Low Complexity MMSE Channel Estimation by Weight Matrix Elements Sampling for Downlink OFDMA Mobile WiMAX System

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Summary
Channel estimation is one of key problems in IEEE 802.16e Orthogonal Frequency Division Multiplexing Access (OFDMA) downlink system. Minimum Mean Square Error (MMSE) channel estimation has been known as a superior performance channel estimation. However, this algorithm has high computational complexity. In this paper, we present low complexity partial-sampled MMSE channel estimation for compromising between complexity and performance. We reduced MMSE channel estimation complexity by partially sampling the MMSE weight matrix. The simulation results show that the bit error rate (BER) performance and equalized signal constellation scatter plot significantly improved over the least square channel estimation and has comparable BER performance with MMSE channel estimation. Depending the size of sampling, significant decrease 57 % to 64 % in computational complexity can be achievedthe.

Key words:
Channel estimation, OFDMA, mobile WiMAX, MMSE.

1. Introduction

IEEE 802.16e has come forward as a primary standard for incoming wireless system mostly due to its high mobility.

In order to facilitate the correct detection of transmitted symbols, the effects of the channel must be removed from the received signal. This is the task of equaliser block which perform normalisation of the signal received by each subcarrier with its channel transfer function estimate. Channel transfer estimates are generated within the channel estimation block. The approach for OFDMA mobile WiMAX system channel estimation is pilot –assisted or training symbol based channel estimation which employ pilot symbols known both to the receiver and the transmitters

In recent years numerous research contributions have appeared on the topic of channel transfer function estimation. MMSE channel estimation has been known as a superior performance channel estimation especially in low SNR. However, this algorithm has high computational complexity which prohibits its direct real-time implementation.

Low complexity MMSE channel estimation was proposed by Beek [11] used FFT and IFFT operation to reduce the complexity. A further improvement of complexity reduction offered by Edfors et al [10] with Singular Value Decomposition (SVD) Based low-rank approximation techniques. SVD based method was not possible to implement with acceptable performance for the system model considered [3]. Noh et al.[12] and Mehlfuhrer et al.[13] proposed to partition the channel autocorrelation matrix in the MMSE estimator into small sub-matrices to reduce the complexity. This partition methods, actually only do the channel estimation each sub-symbol instead of each one symbol but have same overall complexity for one symbol as original MMSE channel estimation.

In this work, we proposed low complexity partial-sampled MMSE (Minimum Mean Square Error) channel estimation for IEEE 802.16e (mobile WiMAX) Orthogonal Frequency Division Multiplexing Access (OFDMA) downlink system. We reduce original MMSE channel estimation complexity by reduce the size of MMSE weight matrix. This could be done by partially sampling the subcarrier positions in one symbol that used for generate correlation matrix, instead of use all subcarrier positions. The simulation results show that the bit error rate (BER) performance significantly improved over Least Square (LS) Interpolation, and had comparable BER with MMSE channel estimation especially at low SNR with significant decrease (57% -64%) in computational complexity.

2. System Model

The OFDM/OFDMA system with pilot based channel estimation is given in figure 1.

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The data bits provided from the source are converted from serial to parralel to form parallel data of some subchannels[7].

Each parallel subchannel modulated to complex QAM symbols of N_u active subcarriers. The modulated data with other null carrier as guardband and DC form N subcarriers.

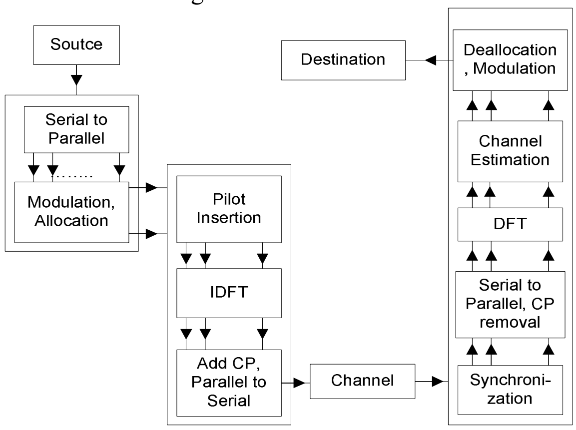

Fig. 1 Blog Diagram of The Pilot Based OFDM System

This data sequence of length N $\{X_k\}$ are then fed into IDFT block symbol by symbol to transform them into time domain and generate an OFDM signal $\{x_n\}$ with the following equation :

$$x_n = IDFT\{X_k\} = \sum_{k=0}^{N-1} X_k \, e^{j2\pi kn/N}, \qquad (1)$$
$$n = 0,1,..., N-1$$

Where N is the DFT length or the number of subcarriers. To prevent inter-symbol interference (ISI), a cyclic prefix of N_g samples is inserted at the beginning of every symbol. After D/A conversion, the signal is transmitted through the frequency selective time varying fading channel with additive noise.

Assumed that the impulse response of the multipath fading channel is given by [1]:

$$h(t,\tau) = \sum_r h_r(t).\delta(\tau - \tau_r), \qquad (2)$$

Where $h_r(t)$ and τ_r are the gain and delay of the r-th path, respectively. The received signal, which has been corrupted by the multipath fading channel and contaminated by the additive white Gaussian noise can be formulated as

$$y(\tau) = \sum_r h_r(t).x(\tau - \tau_r) + w(\tau), \qquad (3)$$

Where $x(\tau)$ is the continuous-time representation of the transmitted discrete-time signal, x_n. The received continuous-time signal then convert back to a discrete – time signal y_n, the receiver do synchronization, downsampling, and removes the cyclic prefix. The simplified baseband model of the received samples takes the form of :

$$y_n = \sum_{l=0}^{L-1} h(l)x(n-l) + w(n) \qquad (4)$$

Where L is the number of sample-spaced channel taps, w(n) is additive white Gaussian noise (AWGN) sample with zero mean and variance of σ^2_w, and h(l) is the time domain channel impulse response (CIR) for the current OFDM symbol. It is assumed that time and frequency synchronization is perfect.

FFT transforms y_n to the frequency domain received base band data :

$$\begin{aligned} Y_k &= FFT(y_n) \\ &= X_k H_k + W_k \end{aligned} \qquad (5)$$

Where H and W are FFT of h and w repectively.

Following FFT block, the pilot signals are extracted and the Channel Estimation is carried out to obtain estimated channel response $\hat{H}_k$ for the data sub-channels. Then the transmitted data is estimated by equalization process :

$$\hat{X}_k = \frac{Y_k}{\hat{H}_k} \qquad (6)$$

After signal demapping, the sourece binary information data are re-constructed at the receiver output.

OFDMA is based on OFDM modulation. Based on the OFDM principle, the pilot both in time domain and in frequency domain is assigned for channel estimation calculation process [2] . The OFDMA downlink IEEE 802.16e symbol structure is using pilots, data, and zero subcarriers. The symbol is first divided into basic clusters and zero carriers are allocated. Pilots and data carriers are allocated within each cluster. Figure 2 below depicts the cluster structure

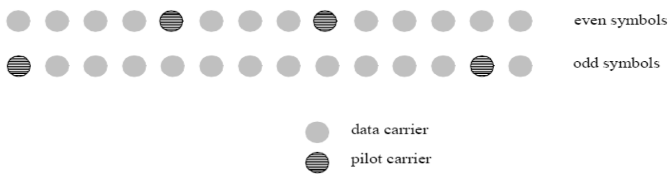

Fig. 2 Downlink OFDMA 802.16e cluster/tile structure

3. Channel Estimation

In this section, the different types of channel estimators considered in this paper are explained. After channel estimation process at pilot subcarrier position, the channel responses at the rest of data subcarrier are estimated by interpolation. First is interpolation at time domain which has 2 symbols time spacing. In this paper we use linear interpolation for time domain interpolation because it is sufficient for small time spacing. H is estimated by vertically 1D linear interpolation, after vertical time interpolation, tile structure is described at figure 3.a

3.1 LS Channel Estimation

In the simple case, the channel estimates, are found by straightforward multiplying the received pilot by the inverse of the known transmitted pilot. This method is called least square (LS) estimator, given by[8] :

$$\hat{H}_{P,LS} = X_P^{-1} Y_P = \left[\frac{Y_P(1)}{X_P(1)} \frac{Y_P(2)}{X_P(2)} \cdots \frac{Y_P(N_P)}{X_P(N_P)} \right]^T \qquad (6)$$

Without using any knowledge of the statictics of the channels, The LS Estimator has very low complexity , but they suffer from a high mean-square error[2].

2.2 MMSE Channel Estimation

The MMSE channel estimator employs the second order statistics of the channel condition to minimize the mean-square error. The major disadvantage of the MMSE estimator is its high complexity, which grow exponentially with the observation sample. The frequency domain MMSE estimate of channel response is given by[6]:

$$\hat{H}_{P,MMSE} = R_{H_P H_P} \left(R_{H_P H_P} + \sigma_n^2 \left(X_P X_P^H \right)^{-1} \right)^{-1} \hat{H}_{P,LS} \qquad (7)$$

Where $H_{P,LS}$ is the LS estimate of channel condition at pilot position, σ_n^2 is the variance of noise , X_P is a matrix containing the transmitted pilot on its diagonal, $R_{H_P H_P}$ is the channel autocorrelation matrix defined by

$$R_{H_P H_P} = E\left\{ H_P H_P^H \right\} \qquad (8)$$

For this case, the correlation function between the channel frequency response value is given by[5] :

$$E\left\{ H_m H_n^* \right\} = \begin{cases} 1, & m = n \\ \dfrac{1 - e^{-j2\pi (N_g (m-n)/N)}}{j2\pi \left(N_g (m-n)/N \right)}, & m \neq n \end{cases}$$

(9)

From equation (9) we can get $R_{H_P H_P}$.

MMSE interpolation for all subcarrier can be perform by modifying the MMSE estimator at equation (7) to obtain all data subcarrier's channel responses, with this equation[3] :

$$\hat{H}_{MMSE} = R_{HH_P} \left(R_{HH_P} + \sigma_n^2 \left(X_P X_P^H \right)^{-1} \right)^{-1} \hat{H}_{P,LS} = Q.\hat{H}_{P,LS} \qquad (10)$$

The MMSE estimator (7 and 10) uses a priori knowledge of σ_n^2 (or *SNR*) and R_{HH} , and is optimal when these statistics of the channel are known. As will become clear from the further discussion, *SNR* value can be predefined: higher target SNRs are preferable to obtain more accurate estimates. Also the robust estimator design necessitates account for the worst correlation of the multipath channel, namely when the channel power-delay profile (PDP) is uniform [14].

3.3 Proposed Down-sampled MMSE Channel Estimation

To reduce the complexity of the MMSE estimator in equation (10), we must reduce the size of correlation matrix and thus also the size of the weight matrix Q. This can be achieved by sampling the subcarrier positions in one symbol that used for generate cross correlation matrix, R_{HH_P} , instead of use all subcarrier positions.

For the original MMSE channel estimator, one element of the cross correlation matrix is given by

$$r_{H_m H_{pn}} = E\left\{ H_m H_n^* \right\} = \begin{cases} 1, & m = n \\ \dfrac{1 - e^{-j2\pi (N_g (m-n)/N)}}{j2\pi \left(N_g (m-n)/N \right)}, & m \neq n \end{cases} \qquad (11)$$

where m= 1 to N , n= P_0 to P_L (P_i=subcarrier index of i^{th} pilot), N= number of subcarrier used in one OFDMA symbol and L= number of interpolated pilot in one OFDMA symbol.

Our proposed Down-sampled MMSE Estimator (DMMSE) only use d indexed subcarrier where d=1 to D with D < N. D=360 is the number of down-sampled subcarriers from N =840 original subcarriers in one OFDMA symbol.

$$r_{H_d H_{pn}} = E\{H_d H_n^*\} = \begin{cases} 1, & d = n \\ \dfrac{1 - e^{-j2\pi(N_g(d-n)/N)}}{j2\pi(N_g(d-n)/N)}, & d \neq n \end{cases} \quad (12)$$

Where $d = 1$ to D, $n = P_0$ to P_L (P_i=subcarrier index of i^{th} pilot), D= number of down-sampled subcarrier in one OFDMA symbol and L= number of time interpolated pilot in one OFDMA symbol.

We used subcarrier number 1, 3, 5, The picture below shows which subcarrier is sampled to form down-sampled MMSE weight matrix. This picture display m, n and d indexed subcarriers from two subsequent tile of one OFDMA frame.

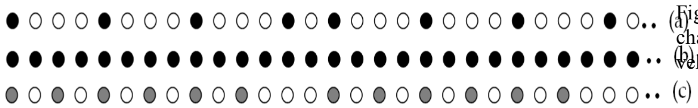

Fig. 3. (a) l indexed pilot subcarriers after time interpolation (number of time interpolated pilot = L=240), (b) n indexed subcarrier or all subcarriers in one symbol (number of all subcarriers = N=840), (c) d indexed subcarrier or down sampled subcarrier in one symbol (number of down-sampled subcarriers=D=360)

The computational complexity of original or full rank MMSE estimator is reduced from N x L (N=840) to Dx L (D=360 and 300) or 57 % and 64 % reduced .

3. Simulation

In this section, we report computer simulation carried out to evaluate and compare performance of the considered channel estimation. We used downlink OFDMA system of IEEE 802.16e. The OFDMA system parameters used in the simulation are indicated in Table I.

The channel models used in the simulation is ITU-R B channel for vehicular environment which closely represent mobile channel for IEEE 802.16e mobile WiMAX. We set the vehicle speed of user to 60 km/h.

Table 1. Parameter Used in The Paper

Three channel estimation method, i.e LS channel estimation , full rank MMSE channel estimation and proposed partial-sampled MMSE channel estimation, are simulated and compared. The result are shown in Fig. 4- 5. The horizontal variable is signal to noise ratio and the vertical variable is Bit Error Rate.

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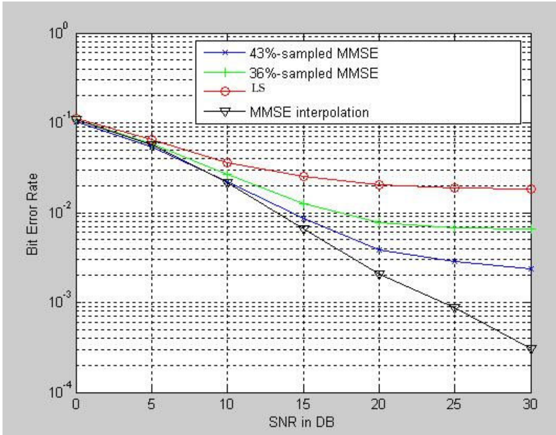

Fig.4. BER performance of LS, MMSE and Partial Sampled MMSE channel estimation methods for QPSK modulation under ITU B vehicular channel model.

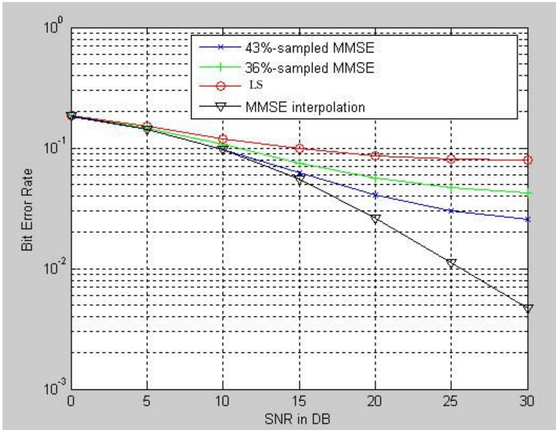

Fig.5. BER performance of LS, MMSE and proposed Partial Sampled MMSE channel estimation methods for 64 QAM modulationunder ITU B vehicular channel model.

At ITU B vehicular channel condition (figure 4 and 5), the Partial sampled MMSE estimator show comparable performance with MMSE estimator at low SNR and slightly under the MMSE estimator's performance at high SNR, on the other hand, has definitely better performance than the LS estimator. The knowledge of the channel corelation in more subcarriers alows better performance

Parameter	Value	Note
N	1024	FFT Size
N_u	840	Number of used subcarrier
N_g	256	CP Size
N_P	120	Number of pilot subcarrier
$N_{PPREAMBLE}$	280	Number of used/pilot subcarriers in preamble

because highly selectivity channel condition at ITU-B vehicular channel make linearly assumption is not valid anymore. Partial sampled MMSE estimator which has moderate channel correlation knowledge also perform moderate BER performance compare to MMSE estimator performance but outperforms the LS estimator (Linear interpolation) significantly.

Fig. 6 – 8 show the scatter plot of received signal and equalized signal for the three different algorithms with 64 QAM modulation. It is shown that the 43% and 36% sampled MMSE interpolation has close equalized signal constellation and better signal constellation than linear interpolation signal constellation.

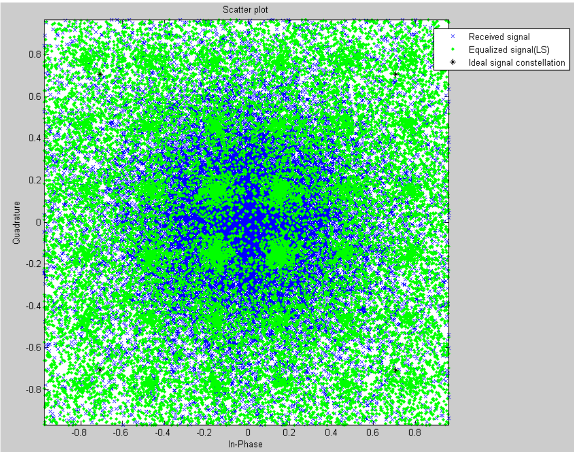

Fig.6 Scatter Plot of equalized signal (green) for LS channel estimation with 64 QAM modulation under ITU B vehicular channel model.

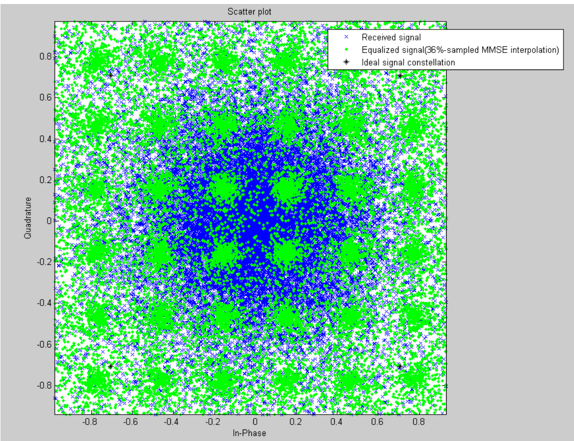

Fig.7 Scatter Plot of equalized signal (green) for 36% sampled MMSE channel estimation with with 64 QAM modulation under ITU B vehicular channel model.

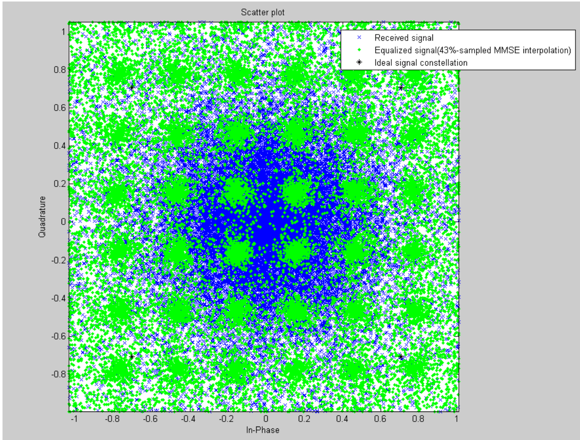

Fig.8 Scatter Plot of equalized signal (green) for 43% sampled MMSE channel estimation with with 64 QAM modulation under ITU B vehicular channel model.

5. Conclusion

In this paper we introduce a low complexity partial samples MMSE channel estimation method for downlink OFDMA IEEE 802.16e (mobile WiMAX) system. The Partial Sampled MMSE channel estimation decrease the original MMSE channel estimation complexity by down-sampling the subcarrier positions in one symbol that used for generate correlation matrix. The simulation results show that the bit error rate (BER) performance significantly improved over Least Square (LS) Interpolation, and had comparable BER with MMSE channel estimation especially at low SNR with significant decrease (57%) in computational complexity.

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