Reconstruction of a Time Varying Signal in a Nonlinear Environment

Hichem Arfa†, Sadok El Asmi††, and Safya Belghith†

†6'Tel Unit Research SupCom Tunis, Tunisia
††Research Unit T.E.C.H.T.R.A SupCom Tunis, Tunisia

Summary
The nonlinearity of a power amplifier or loudspeaker in a large-signal situation gives rise to a nonlinear distortion of acoustic signal. However, recent research has shown that the acoustic echo path is better to be modeled as a nonlinear system due to the nonlinear characteristic of a loudspeaker. This paper deals with the reconstruction of a time varying input in a nonlinear environment. The time varying signal used in this work is a (TV-AR) (Time Varying Auto-Regressive) model with stochastic parameters and power variations. The synthetic signal generated by the (TV-AR) model presented here can be used to simulate the speech signal temporal variations. In order to equalize a nonlinear channel excited by such signal, we use a modified version of NLMS (Normalized Least Mean Square) algorithm which depends of instantaneous power of the (TV-AR) signal. The simulation results shows that this modified version of NLMS offers a better solution for equalizing a nonlinear channel excited by a time varying input.

Key words *nonlinear system, time varying signal, equalization, adaptive algorithms*

1. Introduction

Most of the adaptive echo cancelation techniques proposed in literature are based on an assumption that the echo path is linear, in which both the room impulse response and the power amplifier/loudspeaker considered linear [1]. However, harmonic distortion caused by non-ideal behavior of loudspeakers can significantly affect the perceptual quality of audio signals reproduced by the loudspeaker. Previous work [2] has shown that loudspeaker nonlinearities can be efficiently modeled with good accuracy using low-order, truncated Volterra systems. The main causes of harmonic distortions in loudspeakers are the no uniform flux density of the permanent magnet and the nonlinearity of the suspensions. Such distortions can be controlled by a careful design that imposes expensive constraints or by limiting the output power. Another approach that is less expensive and also does not limit the output power is to use digital equalization techniques. Many techniques are proposed to combating intersymbol interference in digital communication using nonlinear channel [3], [4]. Meanwhile, in an acoustic environment the input signal is a speech or audio signal which presents high dynamics. In this paper, we use a Time Varying Autoregressive (TV-AR) model with stochastic parameters variations and stochastic power variations. The synthetic signal generated by the (TV-AR) model presents characteristics similar to that of a speech signal. Hence, this model seems to be a good candidate to analyze adaptive filter in speech environment. In presence with such signal, the Normalized stochastic gradient algorithms family (NLMS, PNLMS, Sign LMS …) [5] [6] are usually used in real time applications. So, in this paper we present a modified version of NLMS algorithm which uses the instantaneous power of signal in order to give us a good capacity to tracking the variations of input signal and then better equalize a nonlinear channel. Computer simulations show that the proposed algorithm yields, a good equalization performance. The rest of this paper is organized as follows: in section 2, we present the time varying model. A conception of the Normalized stochastic gradient algorithm is given in section 3. The equalization algorithm and the performance of the proposed NLMS algorithm are given respectively in section 4 and 5. Finally, the section 6 summarizes the main conclusions of this work.

2. Time Varying Autoregressive (TV-AR) Model

The speech, at discrete time n, can be modeled as the output of an AR (Autoregressive) filter of order p, where the parameter $\{a_i(n)\}_{i=1:p}$ are time varying and where the power σ^2_ε (n) of the innovation sequence is also time-varying [7]. We have then:

$$x_n = \sum_{i=1}^{p} a_i(n) x_{n-i} + \varepsilon_n \qquad (1)$$

We note $A(n) = (a_1(n),\ldots,a_p(n))^T$ is the AR parameters vector of order p, and ε_n is the Gaussian excitation with time varying variance $\sigma_\varepsilon^2(n)$. A first-order Markov variation is supposed for the vector parameters:

$$A(n+1) = \gamma\, A(n) + \Omega_n \qquad (2)$$

Where Ω_n is a stationary white noise characterized by $E(\Omega_n \Omega^T{}_n) = \sigma_\omega^2 I$ (I is the p order identity matrix). A first-

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order Markov model is used to describe the logarithm variation of the excitation variance. Then, with

$$\Phi_\varepsilon(n) = \log(\sigma_\varepsilon^2(n)) \quad (3)$$

We have

$$\Phi_\varepsilon(n+1) = \beta\,\Phi_\varepsilon(n) + \log(\alpha) + u_n \quad (4)$$

where u_n is a Gaussian noise with variance σ_u^2.

By varying ($\alpha, \beta, \gamma, \sigma_u^2, \sigma_w^2$) parameters, we can model various contexts of speech non stationarities, slow or rapid. Indeed, on the figure 1, we can represent the evolution of a synthetic signal x_n for α=0.955, β=0.998, γ=0.98, σ^2_u=0.0089, σ^2_w=0.5.

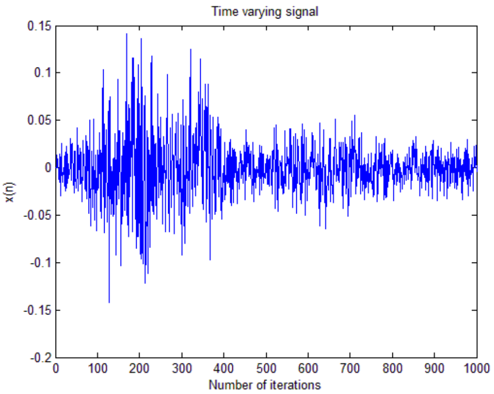

Fig.1 Time Varying Signal

3. The conception of the Modified Normalized Stochastic Gradient Algorithm

In this section, we analyze essentially the influence of signal power variation, onto the compute of the critical step size which will govern the modified NLMS (Normalized Least Mean Square) equalizer algorithm.

3.1 The critical step size

The time evolution of an adaptive transversal equalizer filter W_n of length q is ensured by stochastic gradient family algorithm according to the following equation:

$$W_{n+1} = W_n + \mu_n e_n Y_n \quad (5)$$

$$e_n = x_n - W_n^T Y_n \quad (6)$$

Where $W_n=(w_1(n),\dots,w_q(n))$, $Yn=(y_1(n),\dots,y_q(n))$ is the output vector of length q. The $\mu_n^{LMS} = \mu$ is a constant step size for the LMS algorithm and $\mu_n^{NLMS} = \dfrac{\mu}{Y_n^T Y_n}$ for the standard Normalized LMS algorithm (NLMS). The convergence analysis of this algorithm is made through the analysis of the time evolution of the deviation vector between the adaptive and optimal equalizer filter given by the following expression

$$V_n = H_n - H_{opt}$$

We deduce

$$V_{n+1} = (I - \mu_n Y_n Y_n^T)V_n + \mu_n e_n^{opt} Y_n$$

where the optimal error is $e_n^{opt} = x_n - H_{opt}^T Y_n$. If we compute the mean of the deviation vector, we find

$$E(V_{n+1}) = (I - \mu_n (Y_n Y_n^T))E(V_n)$$

When the classical independence between observation vectors Y_n is supposed. Note that in the case of non stationary inputs, the autocorrelation matrix $R_y(n) = E(Y_n Y_n^T)$ is not Toeplitz because we haven't a wide sense stationary input. For example, for white input $(\sigma_w^2 = 0)$ and q=2 we have:

$$R_y(n) = \begin{pmatrix} \sigma_y^2(n) & 0 \\ 0 & \sigma_y^2(n-1) \end{pmatrix}$$

If we suppose $\lambda_i, i = 1\dots,q$ are the eigenvalues of $R_y(n)$, then the convergence in mean value was assured by $0 < \mu_n < \dfrac{2}{\lambda_{\max}}$ and since $Trace(R_y(n)) = \sum_{i=0}^{q-1} \sigma_y^2(n-i)$ we can deduce the expression of critical step size in non stationary case:

$$0 < \mu_n < \frac{2}{\sum_{i=0}^{q-1} \sigma_y^2(n-i)} = \mu_c^{NS} \quad (7)$$

The μ_c^{NS} is the critical step size in non stationary case. This analysis shows that the classical normalization by $q\sigma_y^2$ or by its rough estimation $Y_n Y_n^T$ is false in particular for a long impulse response of the equalizer i.e. as q increases. That is that the use of the classical NLMS algorithm is not suited for temporal variation of the signal is close to that of the speech signal. In order to compute the critical step size, we need to compute the instantaneous power of the input signal of the equalizer for having a good performance.

3.2 The power of TV-AR signal y_n

We have

$$y_n = X_n^T H + b_n \quad (8)$$

$$y_n^2 = X_n^T H H^T X_n + 2H^T X_n + b_n^2 \quad (9)$$

We compute the expectation of equation 9, we have

$$\begin{aligned} E(y_n^2) &= E(X_n^T H H^T X_n) + \sigma_b^2(n) \\ &= E(X_n^T E(H H^T) X_n) + \sigma_b^2(n) \end{aligned} \quad (10)$$

If we note, $E(HH^T)$ by σ_H^2 and pH the order of the filter H, we will have the following expression,

$$\sigma_y^2(n) = \sigma_H^2 \sum_{i=0}^{p_H-1} \sigma_x^2(n-i) + \sigma_b^2(n) \qquad (11)$$

As we can see in the expression 11 the instantaneous power of the input of the equalizer depends to the instantaneous of the input signal x_n which is a TV-AR signal. For that, we need also computing the instantaneous power of the signal x_n. We will give afterward an explicit expression of the power of the TV-AR input signal x_n.

The matrix formulation of the equation 1 gives:

$$x_n = \sum_{i=1}^{p} a_i(n) x_{n-i} + \varepsilon_n \qquad (12)$$

$$A(n+1) = \gamma\, A(n) + \Omega_n \qquad (13)$$

where $0 < \gamma < 1$. So,

$$x_n^2 = A_n^T X_n X_n^T A_n + 2A_n^T X_n \varepsilon_n + \varepsilon_n^2 \qquad (14)$$

If we compute the expectation of equation 14,
we have:

$$E(x_n^2) = E(X_n^T A_n A_n^T X_n) + \sigma_\varepsilon^2(n) \qquad (15)$$

and

$$A_n A_n^T = (\gamma A_{n-1} + B_{n-1})(\gamma A_{n-1} + B_{n-1})^T \qquad (16)$$

$$E(A_n A_n^T) = \gamma^2 E(A_{n-1} A_{n-1}^T) + \sigma_b^2 I \qquad (17)$$

where I is the identity matrix of order p.

For a great value of N, we have: $E(A_n A_n^T) = \dfrac{\sigma_b^2}{1-\gamma^2} I$

And we can obtain the following formula:

$$E(x_n^2) = E(X_n^T E(A_n A_n^T) X_n) + \sigma_\varepsilon^2(n)$$
$$= \sum_{i=0}^{p-1} \sigma_x^2(n-i) E(A_n A_n^T) + \sigma_\varepsilon^2(n) \qquad (18)$$

Then,

$$\sigma_x^2(n) = \frac{\sigma_b^2}{1-\gamma^2} \left(\sum_{i=0}^{p-1} \sigma_x^2(n-i)\right) + \sigma_\varepsilon^2(n) \qquad (19)$$

The figure 2 shows the input TV-AR signal with length 10000 and the figure 3 the associated critical step size.

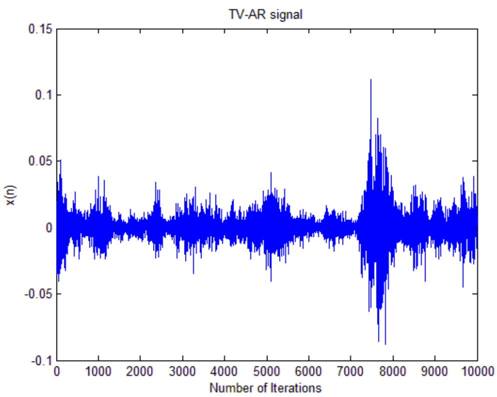

Fig.2 The original Time Varying input Signal

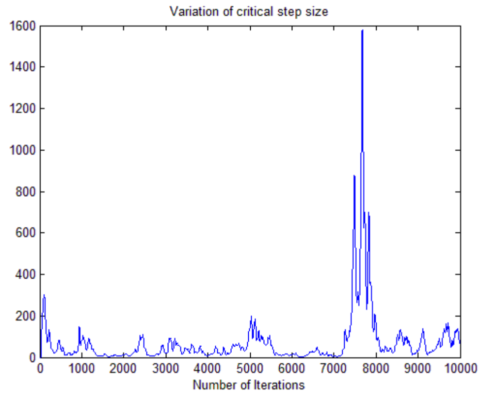

Fig.3 Variation of the critical step size

3.3 The modified NLMS algorithm updates equations

Based on the results from the estimate of the power of the time varying signal given in (19) which will govern the expression of the critical step size in a non stationary environment given in (7), the modified NLMS equalizer algorithm can be implemented. The update equations of the proposed algorithm are given by the following equations:

$$e_n = x_n - Y_n W_n \qquad (20)$$

$$W_{n+1} = W_n + \frac{\mu}{\sum_{i=0}^{q-1} \sigma_y^2(n-i)} e_n Y_n \qquad (21)$$

Where x_n the original input, Y_n is a vector formed by the outputs of nonlinear system and W_n is a vector which contains the equalizer coefficients.

4. Equalizability Conditions

In this section, we will present the conditions of equalization for the nonlinear channel excited by a time varying input. For that, we will use the relaxed conditions given in [8]. This deterministic approach gives conditions of existence of left inverse for a nonlinear system. In fact, in this paper we consider a nonlinear system modeled by a Volterra representation whose input-output relation is given by:

$$y_n = h_0 x(n) + h_1 x(n-1) + h_2 x^3(n) + h_3 x^3(n-1) + w(n)$$

Where the coefficients h_i are constant real numbers and $w(n)$ is an additive white Gaussian noise. The above equation can be used to model a satellite channel. The tangent system, obtained using the Kähler differentials is then given by

$$dy(n) = \alpha_n dx(n) + \beta_n dx(n-1) \qquad (23)$$

where

$$\alpha_n = h_0 + 3h_2 x^2(n) \qquad (24)$$

$$\beta_n = h_1 + 3h_3 x^2(n-1) \qquad (25)$$

Recalling the filtration (H_r) associated to the above time varying system, we have for r =0,

$$\boldsymbol{H_0} = \text{span}_{K(yn)}\{dy(n)\},$$

Now

$$\textbf{dim } \boldsymbol{H_0} = \text{rk}\,[\alpha_n\ \beta_n] = 1,$$

since α_n and $\beta_n \neq 0$. Next, for r=1, we have $\boldsymbol{H_1}=\text{span}_{K(yn)}\{dy(n),dy(n-1)\}$ and ***dim H₁*** is equal to the rank of the sylvester matrix

$$\begin{bmatrix} \alpha_n & \beta_n & 0 \\ 0 & \alpha_{n-1} & \beta_{n-1} \end{bmatrix}$$

Also, since $\alpha_n\alpha_{n-1} \neq 0$ or $\beta_n\beta_{n-1} \neq 0$, this rank is equal to 2. We may check that $\alpha_n\alpha_{n-1}\neq 0$ or $\beta_n\beta_{n-1}\neq 0$, for all integer n, then

$$\textbf{dim } \boldsymbol{H_r} = r+1,\ \forall r$$

and therefore, $\textbf{dim } \boldsymbol{H_{r+1}}$ - $\textbf{dim } \boldsymbol{H_r}$=1=number of inputs. So, we can say that the nonlinear left invertibility of system is achieved and we can equalize the concerned system.

5. Simulation Results

In this section, we consider the following nonlinear channel such that input-output relationship is given by:
$y(n)=h_0u(n)+h_1u(n-1)+h_2u^3(n)+h_3u^3(n-1)+w(n)$,
$w(n)$ is an additive white Gaussian noise with SNR = 25 dB. The channel coefficients are given by $H = [-1.31\ \ \text{-}0.43\ \ 0.2\ \ \text{-}0.5]$. The time varying input of this nonlinear channel is a TV-AR signal which is composed by 10000 iterations as depicted in figure 2. The output of equalizer, denoted by $\hat{u}(n)$
is given as follow:

$$\hat{u}(n) = \sum_{l=0}^{10} W(l)y(n-l) + \sum_{l=0}^{5}\sum_{k=l}^{5} W(k,l)y(n-l)y(n-k)$$

Since, we use a modified version of the NLMS algorithm the value of the step size μ is equal to 0.0006 in equation 21. The performance of equalization is given in term of the Mean Square Error (MSE) criterion, $E((x(n)-\hat{u}(n))^2)$, obtained over 100 independents trials. As we can show in figure 4. which represents the MSE in dB versus number of iterations, we can say that this modified version of NLMS compared to the classical NLMS equalizer, can be considered as a good candidate in order to perform the equalization of nonlinear channel excited by a time varying input. In addition, we have presented in figure 5. the output of the equalizer governed by the modified NLMS equalizer and we can see that the equalization performance is good in term of tracking the time variation of the input signal. Finally, the figure 6. shows the gain between the classical NLMS equalizer and the modified NLMS equalizer. In fact, such simulation results show that the use of the modified version of the NLMS equalizer is more appropriate in the case of time varying inputs.

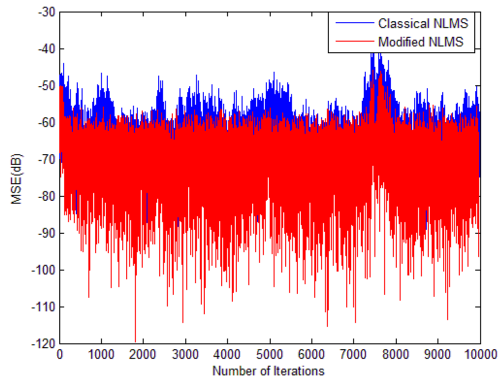

Fig 4. MSE (dB) versus Number of iterations

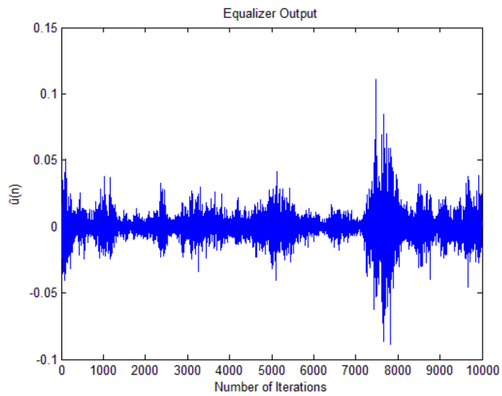

Fig 5. Equalizer output

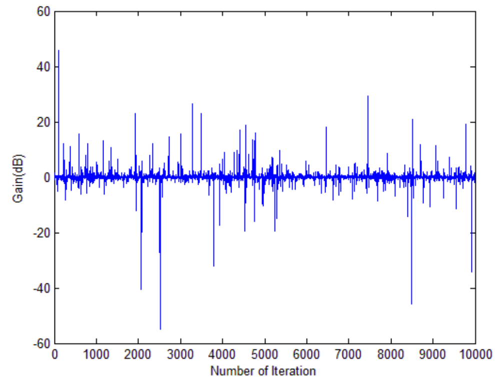

Fig 6. The gain between the classical NLMS and Modified NLMS equalizers

6. Conclusion

In this paper, we have presented a modified version of NLMS algorithm which can be used in order to equalize a nonlinear channel excited by a time varying input. The main advantage of this algorithm is the use of the instantaneous power of the time varying input signal. The time variation of the input is according to the (TV-AR) model, which presents a similar characteristic of a speech or audio signal. The simulation results show that this algorithm is more appropriate to equalize a nonlinear channel excited by a time varying inputs.

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Hichem Arfa He received the M.Sc. degree in Communication systems specialty Multimedia in 2003, from the National Engineer School of Tunis (ENIT), Tunisia. He is currently a research assistant in ISI Mahdia, Tunisia. His current research interests include Nonlinear Systems, Nonlinear Channel equalization, and Adaptive Algorithms.

Sadok El Asmi He obtained the B. S. in Applied Mathematics from the University of Paris XI-Orsay, in 1988, and the M.E. and Ph. D. degrees in Automatic Control Theory from the University of ParisXI-Orsay, in 1999 and 1992, respectively. From 1993 to 1994, he was a Research Associate at the "LSS-CNRS, Supeléc, Paris". Since 1995, he joined the "Ecole Supérieure des Communications de Tunis", where he is currently an Associated Professor. His research interests include MIMO Channel Identification and MIMO Channel Equalization.