

# An Efficient dimension reduction algorithm to extract dorsal hand vein pattern based on Generalized Method of Moments and Moore-Penrose generalized inverse procedure

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## Summary

In this paper, we present a Cartesian-based co-ordinate system (CBM) to represent dorsal hand vein pattern and also introduce a novel method to extract the dorsal hand vein pattern that is based on the generalized method of moments. However, this method leads to high dimensional covariance matrix which is singular or rank-deficient. To overcome this problem, we use a fast and accurate algorithm to compute the inverse using the Moore-Penrose method. The biometric system developed is tested on a database of 100 images. The false acceptance rate (FAR), false rejection rate (FRR) and the matching time are being computed.

## Key words:

*Dorsal hand vein pattern, generalized method of moments, Moore- Penrose inverse.*

## 1. Introduction

Biometric which involves the analysis of human biological, physical and behavioral characteristics is being developed to ensure more reliable security and to overcome the problems encountered in traditional security system such as password. The most popular biometric features that are used are fingerprints, hand geometry, iris scans, faces, as well as handwritten signatures. Recently dorsal hand vein pattern biometric is attracting the attention of researchers and is gaining momentum. Anatomically, aside from surgical intervention, the shape of vascular patterns in the back of the hand is distinct from each other [1], [2]. It is a randotypic trait, which is formed during the early phases of embryonic development and hence unique to everyone [3]. Veins are found below the skin and cannot be seen with naked eyes. Its uniqueness, stability and immunity to forgery are attracting researchers. These feature makes it a more reliable biometric for personal identification [4]. Furthermore, the state of skin, temperature and humidity has little effect on the vein image, unlike fingerprint and facial feature acquirement [5]. The hand vein biometrics principle is non- invasive in nature where dorsal hand vein

pattern are used to verify the identity of individuals [7]. Vein pattern is also stable, that is, the shape of the vein remains unchanged even when human being grows.

Extensive researches are carried out on vein patterns and researchers are striving hard to find methods and techniques to develop dorsal hand vein security system. Any biometric system consists of four main steps namely the preprocessing, feature extraction, processing and matching phase. Feature extraction is a crucial step in biometric system and its capability directly influence the performance of the system. Different methods such as Principle Component Analysis (PCA) [6], modified Principle component analysis with Lanczos and Cholesky decomposition [7], and quadratic inference function [8] have been explored to extract and represent dorsal hand vein features. However, to improve the accuracy of the dorsal hand vein verification system, more methods are being scrutinized.

In this work, the method proposed aims at reducing the dimension of the training set by building an adaptive estimating equation or a quadratic inference function [9],[10] that combines the covariance matrix and the vectors in the training set while at the same time the inverse of the covariance matrix is calculated by an efficient algorithm based on Moore-Penrose inverse following Courrieu [11]

The organization of the paper is as follows: in section 2 we describe the pre-processing phases applied on the dorsal hand vein pattern and a Cartesian-based block matrix representation of the dorsal hand vein pattern, section 3 explains generalized method of moment, in section 4 we perform the vein pattern matching, experimental results are presented in section 5 and finally section 6 conclude the paper.

## 2. A Cartesian-based block matrix representation of the dorsal hand vein pattern

In this section, we present a novel approach of representing dorsal hand vein pattern. This approach involves the construction of a block matrix training set to represent the dorsal hand vein features. Firstly, it is

important to obtain the vein pattern in the image captured. This procedure requires image acquisition, hand segmentation, vein pattern segmentation, noise filtering and thinning of the vein pattern. After these preprocessing techniques, we obtain an image consisting of a background represented in black and a thinned vein pattern in white. The following figure shows the resulting image after applying all the preprocessing steps.

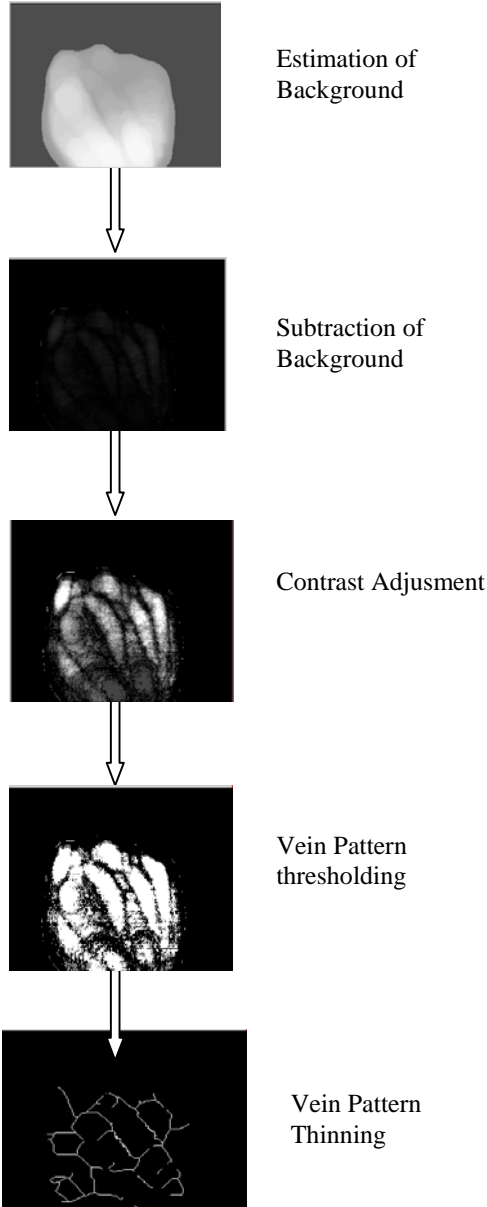


Fig. 1 Biometric Procedure.

The resulting image is represented as a binary matrix of size  $320 \times 240$  where the black color is coded as 0 and the white color is coded as 1. We provide an alternative

matrix representation that converts these binary codes into a two-dimensional cartesian coordinate system where the black color takes value 0 for both the  $x$  and  $y$  coordinates and the white color is indexed by its  $i^{th}$  and  $j^{th}$  position in the binary matrix. We illustrate this concept through the following examples: Assume a  $3 \times 3$  sub-matrix from the  $320 \times 240$  binary image matrix

$$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \quad (1)$$

We convert this matrix in cartesian system as follows:

$$\begin{pmatrix} 0 & 0 & 1 & 2 & 1 & 3 \\ 2 & 1 & 2 & 2 & 2 & 3 \\ 0 & 0 & 0 & 0 & 3 & 3 \end{pmatrix} \quad (2)$$

Ultimately, the size of the two-dimensional cartesian coordinate based image matrix will be of size  $320 \times 240 \times 2$ .

Assuming assume  $I$  images for the training set  $X$ , i.e,

$$X = [X_1, X_2, \dots, X_i, \dots, X_I] \quad (3)$$

$$X_i = \begin{bmatrix} X_{i,1,1} & X_{i,1,2} & \dots & X_{i,1,240 \times 2} \\ X_{i,2,1} & X_{i,2,2} & \dots & X_{i,2,240 \times 2} \\ \vdots & \vdots & X_{i,j,k} & \vdots \\ X_{i,320,1} & \dots & \dots & X_{i,320,240 \times 2} \end{bmatrix} \quad (4)$$

where

$$X_{i,j,k} = (x_{ij}, y_{ik}) \quad (5)$$

where  $i$  is the index for the  $i^{th}$  image,  $j$  is the corresponding index for the  $x$  co-ordinate of the  $i^{th}$  image,  $k$  is the corresponding index for the  $k$  co-ordinate of the  $i^{th}$  image where  $i = 1, \dots, I$ ,  $j = 1, \dots, 320$  and  $k = 1, \dots, 240 \times 2$ . Thus, the training block matrix  $X$  is of dimension  $320 \times 2 \times I \times 240$ . As it can be noted, the size of this matrix is very large especially when  $I$  is large. Thus, it becomes difficult to work with the original matrix  $X$ . To overcome this problem, we combine the cartesian-based co-ordinate matrices  $X_i$  via a quadratic inference function that is independent of the number of images  $I$ .

### 3. Generalized method of moments (GMM)

Generalized method of moments developed by Hansen [12] is a powerful statistical tool that yields reliable and robust parameter estimates especially in regression analysis. It has been used in a variety of research applications particularly in the field of econometrics [13], in the analysis of longitudinal data and in solving generalized estimating equations [9], [10]. So far, it has not been applied in the field of biometric security. In this paper, we introduce GMM to detect vein pattern. Conceptually, it is made up of moment functions and an empirical covariance matrix of these moments. Both are combined to form a quadratic inference function (QIF) of the form

$$Q = g^T C^{-1} g \quad (6)$$

where  $g$  is a matrix comprising of moments  $\phi_i$  and  $C$  is the covariance matrix given by

$$C = \frac{1}{I} \sum_{i=1}^I \phi_i \phi_i^T \quad (7)$$

In our context, we define  $\phi_i$  as a  $320 \times 240$  matrix where the  $(j, k)^{th}$  element of  $\phi_i$  are given by

$$\phi_{ij} = x_{ij} - \psi_j \quad (8)$$

and

$$\phi_{ik} = y_{ij} - \psi_k \quad (9)$$

where

$$\psi_j = \frac{1}{I} \sum_{i=1}^I x_{ij} \quad (10)$$

and

$$\psi_k = \frac{1}{I} \sum_{i=1}^I y_{ik} \quad (11)$$

where  $x_{ij}$  and  $y_{ik}$  are the corresponding  $x$  and  $y$  coordinates from equation (5). Thus, the dimension of  $g$  is  $320 \times 240 \times 2$  and  $C$  is of  $320 \times 320$ . Ultimately, the GMM based moment objective function is of size  $2 \times 240$  by  $2 \times 240$  following equation (6). Note the dimension of the training set has been reduced considerably from  $320 \times 240 \times 2$  to  $2 \times 240 \times 2 \times 240$  and equation (6) does not depend on the number of images  $I$ . Thus, even if  $I$  is very large, this will not affect the GMM based QIF. However, this method has a drawback. From simulation studies, we note that the covariance matrix  $C$  in equation

(6) is close to singularity and is ill-conditioned. We used the pseudo-inverse function (*pinv*) in Matlab to compute its inverse but it does not provide a reliable estimate of the matrix. In this context, Pierre Courrieu [11] has developed a fast computing Moore-Penrose inverse algorithm that solves such matrices even with rank deficiency. We use his algorithm to compute the inverse of  $C$ . The steps can be summarized as follows:

1) Let  $A = CC^T$

2) Let  $L = \text{zeros}(\text{size}(A))$

3) Initialize  $r = 0$  and compute

$$L(k : n, r) = A(k : n, k) - L(k : n, 1 : (r-1))L(k, 1 : (r-1))^T$$

for  $k = 1, \dots, 320$  and  $r = r + 1$  at each iteration until

$$L(k, r) > 10^{-9}.$$

4)  $M = \text{inv}(L^T L)$

5)  $C^{-1} = LMML^T G^T$

To generate the space of vein pattern, we use

$$Qv_i = \mu_i v_i \quad (12)$$

For each eigenvector, a family of eigenvein has to be generated. However, many eigenveins are being generated. In order to determine how many eigenveins are required, the following formulae are being used. We have accounted for 90 % and 95% of the variation in the training set.

$$\frac{\sum_{i=1}^{2N'} \mu_i}{480} > 0.9 \quad (13)$$

$$\frac{\sum_{i=1}^{2N'} \mu_i}{480} > 0.95 \quad (14)$$

We have already obtained  $2N'$  eigenveins. For each element in the training set, the weight is calculated. This weight will demonstrate the contribution of each eigenvein to respective training element. If the weight is bigger, then the eigenvein has shown the real vein. If the value is less, there is no big contribution with the real vein for that particular eigenvalue. The following operation shows how each element in the training set is projected onto the vein space:

$$\omega_k = \frac{1}{320 \times 240} \sum_{i=1}^{320} \sum_{j=1}^{240} (Qv_k)^T (X_{ij}^T - \phi_j^T) \quad (15)$$

where  $1 < k < 2N'$

Each element in the training set has a weight to determine their contribution to the vein space.

#### 4. Vein pattern matching

To test the accuracy and efficiency of our proposed method, we need to perform vein pattern matching. Vein pattern matching involves recognizing an image, which means checking whether the image exist in the database. When a person wants to get access to the system, the picture of the vein, known as the test image is captured. The coordinates of the test image are obtained and represented as the training set. The weight of the new image is calculated and projected on the vein space [6],[7],[8]. The vein space contains all the vein images. Thus, we have to check whether the input image exist in that space. The Euclidean distance between the projected image and those stored is being calculated. First of all, our system checks whether the test image is a vein by testing it with an arbitrary value. Then the Euclidean distance is computed to check whether the test image exist in the database. If it is vein image, then it is accepted. The results were recorded and analyzed.

#### 5. Experimental results

The hand dorsal vein biometric was tested using pixel by pixel method and the generalised method of moment discussed in this paper. It is to be noted that pixel by pixel method test each individual pixel by counting the number of overlapped pixel in the test image and that of the template found in the database.

In order to test the efficiency and accuracy of the method proposed, false acceptance rate (FAR) and false rejection rate(FRR) are computed. False Acceptance Rate refers to the total number of unauthorized persons getting access to the system over the total number of people attempting to use the system. False Rejection Rate refers to the total number of authorized persons not getting access to the system over the total number of people attempting to get access to the system. The table below shows the FAR and FRR for 20,40,60,80 and 100 images tested.

Table 1: FAR and FRR using pixel by pixel method

Number of images	FAR(%)	FRR(%)
20	0.1000	0.1500
40	0.0250	0.0750
60	0.0340	0.0670
80	0.0375	0.0250
100	0.0400	0.0300

Table 2: FAR and FRR using Generalized Method of Moment

Number of images	FAR(%)	FRR(%)
20	0.0500	0.0600
40	0.0250	0.0500
60	0.0340	0.0340
80	0.0250	0.0125
100	0.0200	0.0300

According to the results obtained, the FAR and FRR is less when using generalized method of moment compared to pixel by pixel method. In order to test the efficiency of our proposed method, we have computed the matching time of the method illustrated in the table below:

Table 2: Comparison of matching time

Number of images	Matching time using pixel by pixel(in second)	Matching time using Generalized method of moment (in second)
20	275	131
40	580	300
60	843	452
80	1130	576
100	1400	703

From the results obtained, it is noticed that the matching time of the proposed method is less compared to the pixel by pixel method.

#### 6. Conclusion

The new method proposed that is the generalized method of moments based on Moore- Penrose an algorithm was successfully applied on hand dorsal vein pattern providing satisfactory results. The FRR and FAR were computed and are found to be less when using our proposed method. It also reduces the dimension of the matrices which

consequently has an impact on matching time. The matching time is improved in our proposed method and this is desired in all biometric security system. We compared our method with pixel by pixel based method.

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