Computations of "Radioactive decay" and "Electron trapped in an infinite potential well" using Lab View

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Summary

A number of scientific simulations are explained which were programmed in Lab view .We choose Lab view because of its convenience for visualization. The scientific topics covered in the paper are "Radioactive decay" and "Electron trapped in an infinite potential well", with a brief theory and background of both the processes along with the mathematical equations. The algorithm used to produce the decay simulation is based on random process for which we used Monte Carlo method. Specifically we are to determine when radioactive decay looks exponential and when it looks stochastic (i.e. determined by chance).Where as the other simulation is based on the analytical equation. Every probable aspect in the programs is elaborated in terms of charts and graphs for better understanding.

Keywords: *Lab view, electron, potential well, Schrödinger equation, simulation, decay law, graphs*

1. Introduction

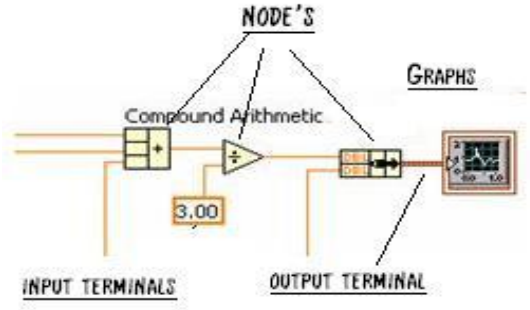
The investigation of certain physical mechanisms by numerical modeling i.e. simulating nature by applying the laws of Physics to virtual processes is becoming increasingly important. The method can lead to an understanding of the overall impact when specific parameters are selectively modified. If the parameters can be changed and adjusted interactively while their effect on a given system is visualized, a student may gain an understanding of the process by observing the effects of the changes.

Also it allows us to produce interactive software for all major computer platforms (Windows, Linux, and Sun UNIX).

2. Lab View

In Recent years the programming concept has evolved with great significance and priorities due to their reliability and machine based measurements.

The pattern of text-based languages emerged gradually which summed up with into a huge collection and classification, for example C/C++, FORTRAN, Java, Pascal etc. Lab view has graphical interface for the programmer. Commands of this language are visually designed functions therefore it is termed as "VISUAL-BASED" programming language. The programs created in Lab view environment are known as Virtual Instruments (or simply VI's). Lab view follows a dataflow model for running VI's. A block diagram "node" executes when all its inputs are available from its input terminals. When a node completes execution, it supplies data to its output terminals and passes the output data to the next node (if available for calculation) in the dataflow path or simply for visualization.



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Why to use Lab View?

1. Lab VIEW provides extremely efficient graphical programming environment, in which data acquisition, data storage and visual presentations are governed easily.

2. Lab VIEW accompanies a utility software used for handling data acquired through external devices(DAQ) called MAX(Measurement & Automation Explorer).This utility software meets the industry standards.

3. Lab VIEW provides DAQ devices that are cost effective.

3. Electron trapped in an infinite potential well

It is a problem that provides several illustrations of prosperities of wave functions and also is one of the easiest problems to solve using time-independent one-dimensional Schrödinger equation (1) is that of the infinite square (particle in a BOX). A macroscopic example is a bead moving on a frictionless wire b/w two massive stops clamped to the wire. Here heights of the barriers between which the particle is bound, are infinite, so particle cannot penetrate through it, but rebounds from barrier.

$$-\frac{h^2}{2m}\frac{d^2(x)}{dx^2}+V(x)\psi(x)=E\psi(x) \quad (1)$$

$\psi(\mathrm{x})$ Must have zero value at walls and all points beyond the walls, signifying that probability of finding the particle in those locations is zero. So standing waves can be setup in the string subject to boundary condition that displacement of string is zero at two rigid supports. We can ease our introduction to Q-Mechanics by exploring analogy b/w mechanical waves propagating along a stretched string and matter waves associated with an electron trapped in infinite well.

3.1 Energy Levels

The quantized Energy values or Eigen values are found from equation:

$$E_n = n^2 E_1$$

Ground state energy is $E1 = \hbar^2\pi^2 / 2m L^2$

The nth state of potential is called Eigen state of total energy with Eigen value E_n.

Constant 'A' in Wave-Function (ψ (x) =A Sin kx) is determined by Normalization condition:

$$1=\int_{-\infty}^{\infty}|\psi(x)|^2\,dx=|A|^2\int_0^L \sin^2(kx)\,dx=|A|^2\frac{L}{2}$$

Then Eigen functions are:

$$\psi_n(x)=\sqrt{\frac{2}{L}}\sin\left(n\pi\frac{x}{L}\right) where\ n=1,2,3,4\ldots.$$

4. Radioactive decay

One nucleus changes into another with the emission of radiation.

4.1 Decay constant

The Probability of decay of a nucleus per unit time is denoted by λ and is called Decay constant. If N is the total number of nuclei present in a sample, then the number of nuclei decaying per unit time is the product of the number of radioactive nuclei and the decay probability of the nucleus.

$$\frac{dN}{dt}=|\lambda N|$$

The Decay constant is a characteristic of the nucleus. This means no two nuclei with different constituents have the

same Decay constant. Therefore the determination of the Decay constant leads to the qualitative analysis (identification) of material and determination of the activity leads to the quantitative analysis (composition) of the material.

4.2 Exponential Decay Law

According to Equation: $\frac{dN}{dt} = |\lambda N|$

Or $$\frac{dN}{dt} = -\lambda N$$

Where the negative sign indicates that N is decreasing with time. If at t=0, the number of radioactive nuclei present in a sample are No , then the number of nuclei N at time t can be determined by integrating the above equation with respect to time and we get:

$$N = No\ e^{-\lambda t}$$

And the activity is:

$$A = \lambda N = -\lambda\ No\ e^{-\lambda t}$$

$$A = Ao\ e^{-\lambda t}$$

This means that Activity or the number of radioactive nuclei decreases exponentially with time.

4.3 Half Life (T) of a Substance

It is defined as the time interval in which the number of radioactive nuclei present in the substance is reduced by a factor of 2.

According to

$$N = No\ e^{-\lambda t}$$

Therefore at t =T, N = No / 2 and substituting in the above equation

$$1/2 = e^{-\lambda t}$$

Or

$$\lambda T = \ln (2)$$

$$\Rightarrow \quad T = \ln (2) / \lambda = 0.693 / \lambda$$

Since λ is a characteristic of a nucleus, so T is also a characteristic of a nucleus. This important fact is used to distinguish different types of nuclei.

The Half-Life remains constant whatsoever may be the change in the chemical and physical shape of the materials.

The number of atoms remained after different half lives are:

At t=1T, N/No = 1/2

At t=2T, N/No = $(½)^2$

At t=3T, N/No = $(½)^3$

After n half lives, where n = t/T

$$\frac{N}{N0} = (½)^n$$

5. Monte Carlo Method

- This method is used to find the solution of such physical phenomena whose Mathematical model depends on probability.
- Monte-Carlo calculation depends on the random number generator

Spontaneous decay is a natural process in which a particle decays into other particles. Because the exact moment of when any one particle will decay is random, it doesn't matter how long the particle has been around or what is happening to the other particles. In other words the probability of any particle decaying per unit time is a constant, also when that particle decays, it gone forever. As the number of particles decreases with time, so will the number of decays.

6. Implementation

6.1 Radioactive Decay Simulation

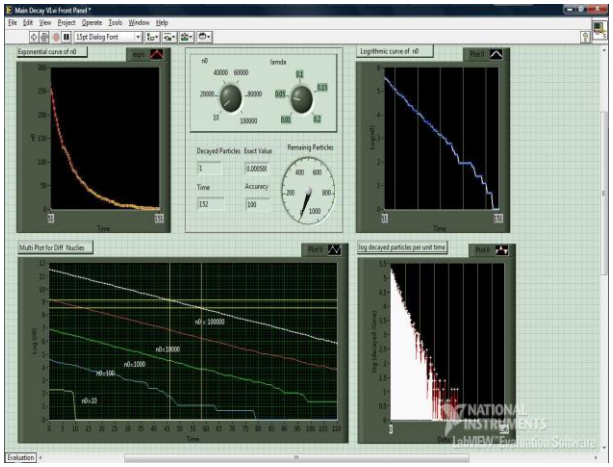

Fig 6.1.1

6.1.1 Decay Simulation Algorithm

Start

1. input N, lambda;
2. initialize delN = 0,decay
3. loop (while N !=0)
   decay = Random number b/w 0-1
   If (decay < lambda)
   delN = delN +1
   N = N-delN.
   End loop
4. Output Display graph of N , DelN

End

6.1.2 Procedure to use the Decay VI:

1. Select the total number of nuclides in the sample using the KNOB "n0".
2. Select the Decay parameter of the element used in the sample using the KNOB "lambda".
3. Press Ctrl+R to run the simulation.
4. Read data from indicators.
5. Read Data from the graph (Logarithmic curve of n0) using the cursor legends, drag the yellow cursors upon any two diff places on the curve.
6. Determine the slope of the curve, where slope = decay parameter.
7. Put the value of slope in slope control and it will calculate the half life of the element.

6.1.3 Charts & Graphs in the Decay VI:

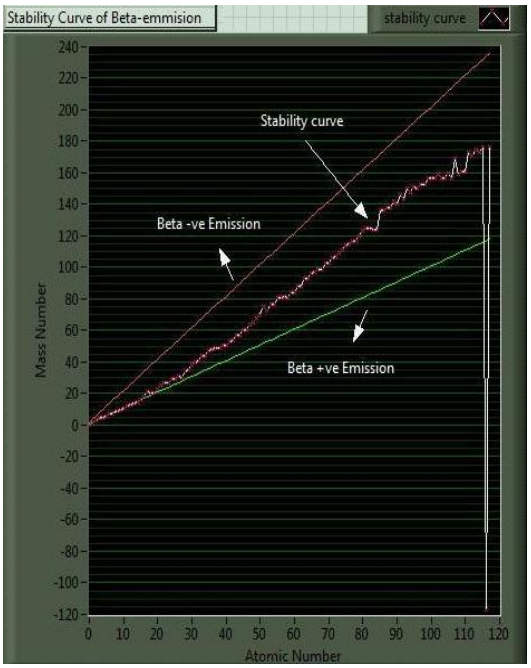

Fig 6.1.2 this plot is generated by reading the entire periodic table from a file, the curve provides vital information about the elements that the isotopes above the curve give beta +ve emitters and below are beta –ev emitters.

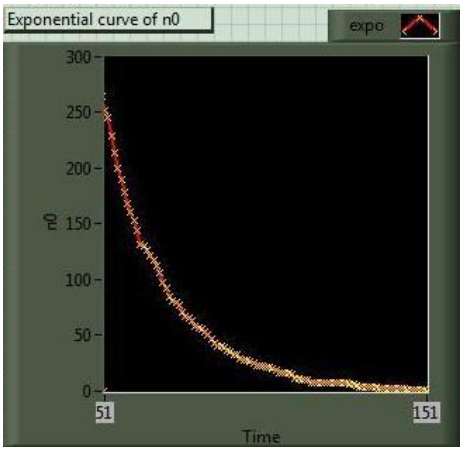

Fig 6.1.3 displays how n0 decays exponentially

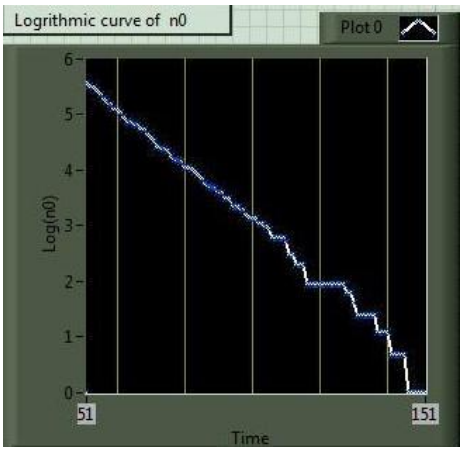

Fig 6.1.4 fits the exponential data into straight line to show the stochastic & continuous behavior in del t.

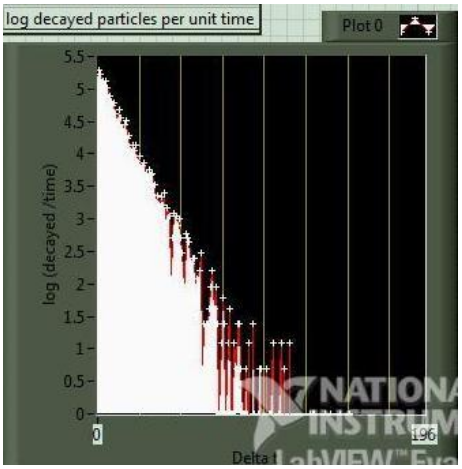

Fig 6.1.5 displays decayed particles per unit time interval, this curve is required to be proportional to the Log curve of n0.

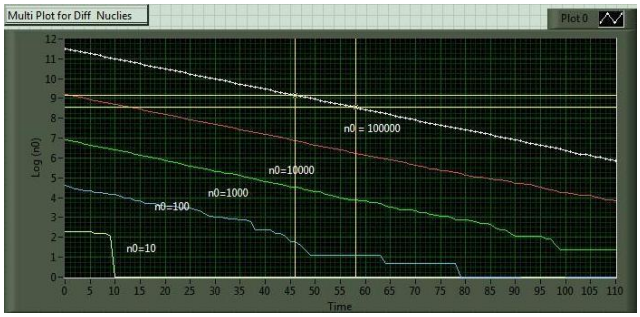

Fig 6.1.6 this graph displays the different curves for different number of nuclei's in different samples belonging to the same element type, every element has unique decay parameter.

Fig 6.1.7 fits the exponential data into straight line to show the stochastic & continuous behavior in given time interval; also we can determine the slope of the curve using cursor legend by dragging the yellow indicators on the curve to find the half life of the element.

6.2 Electron trapped in an infinite potential well Simulation

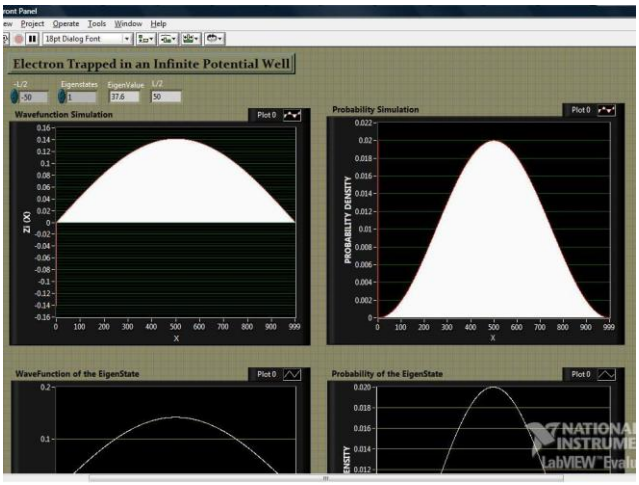

Fig 6.2.1

6.2.1 Potential Well algorithm:

Start

1. Input width, Eigen state.
2. Calculate the Eigen value.
3. Solve Schrödinger equation and generate Eigen function for corresponding Eigen value.
4. Calculate the probability density using normalization condition.
5. Output Graph Eigen function, Probability Density.

End

6.2.2 Procedure to use the VI:

1. Input width of the potential well in the control "-L/2".
2. Input Eigen state (quantum number) in the control "Eigen states".
3. Press Ctrl+R to run the simulation.
4. The "Eigen value" indicator gives output corresponding to the quantum number.
5. Read data from plots by placing cursor on different points to find energy at that point.
6. Also use multi-state plot to find variations in different Eigen functions.

6.2.3 Charts & Graphs in the Well VI:

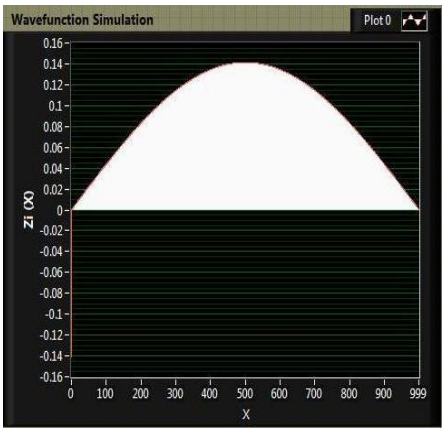

Fig 6.2.2 displays how Eigen function simulates to corresponding Eigen value.

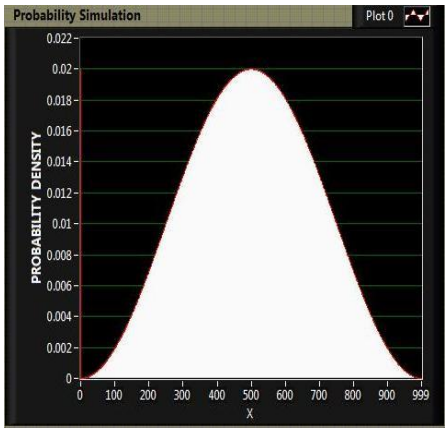

Fig 6.2.3 fits the Eigen function to find the %position of electron (particle) in a particular region.

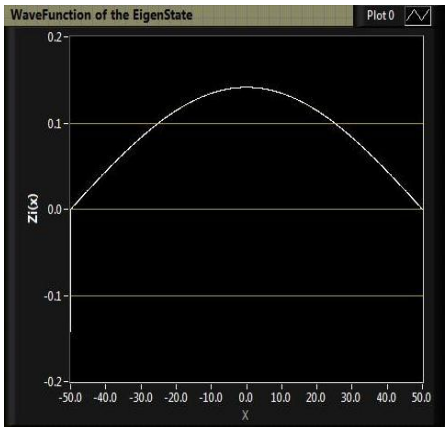

Fig 6.2.4 this graph displays the wave function obtained for certain Eigen value (energy), uses the cursor legend to display the particular value of wave function at any particular point.

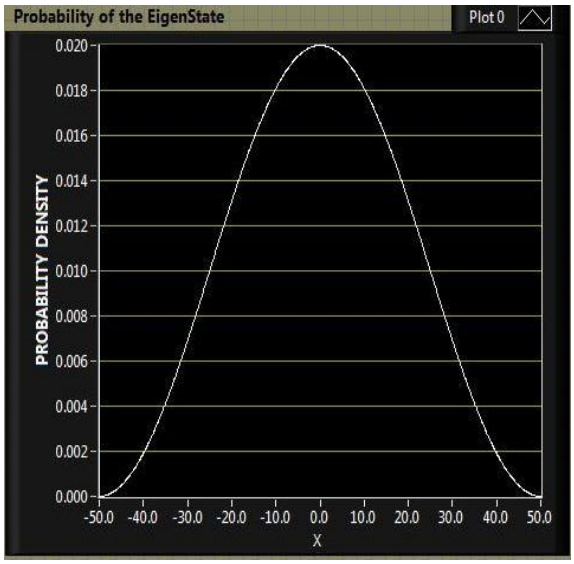

Fig 6.2.5 this graph displays the Probability obtained from wave function expression, uses the cursor legend to display the particular value of probability at any particular point.

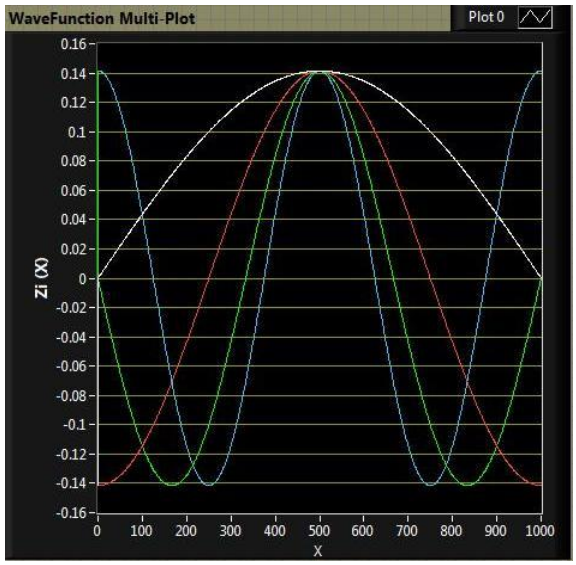

Fig 6.2.6 this graph displays all the wave functions starting from gourd state to the "Eigen state" (control), use cursor legend to determine variation among different wave function.

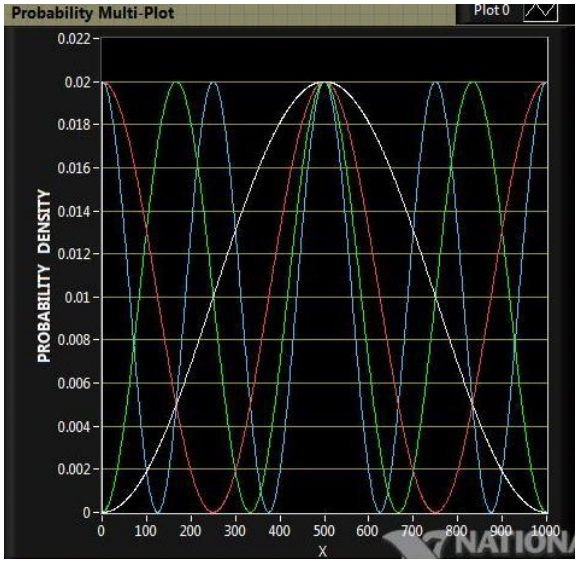

Fig 6.2.7 This graph displays all the Probabilities corresponding to all the wave functions starting from ground state to the "Eigen state" (control), use cursor legend to determine variation among different probabilities.

Conclusion

Our aim was to get a hand full experience in computing in lab view, now we hope to extend our work from general to more specific calculations using the same platform in more computing applications, for example numerical simulations of specific elements in the periodic table, distinguishing between the radiation emission from different elements, applications of numerical solution of Schrödinger's equation, applications for image processing using neural networks etc.

Acknowledgments

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