Comments on "Computation of the Range of Intervals between in which the roots lie by Given's technique"

T.D.Roopamala† and S.k.Katti††,

University of Mysore

Summary
In this technical note, a computationally superior The following procedure has been adopted in the Given's method.Firstly, the Strum sequence has been computed as given below method via Gerschgorin's technique to compute the range of intervals between which the root that lie of a tridiagonal matrix over the existing Given's technique is discussed.

Key words: Strum sequences, eigenvalues, Range of intervals Gerschgorin's circles

1. Introduction

The concept of stability plays very important role in the analysis of systems. A system matrix can be used [1] for the stability studies. In literature, there exists Given's method for the computation of bounds of the eigenvalues and it takes lots of computation [2]. It has been found that by using Gerschgorin's technique the bounds of the eigenvalues can be obtained graphically, which takes no computations**.**

2. In the following, the Given's method and Gerschgorin's method have been explained to compute the intervals under which the roots of the given matrix A lie and both the methods are compared

2.1 GIVEN'S TECHNIQUE

In [2], the following tridiagonal matrix is considered to compute range of intervals for the eigenvalues.

$$A = \begin{bmatrix} 4.000 & 3.000 & 0.000 & 0.000 \\ 3.000 & 3.334 & 1.666 & 0.000 \\ 0.000 & 1.666 & -1.22 & 0.907 \\ 0.000 & 0.000 & 0.907 & 1.907 \end{bmatrix}$$

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The following procedure has been adopted in the Given's method.

Firstly, the Strum sequence has been computed as given below.

$P_0(\lambda) = 1$

$P_1(\lambda) = 4 - \lambda$

$P_2(\lambda) = (3.334 - \lambda) P_1(\lambda) - 9$

$P_3(\lambda) = -(1.32 + \lambda) P_2(\lambda) - 2.776 P_1(\lambda)$

$P_4(\lambda) = (1.987 - \lambda) P_3(\lambda) - 0.823 P_2(\lambda)$

Then, the table 1 can be constructed.
V (-2) – V (-3) = 1, one root lies between $-3 \le \lambda_1 \le -2$
Similarly other roots ranges are $1 \le \lambda_2 \le 2$, $2 \le \lambda_3 \le 3$ and $6 \le \lambda_4 \le 7$ We now obtain V (-2.5) as 1 and comparing it with V (-3) and V (-2) it is concluded that $-3 \le \lambda_1 \le -2.5$

Table 1

λ	P_o	P_1	P_2	P_3	P_4	V (λ)
-3	+1	7	35.338	39.396	170.077	0
-2	+1	6	23.004	-1.0133	-18.945	1
1	+1	3	-1.998	-3.693	-2.0	1
2	+1	2	-6.337	15.47	5.01	2
3	+1	1	-8.667	34.661	-27.98	3
6	+1	-2	-3.668	32.402	-127.01	3
7	+1	-3	1.989	-8.22	39.57	4

Then the range under which the roots are calculated as This procedure can be continued.. The range $-3 \le \lambda_1 \le -2.5$ can be further bisected and a new lighter bound for λ_1 be determined. Continued application of the bisection technique will keep reducing the interval in which λ_1 lies until a desired accuracy of λ_1 is attained. However in this Given's method the calculation of the Strum sequences need a lot of computations. Authors in this article used Gerschgorin's circle technique to do the same and it is described as follows.

(ii) GERSCHGORIN'S THEOREM

For a matrix A of order (nxn), let P_k be the sum of the moduli of the elements along the k th row excluding the diagonal elements a_{kk}. Then every eigenvalues of A lies inside the boundary of atleast one of the circles.

$$\left|\lambda - a_{ii}\right| \le \sum_{\substack{i=1 \\ i \ne j}}^{n} \left|a_{ij}\right| = r_j$$

2.2 From the above, Gerschgorin circles are drawn as follows

Consider the same tridiagonal matrix as in the method (i) and the Gerschgorin's circles have been shown in figure (1) from software developed by the Authors.

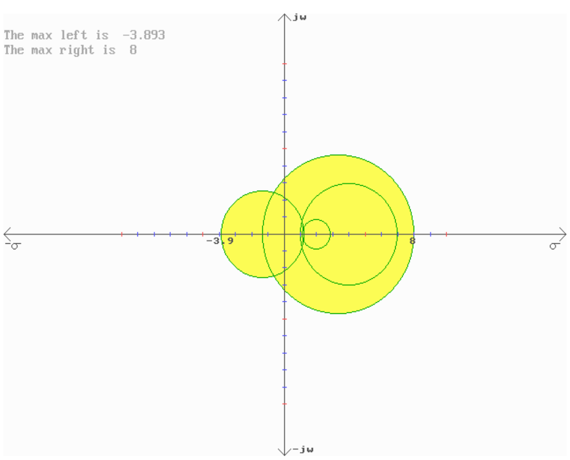

Fig (1): Gerschgorin's bound [3.9, 8]

From the Gerschgorin circle diagram it is easy to decide the bound and the interval in which the eigenvalues lie. The Gerschgorin's bound is [3.9, 8]. Intervals are as follows.

$-3.9 < \lambda < -1.4$, $-1.4 < \lambda < 0$, $0 < \lambda < 0.9$, $0.9 < \lambda < 1$, $1.1 < \lambda < 3$, $3 < \lambda < 3.9$, $3.9 < \lambda < 7$, $7 < \lambda < 8$

3. Conclusions

In this paper, we have proposed a simple technique for calculating the intervals under which the eigenvalues lie using the Gerschgorin's circles. It does not need any computation vis-à-vis the Given's method

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T. D. Roopamala was born in India. She has been graduated from Mysore University in B.Sc (Electronics – in 1984), M.Sc (Mathematics- 1986) , PGDCA- (6th rank - 1991) and Ms(Software Systems–BITS pilani – 1998) . She is presently working in S.J.C.E., Mysore with a teaching experience of 21 years. Her area of interest is computational techniques, computer engineering.

S. K. Katti was born in 1941 in India. He has graduated in B.E.(Tele-com –(1964)) , B.E.(Elect- (1965)) and M.E. (in control systems-(1972)) from Pune University (India). He has obtained his Ph.D degree in.' Systems Science', from Indian Institute of Science Bangalore in 1984. He has a teaching experience of 42 years. He has worked as a Professor of Electrical Engineering at Pune Engineering College during 1994-1999 and finally he has retired from the college. Presently he has been working as the Professor of Computer science and Engineering at S.J.C.E, Mysore since 2001. His areas of Research interest are: Multivariable control system designs, Artificial intelligence, Digital signal processing, Cognitive Science, Fussy logic and Speech Recognition via HMM models. He has 7 International publications, 2 International Conferences and 7 papers at National level. He has worked as a Reviewer for IEEE transaction on Automatic Control and also he was reviewer for Automatica. He has worked as external examiner for few Ph.D thesis in Computer Science. Presently, two Research Scholars are working under him in the area of Computer science for Ph.D studies.