Eigen Structure Based Direction of Arrival Estimation Algorithms for Smart Antenna Systems

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Abstract:
Smart antenna is recognized as promising technologies for higher user capacity in third generation (3G) wireless networks. The core of this technology is the selection of smart algorithms in adaptive array. As signals are received from each antenna element, the direction-of-arrival (DOA) algorithm computes the angle of arrival of all impinging signals. A systematic study and comparison of the performance of two famous Eigen structure based DOA algorithms known as the MUltiple SIgnal Classification (MUSIC) and the Estimation of Signal Parameter via Rotational Invariance Techniques (ESPRIT) has been extensively studied in this research work. Simulation results revealed that MUSIC algorithm exhibits high resolution, but it can be computationally intensive. Whereas ESPRIT algorithm also provides the high resolution of MUSIC, but it does not require a costly search and can be used with different array geometries.

Key words:
MUltiple SIgnal Classification (MUSIC), Estimation of Signal Parameter via Rotational Invariance Techniques (ESPRIT), Eigen structure, correlation matrix, Direction -of –arrival.

1. Introduction

.Smart antenna technologies are set to revolutionize the wireless communication industry with their enormous potential to enable the development of more powerful, cost effective and efficient wireless systems. In propagation channel, even for one source there are many possible propagation paths and angle of arrival. If several transmitters are operating simultaneously, each source potentially creates many multipath components at the receiver. Therefore it is important for a receive array to be able to estimate the angle of arrival in order to decide which emitter are present and what are their possible angular locations. This information can be used to eliminate or combine signals for greater fidelity, suppress interferers.

Consider an array of M elements with M potential weights. Each received signal $x_m(k)$ includes additive,

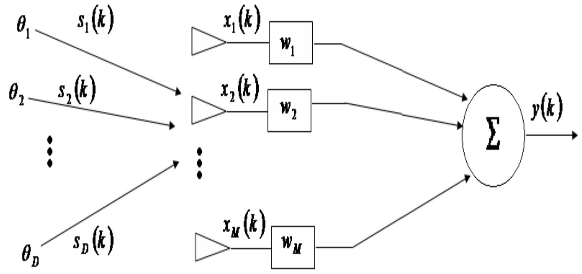

Fig. 1. M-element array with arriving signals

zero mean, Gaussian noise time id represented by the k[th] time sample. Thus, the array output y can be given as

$$y(k) = \bar{w}.\bar{x}(k) \qquad \text{---------- (1)}$$

Where

$$\bar{x}(k) = [\bar{a}(\theta_1)\bar{a}(\theta_2)....\bar{a}(\theta_D)].\begin{bmatrix} s_1(k) \\ s_2(k) \\ . \\ s_D(k) \end{bmatrix} + \bar{n}(k) \qquad \text{----- (2)}$$

$$= \bar{A}.\bar{s}(k) + \bar{n}(k)$$

and

$\bar{w} = [w_1 \quad w_2 \quad . \quad w_M]^T$ = array weights

$\bar{s}(k)$=vector of incident complex signals at time k

$\bar{n}(k)$=noise vector at each array element m, zero mean, variance σ_n^2

$\bar{a}(\theta_i)$=M-element array steering vector for the θ_i direction of arrival

$\bar{A} = [\bar{a}_1(\theta_1) \quad \bar{a}_2(\theta_2) \quad . \quad a_3(\theta_D)]$M X D matrix of steering vectors $\bar{a}(\theta_i)$

The arriving signals are time varying and calculations are based upon time snapshots of the incoming signal.

Then, the M X M array correlation matrix $\bar{R}_{xx}$ can be defined as follows

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$$\overline{R}_{xx} = E\left[\overline{x} \quad . \quad \overline{x}^H\right] = E\left[\left(\overline{A}\overline{s} + \overline{n}\right)\left(\overline{s}^H \overline{A}^H + \overline{n}^H\right)\right.$$
$$= \overline{A}\overline{R}_{ss}\overline{A}^H + \overline{R}_{nn} \qquad \text{---------- (3)}$$

where
$\overline{R}_{ss}$ =D X D source correlation matrix
$\overline{R}_{nn} = \sigma_n^2 \overline{I}$ =M X M noise correlation matrix
$\overline{I}$ =N X N identity matrix

2. Eigen Structure Methods

Eigen structure methods depend on the following properties of correlation matrix R

1. The space spanned by its Eigen vectors may be partitioned into two subspace, namely the signal subspace and the noise subspace.
2. The steering vectors corresponding to the directional sources are orthogonal to the noise subspace. As the noise subspace is orthogonal to the signal subspace, those steering vectors are contained in the signal subspace.

The array correlation matrix has M Eigen values $(\lambda_1 \quad \lambda_2 \quad . \quad \lambda_M)$ along with M associated Eigen vectors $E = (\overline{e}_1 \overline{e}_2 .....\overline{e}_M)$. If the Eigen values are sorted from smallest to largest, matrix E can be divided into two sub space such that $\overline{E} = \left[\overline{E}_N \quad \overline{E}_s\right]$. The first subspace $\overline{E}_N$ is called the noise subspace and is composed of M-D Eigen vectors associated with the noise, the Eigen values are given as $\lambda_1 = \lambda_2 = ... = \lambda_{M-D} = \sigma_n^2$. The second subspace $\overline{E}_s$ is called the signal subspace and is composed of D Eigen vectors associated with the arriving signals.

3. MUSIC Algorithm

3.1 Introduction:

MUSIC is an acronym which stands for MUltiple SIgnal Classification. This approach was first posed by Schmidt. It is a simple, popular high resolution and efficient Eigen structure method. It promises to provide unbiased estimates of the number of signals, the angles of arrival and the strengths of the waveforms.

From array correlation matrix $\overline{R}_{xx}$ can find D eigen vectors associated with the signals and M-D eigenvectors associated with the noise. Then choose the eigen vectors associated with the smallest eigen values. Noise eigenvectors subspace of order M X (M-D) is constructed and is given as

$$\overline{E}_N = [\overline{e}_1 \quad \overline{e}_2 \quad . \quad . \quad \overline{e}_{M-D}] \text{------------------(4)}$$

The noise subspace eigen vectors are orthogonal to the array steering vectors at the angles of arrivals $\theta_1, \theta_2, ..., \theta_D$. The pseudo-spectrum- a function that gives an indication of the angle of arrival based upon maxima versus angle for MUSIC is given as

$$P_{MU}(\theta) = \frac{1}{\left|\overline{a}(\theta) \quad \overline{E}_N \quad \overline{E}_N^H \quad \overline{a}(\theta)\right|} \qquad \text{------------(5)}$$

3.2 Simulation Results:

For simulation purpose an N element linear array is used with individual elements spaced at half wavelength distance. Binary Walsh like signals of amplitude 1 and Gaussian distributed noise of σ_n^2 =0.1 are assumed with K finite samples. All correlation matrices by time averages are calculated to get R_{xx} as follows

$$\hat{R}_{xx} = \overline{A}\hat{R}_{ss}\vec{A}^H + \overline{A}\hat{R}_{sn} + \hat{R}_{ns}\overline{A}^H + \hat{R}_{nn} \qquad \text{------(6)}$$

Figures (2) shows angular spectra for the number of elements N=6, 8, 12 with spacing between elements of array d=0.5lambda and number of time samples K=100 for arriving angles at -20^o and 10^o. This shows that rather than increase in number of array elements, increasing aperture makes it robust.

Figure (3) shows angular spectra for the different number of time samples K=10, 100, 1000 for N=6. This shows that for large number of time samples the spatial resolution of MUSIC algorithm will be more.

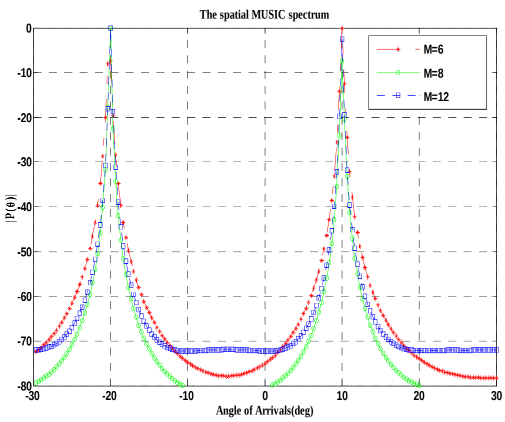

Fig.2. Normalized pseudospectrum plots for MUSIC algorithm for arrival angles at -20^o and 10^o (K=100)

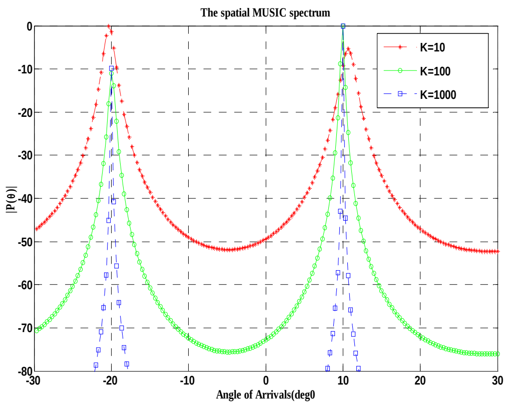

Fig.3. Normalized pseudospectrum plots for MUSIC algorithm for arrival angles at -20^o and 10^o (M=6)

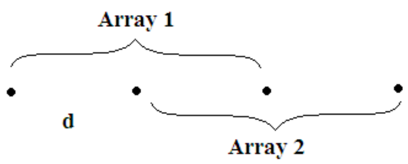

Fig.4. doublet composed of two identical displaced arrays

4. ESPRIT Algorithm

4.1 Introduction:

ESPRIT stands for Estimation of Signal Parameters via Rotational Invariance Techniques and was first proposed by Roy and Kailath in 1989. This technique exploits the rotational invariance in the signal subspace which is created by two arrays with a translational invariance structure.

Assumptions

i) Narrow band sources (D<M) so that one knows the translational phase relationship between the multiple arrays to be used

ii) These sources are assumed to be of sufficient range so that the incident propagating field is approximately planar.

iii) The sources can be either random or deterministic and the noise is assumed to be random with zero mean.

iv) It assumes multiple identical arrays called doublets (These arrays are displaced transitionally but not rotationally).

The complete received signal considering the contributions of both sub arrays is given as

$$\bar{x}(k)=\begin{bmatrix}\bar{x}_1(k)\\ \bar{x}_2(k)\end{bmatrix}=\begin{bmatrix}\bar{A}_1\\ \bar{A}_1.\Phi\end{bmatrix}.\bar{s}(k)+\begin{bmatrix}\bar{n}_1(k)\\ \bar{n}_2(k)\end{bmatrix} \qquad \text{----------(7)}$$

Where $\bar{\Phi}$ is a D X D diagonal unitary matrix with phase shifts between the doublets for each angle of arrivals.

Let $\bar{E}_1$ and $\bar{E}_2$ are M X D matrices whose columns are composed of the D eigenvectors corresponding to the largest eigenvalues of $\bar{R}_{11}$ and $\bar{R}_{22}$. Since the array are translationally related, the subspaces of eigenvectors are related by a unique non-singular transformation matrix $\bar{\Psi}$ such that

$$\bar{E}_1\bar{\Psi}=\bar{E}_2 \qquad \text{--------------------------------(8)}$$

Similarly, these matrices are related to steering vector matrices A and AΦ by another unique nonsingular transformation matrix T, as the same subspace is spanned by these steering vectors. Thus

$$\bar{E}_1=\bar{A}\bar{T} \text{ and } \bar{E}_2=\bar{A}\bar{\Phi}\bar{T} \qquad \text{-------------(9)}$$

Substituting for E_1 and E_2 and the fact that A is of full rank, one obtains

$$\bar{T}\bar{\Psi}\bar{T}^{-1}=\Phi \qquad \text{----------------------(10)}$$

Which states that the eigenvalues of Ψ are equal to the diagonal elements of Φ and that the columns of T are eigenvectors of Ψ. An eigendecomposition of Ψ gives its eigenvalues and equating them to Φ leads to the DOA estimates

$$\theta_i=\sin^{-1}\left(\frac{\arg(\lambda_i)}{kd}\right) \qquad i=1, 2,\dots, D \qquad \text{--------(11)}$$

How one obtains an estimate of Ψ from the array signal measurements efficiently has led to many versions of ESPRIT. One version referred to as total least squares (TLS). This procedure is outlined as follows.

- Estimate the array correlation matrices $\bar{R}_{11}$ and $\bar{R}_{22}$ from the data samples.
- Calculate the signal subspaces $\bar{E}_1$ and $\bar{E}_2$ based upon the signal Eigen-vectors of $\bar{R}_{11}$ and $\bar{R}_{22}$.
- Next form a 2D X 2D matrix using the signal subspace such that

$$\bar{C}=\begin{bmatrix}\bar{E}_1^H\\ \bar{E}_2^H\end{bmatrix}\begin{bmatrix}\bar{E}_1 & \bar{E}_2\end{bmatrix}=\bar{E}_C\Lambda\bar{E}_C^H \qquad \text{---------(12)}$$

  Where the matrix $\bar{E}_C$ is from the eigenvalue decomposition (EVD) of $\bar{C}$ such that $\lambda_1\geq\lambda_2\geq\dots\geq\lambda_{2D}$ and $\bar{\Lambda}$=diag $\{\lambda_1, \lambda_2,\dots, \lambda_{2D}\}$
- Partition $\bar{E}_C$ into D X D submatrices such that

$$\bar{E}_C=\begin{bmatrix}\bar{E}_{11} & \bar{E}_{12}\\ \bar{E}_{21} & \bar{E}_{22}\end{bmatrix} \qquad \text{---------------(13)}$$

- Estimate the rotation operator $\bar{\Psi}$ by $\bar{\Psi}=-\bar{E}_{12}\bar{E}_{22}^{-1}$
- Calculate the eigenvalues of $\bar{\Psi}$, λ_1, λ_2, …, λ_D
- Now estimate the angles of arrival, given that λ_i

$$\lambda_i=|\lambda_i|e^{j\arg(\lambda i)}$$

$$\theta_i = \sin^{-1}\left(\frac{\arg(\lambda_i)}{kd}\right) \qquad i=1, 2, \ldots, D \quad \text{--------(14)}$$

4.2 Simulation Results:

For simulation purpose an N element linear array is used with individual elements spaced at half wavelength distance. Assumptions are made same as MUSIC algorithm simulations.

Figures (5) shows angular spectra for the number of elements N=6, 8, 12 with spacing between elements of array d=0.5lambda and number of time samples K=300 for arriving angles at -20^o and 20^o. This shows that for a linear array with a large number of element and overlapping the spatial resolution of ESPRIT algorithm will be more.

Figure (6) shows angular spectra for the different number of time samples K=10, 100, 1000 for N=6. This shows that the large number of time samples doesn't affect the resolution of ESPRIT resolution.

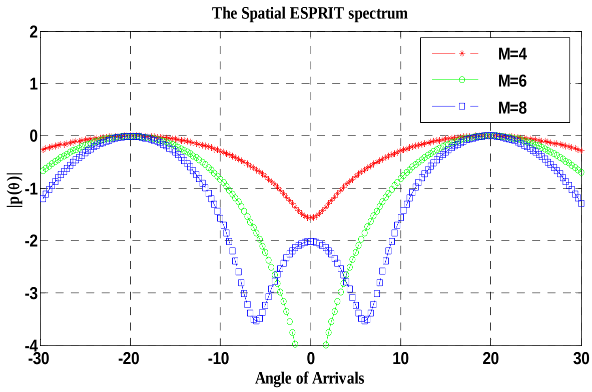

Fig.5. Normalized pseudospectrum plots for ESPRIT algorithm for arrival angles at -20^o and 20^o (K=300)

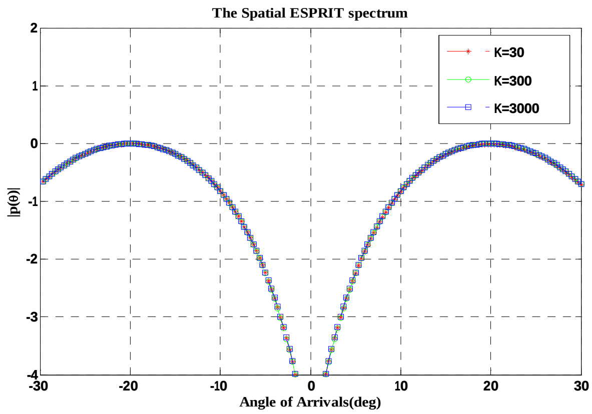

Fig.6. Normalized pseudospectrum plots for ESPRIT algorithm for arrival angles at -20^o and 20^o (M=6)

5. Conclusions:

This paper discusses and compares the performance of two famous eigen structure based algorithms known as the Multiple Signal Classification (MUSIC) and Estimation of Signal Parameter via Rotational Invariance (ESPRIT). The results obtained from simulations shows that MUSIC algorithm exhibits high resolution, but it can be computationally intensive, whereas ESPRIT algorithm also provides the high resolution of MUSIC, but it doesn't require a costly search. ESPRIT allows the DOA's to be computed directly. MUSIC fail to resolve correlated sources, resolution will be more for large number of array elements and time samples and more sensitive to both sensor gain and phase errors. ESPRIT resolution is more for large number of overlapping elements, more sensitive to only phase errors and has been used with different array geometries.

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