Deriving Call Holding Time Distribution in Cellular Network from Empirical Data

Mohammed Alwakeel

Computer Science Department
Faculty of Computers and Information Technology
Tabouk University
Tabouk, Saudi Arabia, P. O. Box 741

Summary
The call holding time distribution in cellular systems is one of the main parameters that are used to study and analyze several system performance measures. Several statistical distributions have been used in the literature to model the call holding time distribution in 3rd and 4th generations cellular systems, such as exponential, Erlang, Gamma, and generalized Gamma. In practice, the call holding time is affected by several factors such as the service plan, the class of the service area, and some of the system design parameters, however, the parameters of the distributions used to model the call holding time were assumed, and the effects of some factors on the call holding time are eliminated. In this paper, we derived the probability density function of the call holding time based on actual data taken at Tabouk city, Saudi Arabia, from the working Aljwal network which is a 3.5G cellular network operated by Saudi Telecommunications Company, then the probability density function of the call holding time is approximated by gamma distribution and its parameters are derived using Maximum Likelihood Estimation.

Key words:
Call Holding Time, Cellular Systems, Exponential Distribution, Maximum Likelihood Estimation

1. Introduction

Cellular networks have evolved into one of the most exciting areas in telecommunications industries. New generations of cellular networks support a wide variety of services such as voice, data, and video. The user of 3.5 G or newer cellular networks can make a voice call or/ and make data connection, while watching an online movie, hence, "Wireless networks should be designed with desired quality-of-service requirements to achieve good performance" [1].

One of the main parameters that are used to study and analyze several system quality-of-service performance measures, such as call dropping probability, call termination probability, and handoff rate is the call holding time (CHT). The call holding time is defined as the time from the moment the call is accepted to the moment the call is completed or terminated due to lack of resources. In many previous researches, it was assumed, for analytical simplicity, that the call holding time is exponentially distributed, this assumption is usually acceptable for systems that support voice and simple data transmission services. However, because of technological advances and the growing interest in personal communications services, which require real time video and multimedia transmissions, the exponential distribution may no longer appropriately model the call holding time of practical 3.5G networks. A more general distribution than the exponential distribution is required to provide a more realistic characterization of the call holding time [2-4].

Several statistical distributions have been used in the literature to model the call holding time distribution in 3rd and 4th generations cellular systems, in [4] exponential distribution is used to model CHT, in [5] exponential, Erlang ,and hyperexponential distributions are used to model CHT, while in [6] generalized gamma is used. In practical situations, the call holding time is affected by several factors such as the service plan (e.g. flat rate plan or free weekend call plan etc.), the class of the service area (e.g. business or residential areas), and some of the system design parameters such as the call termination probability. In most literature the parameters of the distributions used to model the CHT were assumed, and the effects of some factors on CHT are eliminated. In this paper, we derived the probability density function of the call holding time ($f_{CHT}(t)$) based on actual data taken at Tabouk city, Saudi Arabia, from the working Aljwal network which is a 3.5G cellular network operated by Saudi Telecommunications Company (STC), then the $f_{CHT}(t)$ is approximated by gamma distribution and its parameters

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are derived using Maximum Likelihood Estimation (MLE). The rest of this paper is organized as follow, in section 2, the $f_{CHT}(t)$ is found from actual data, in section 3, the parameters of the approximated probability density function of the call holding time ($f_{ACHT}(t)$) are derived, and in section 4, some concluding remarks are made.

2. Actual Data Collection and Call Holding Time

The actual call holding time data is collected from the billing system Aljawal network operated by STC, where a large sample of CHT for large sample of Aljawal customers (businessmen, students, housewives, etc..) from all ages is recorded during a full year duration. From the sample the actual $f_{CHT}(t)$ can be found as shown in Fig. 1, and the mean of the CHT (μ_{CHT}) is found as

$$\mu_{CHT} = \frac{\sum_{i=1}^{n} T_i}{n} \tag{1}$$

where n is the size of the sample, and T_i is the CHT of call i. from the figure we can see that the exponential distribution is not an accurate approximation of the $f_{CHT}(t)$, instead, the more versatile gamma distribution may provides more accurate approximation of the CHT distribution, where $f_{ACHT}(t)$ is defined as [7]

$$f_{ACHT}(t) = \frac{t^{(\alpha-1)} e^{-\left(t/\beta\right)}}{\beta^{\alpha}\Gamma(\alpha)} u(t) \tag{2}$$

with mean given by $\mu_{ACHT} = \alpha\beta$, and the corresponding cumulative distribution function (CDF) is given by

$$F_{ACHT}(t) = \frac{\gamma(\alpha, t/\beta)}{\Gamma(\alpha)} u(t) \tag{3}$$

where α is the shape parameter, β is the scale parameter, $u(t)$ is the unit step function, $\Gamma(x)$ is the gamma function [8], and $\gamma(x,y) = \int_0^y t^{x-1} e^{-t} dt$ is the incomplete gamma function [8].

3. Maximum Likelihood Estimation of the parameters of $f_{ACHT}(t)$

In order to estimate the parameters of $f_{ACHT}(t)$ according to the empirical data the Maximum Likelihood Estimation (MLE) is used. The likelihood function of $f_{ACHT}(t)$ for N independent identically distributed observations $(T_1, ..., T_n)$ is given by

$$L(\alpha,\beta) = \prod_{i=1}^{n} f_{ACHT}(T_i) \tag{4}$$

then the log-likelihood function is given by [5]

$$\ell(\alpha,\beta) = (\alpha-1)\sum_{i=1}^{n}\ln(T_i) - \left(n\ln(\Gamma(\alpha)) + n\alpha\ln(\beta) + \sum_{i=1}^{n}(T_i/\beta)\right). \tag{5}$$

Taking the derivative of (5) with respect to β , and setting it equal to zero to find the maximum we have

$$\beta = \frac{\mu_{ACHT}}{\alpha} \tag{6}$$

substituting (6) in (5) we have

$$\ell = (\alpha-1)\sum_{i=1}^{n}\ln(T_i) - n\alpha - n\alpha\ln(\mu_{ACHT}/\alpha) - n\ln(\Gamma(\alpha)) \tag{7}$$

Taking the derivative of (7) with respect to α and setting it equal to zero to find the maximum we have [8]

$$\ln(\alpha) - \psi(\alpha) = \ln(\mu_{ACHT}) - \frac{1}{n}\sum_{i=1}^{n}\ln(T_i) \tag{8}$$

where $\psi(x)$ is the digamma function given by [5]

$$\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)} \tag{9}$$

and $\Gamma'(x)$ is the derivative of $\Gamma(x)$. There is no close form for α , however, a numerical solution can be found easily, and from (6) and (8) the parameters of $f_{ACHT}(t)$ may be evaluated using the actual data. Fig. 2, shows $f_{CHT}(t)$ and $f_{ACHT}(t)$ and as we can see from the figure gamma distribution provides an accurate approximation to the call holding time of Aljawal cellular network in Tabouk area.

4. Conclusion

Many distributions were used in literature to model the call holding time distribution in cellular systems including exponential distribution, where the main reason behind using exponential distribution is to simplify the analysis. In practice, exponential distribution may be used to model the call holding time in 1st and 2nd generation systems accurately, but the results introduced in this paper show that exponential distribution is not an accurate approximation to the call holding time distribution in 3rd and 4th generation cellular systems, instead, the more versatile gamma distribution provides more accurate and simple approximation.

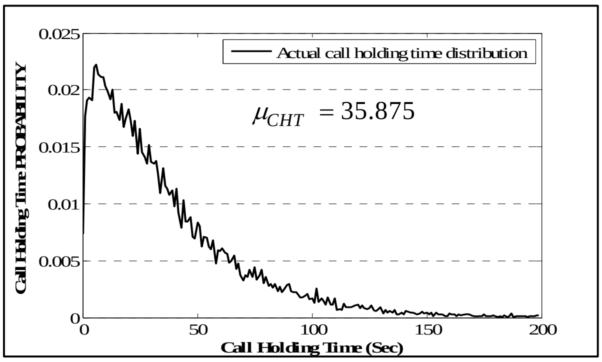

Fig.1. Actual call holding time distribution

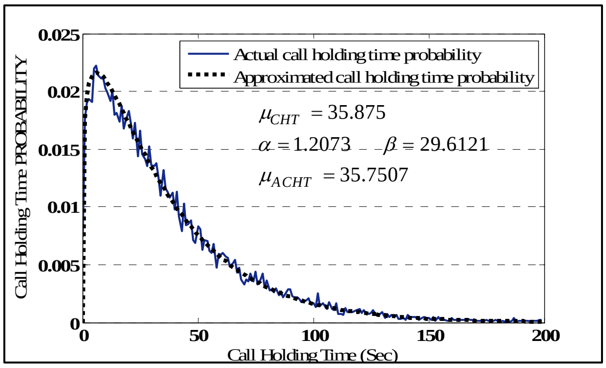

Fig.2. Actual & Approximated call holding time distributions

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Mohammed M. Alwakeel received the B.S. degree in Computer Engineering and the M.S. degree in Electrical Engineering in 1993 and 1998, respectively, both from King Saud University, Riyadh, Saudi Arabia, and the Ph.D. degree in Electrical Engineering in 2005 from Florida Atlantic University, Boca Raton, FL, USA.
From 1994 to 1998, he was employed as Communications Network Manager at The National Information Center in Saudi Arabia. From 1999 to 2001, he was employed by King Adulaziz University as a lecturer and as vice dean of Tabuk Community College. He is now the dean of Computers and Information Technology college at University of Tabuk. His current research interests include teletraffic analysis, mobile satellite communications, and cellular systems.