hierarchically. Closer to the customers lays the BTS (Base Transceiver Station) equipments layer that is the first layer. They are responsible for interfacing the cell phones to the GSM network through radio frequency. These equipments use antennas on top of towers or buildings that are the most visible and known parts of the network by people in general. The BTS are distributed for the whole covering area has a minimum radio-frequency signal that usually guarantees an acceptable conversation quality. It is similar to the power poles with a calculated spacing have an acceptable luminosity. This layer is made of hundreds or thousands of equipments although each one is less expensive. Depending on the traffic demand of a BTS its E1 link can waste its capacity. To avoid this undesirable behavior usually this equipments are shipped with units that allows cross connections between timeslots carried by E1 links. This is a very convenient resource to gather timeslots from E1 links of other BTS and carry then in fewer E1 links with better usage factor. The elected BTS stations to group and cross connect timeslots from a set of BTS are called hubs. Hubs can be considered the second layer of the hierarchy, since it represents the first grouping function in the GSM network that increases the transmission efficiency. Until the second layer, the timeslot handling is deterministic. It is dimensioned in a one-to-one fashion. One cell phone call uses one air voice channel timeslot in the Ater interface to the BTS. One Ater voice timeslot is associated to one sub-timeslot in the Abis interface between BTS and BSC (Base Station Controller). Four sub-timeslots are grouped in one E1 timeslot in this Abis interface. The cross connection always preserves the internal structure of this timeslots. Therefore in a hub, the sum of all traffic timeslots from the grouped BTS side is equals to the sum of all traffic timeslots of the BSC side. There is no intelligence in this operation, only reorganization of the E1 links for better use of its capacities. On the other side, the third layer, BSC equipments use the traffic statistical aspect to significantly reduce the number of channels need for carrying the total traffic. It is no more a one-to-one deterministic association. BSC is the first switch in the GSM network before entering in the network core. Telephony switches recalls the old switchboard that used to connect many subscribers lines to few trunk lines. The trunks are dimensioned based on the subscribers call amount and time period. Agner Krarup Erlang formulated a way of correlate the traffic (in Erlangs), the number of channels and the probability of blocking (also called grade of service or GoS). One of the most used equations to deal with telephony traffic is the Erlang B equation (1).

$$e_b = \frac{\frac{a^n}{n!}}{\sum_{i=0}^{n} \frac{a^i}{i!}} \qquad (1)$$

Although it is an effective way of reducing the transmission E1 lines to MSC, this equipment is significantly more expensive than a BTS. The forth layer of a GSM network is composed by grouping up to tens of BSC in a MSC. MSC is a very complex and expensive switch that accumulates several tasks related to all telecommunication services. It integrates the core of the GSM network among other kind of equipments like HLR

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(Home Location Register), EIR (Equipment Identity Register), SGSN (Serving GPRS Support Node) and others. These core equipments are out of the scope of this paper. One significant part of a GSM network cost comes from the transmission lines. They do the duty of overcoming the distances. Hubs and switches purpose is increasing transmission efficiency and minimize this transmission cost. But one difficulty arises when a designer works on a GSM network: The search for the minimal cost design in a very complex combinational problem. The designer have to elect BTS stations to have the hub function based on the BTS neighborhood. Distance is important in the cost but a large weight comes from the E1 lines from hub to BSC occupation rate. The BTS linked to a hub must sum time slots that maximize the E1 lines occupation rate, but each BTS has a particular time slot demand to be carried. On the third layer, BSC also has to be allocated based on the geographical position that groups the total traffic of a set of BTS and links to MSC with a reduced amount of E1 lines. To turn the combinational problem even harder, BSC equipments have some different models with its respective traffic capacity and acquisition cost. To address this combinational problem this work presents a mathematical model for designing a GSM network with multi-layers that respects the traffic demand and capacities, minimizing the total network cost over a time period of evaluation. In Aguiar et. al. [1] a layer 2 model based on Kubat and Smith [2] and Kubat et. al. [3] is adapted to some scenarios of Brazilian mobile carriers. In Aguiar and Pinheiro [4] a layer 3 integer programming model [5] was developed to determine the association matrix between BTS and BSC, the geographical allocation of BSC nodes, The number and model of BSC and its trunk sizing based on the geographical location and traffic demand of the BTS. This work merges and expands this two isolated layer approaches in a multi-layer design of the GSM network. Ferreira et. al. [6] used Lagrangean Relaxation Method [7] to extend the boundaries for larger network problem instance. Rigolon et. al. [8] extends this line of research with sub gradient methods. Section II shows this mathematical model for the Multi-layer GSM Network Design. Afterwards section III describes the results for simulated networks problems instances. Section IV makes some considerations on the conclusion.

## 2. The Model for the Multilayer GSM Network

This model unifies both model found in Aguiar and Pinheiro [9] and in Aguiar et. al.[10], so it optimizes the designing of a multi-layer GSM Network.

### 2.1 Sets

- $T$ Set of BTS nodes
- $H$ Set of HUB nodes
- $B$ Set of BSC nodes
- $W$ Set of BSC models
- $C$ Set of link capacities

### 2.2 Decision Variables

- $u_{kc}$ Decision variables for choosing the capacity $c \in C$ of E1 lines trunk between BSC $k \in B$ and MSC $r$.
- $v_{kw}$ Decision variables for BSC $k \in B$ model $w \in W$ choice.
- $x_{ijk}$ Decision variables for link allocation between BTS node $i \in T$, HUB node $j \in H$ and BSC node.
- $z_{jk}$ Decision variables for link dimensioning between HUB node $j \in H$ and BSC node $k \in B$.

### 2.3 Constants

- $ct_{ij}$ Link cost between BTS $i \in T$ and HUB $j \in H$ nodes in an analysis time period.
- $ch_{jk}$ Link cost between HUB $j \in H$ and BSC $k \in B$ nodes in an analysis time period.
- $cb_{kc}$ Link cost capacity $c \in C$ between BSC node $k \in B$ and MSC $r$ in an analysis time period.
- $cm_w$ BSC model $w \in W$ acquisition cost considering an analysis time period.
- $ae_i$ BTS node $i \in T$ traffic demand in Erlands.
- $at_i$ BTS node $i \in T$ traffic demand in timeslots.
- $f_c$ Capacity of link $c \in C$ in Erlangs.
- $b$ Capacity ok one E1 link from HUB to BSC in timeslots.
- $e_w$ BSC model $w \in W$ traffic capacity in Erlangs.

### 2.4 Objective Function

The objective function (2) minimizes the HUB nodes, HUB and BSC nodes transmission cost, plus BSC acquisition total cost.

$$\min \sum_{i \in T} \sum_{j \in H} \sum_{k \in B} ct_{ij} x_{ijk} + \sum_{j \in H} \sum_{k \in B} ch_{jk} z_{jk} + \sum_{k \in B} \sum_{c \in C} cb_{kc} u_{kc} + \sum_{k \in B} \sum_{w \in W} cm_w v_{kw} \qquad (2)$$

### 2.5 Constraints

These are the constraints adopted

$$\sum_{j\in H}\sum_{k\in B} x_{ijk} = 0, \forall i \in T \qquad (3)$$

(3) Each BTS must be connected to one and only one HUB.

$$\sum_{i\in T}\sum_{k\in B} at_i x_{ijk} \leq \sum_{k\in B} bz_{jk}, \forall j \in H \qquad (4)$$

(4) Link dimensioning from HUB to BSC

$$x_{ijk} \leq z_{jk}, \forall i \in T, \forall j \in H, \forall k \in B \qquad (5)$$

(5) There must be no allocation through a hub candidate that has no link to a BSC.

$$\sum_{i\in T}\sum_{j\in H} ae_i x_{ijk} \leq \sum_{c\in C} f_c u_{kc}, \forall k \in B \qquad (6)$$

(6) uck dimensioning that allows all BTS assigned to one BSC's traffic flow.

$$\sum_{i\in T}\sum_{j\in H} ae_i x_{ijk} \leq \sum_{w\in W} e_w v_{kw}, \forall k \in B \qquad (7)$$

(7) BSC dimensioning accordingly to the given models and the total traffic demand.

$$\sum_{c\in C} u_{kc} \leq 1, \forall k \in B \qquad (8)$$

(8) Only one trunk capacity can be chosen for one BSC.

$$u_{kc} \in 0,1, \forall k \in B, \forall c \in C \qquad (9)$$

$$v_{kw} \in 0,1, \forall k \in B, \forall w \in W \qquad (10)$$

$$x_{ijk} \in 0,1, \forall i \in T, \forall j \in H, \forall kinB \qquad (11)$$

$$z_{jk} \in Z^+, \forall j \in H, \forall k \in B \qquad (12)$$

## 3. Computational Results

A generator of problem instances was developed to study both individual networks particularities and size impacts. 10 instances of each size class were generated. These classes have the following number of BTS nodes: 5, 10, 15, 20, 25, 30, 35, 40. For each instance this assumptions were taken: The transmission cost is a linear function of distance. The values are local market approximations. There is no price reduction based on the amount of E1 lines. The geographical location of BTS sites is determined randomly with a configurable dispersion. Each BTS site is a candidate for hub and BSC allocation. In real networks, other site locations that have no BTS can be included for candidates. The BTS traffic demand in timeslots is generated randomly from 3 to 10 timeslots that is the approximated value that an E1 line supports. The traffic demand in Erlangs is calculated from the number of voice channels that fits in the number of timeslots or a E1 line, with 2% of GoS. was adopted. There are three BSC models in the simulations with fictitious but reasonable capacity and costs. A small model with 512 Erlangs; a medium model with 1024 Erlangs and a large model with 2048 Erlangs of capacity used is equals to 31 timeslots. These computational tests ran in an AMD Turion 1.8 MHz 64 bits processor with 1 GB of RAM memory. The model was implemented on Ilog OPL integrated environment with CPLEX 10.0 solver library [11]. In Fig. 1, 2, 3 and 4 is shown a solution for a problem instance with 20, 25, 30 and 40 BTS sites respectively. In this solution we can see that there is many hubs concentrating the nearby traffic and linking to the BSC. There are two BSC in the network to handle the traffic generated by subscribers that uses these BTS. They are represented by squares. A large model BSC was chosen to be allocated on the site of BTS 06 (on the right), since it has 21 BTS to deal with. On the other hand, a medium model BSC is allocated on the site of BTS 16 (on the left) because it works with only 9 BTS. Notice that the distance between BSC nodes and MSC (larger circle on top left) is small compared to the average distance of the other links. That happens because of the number of E1 links dimensioned. There is 9 E1 links for the larger BSC and 4 for the medium one, while links between BTS, hub and BSC sites are unitary. In this scenario the links costs increase linearly with the distance but is multiplied by the number of E1 links too. Collected data from computational tests are resumed in tab. 1. Instance size refers to the number of BTS nodes handled by the problem instance, which affects directly the complexity in terms of memory and computational time expended.

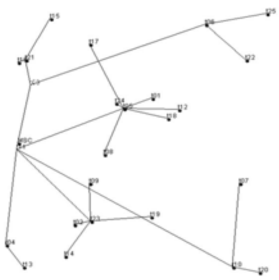

Fig.1 Solution for 20 BTS sites.

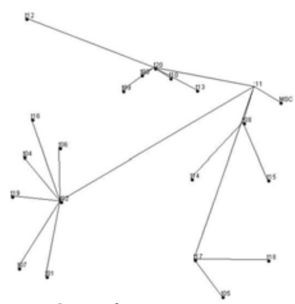

Fig.2 Solution for 25 BTS sites.

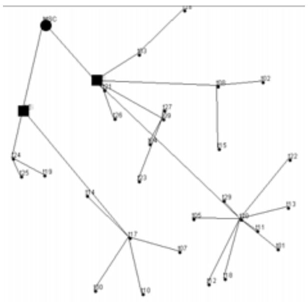

Fig.3 Solution for 30 BTS sites.

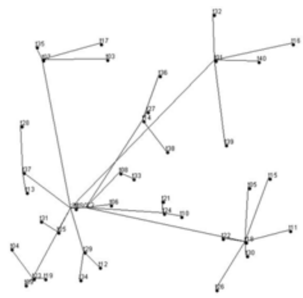

Fig.4 Solution for 40 BTS sites.

Variables and constraints represent the columns and lines of the mathematical model matrix. Non-zero density is calculated as the ratio between the number of matrix coefficients that are not equals to zero and the total number of coefficients of that matrix. Average time and standard deviation where used to describe statistically the amount of time elapsed to solve the problem instances of the integer programming model in seconds. The elapsed time suffers a significant variation depending on the particular problem instance. Despite this fact, the average values tend to an exponential function due to Branch-and-bound algorithm [5].

$$y = 0,851e^{0,244x} \tag{13}$$

Table 1: Computational Results for the Multilayer GSM Network Desing model.

| Instance Size (in BTS) | Variables | Constraints | Non-Zero Density | Average Time (s) | Standard Deviation |
|---|---|---|---|---|---|
| 5 | 211 | 150 | 2,6998% | 0,5785 | 0,3310 |
| 10 | 1241 | 1050 | 0,4866% | 2,8915 | 1,8467 |
| 15 | 3811 | 3450 | 0,1585% | 3,2500 | 1,8929 |
| 20 | 8681 | 8100 | 0,0695% | 10,1820 | 8,5499 |
| 25 | 16601 | 15750 | 0,0363% | 1314,339 | 1211,1110 |
| 30 | 28321 | 27150 | 0,0213% | 1325,010 | 1294,9840 |
| 35 | 44591 | 43050 | 0,0135% | 1080,436 | 478,6856 |
| 40 | 66161 | 64200 | 0,0091% | 1363,216 | 832,9629 |

## 4. Conclusion and Future Works

The model works with effectiveness producing a GSM network from BTS to MSC as an integrated multi-layer design. The total cost of transmission links and BSC acquisition is reduced to the optimal value. It can be far beyond the human limited design in such exponential combinatory problem. However, the complexity of the problem is such that the size of the handled networks is very limited compared to actual mobile carriers networks. This is an issue that can be tackled by approximate methods. A framework that hybridizes exact method and meta-heuristics has presented good results in expanding these boundaries in other classes of problems. Nepomuceno et. al. [12] used this framework to solve container loading problems. In the same problem category, Pinheiro and Coelho [13] presented a variation of the implementation to work with cutting problems. This framework was used too by Aguiar et. al. [14] to solve a model for Wireless Sensor Network. We intend to overcome this challenge using this framework with some innovations aggregated in the future. Other interesting contribution to this work is the possible incorporation of the layer 4, where the MSC allocation and sizing could be optimized together as a wider model.

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