A Suitable Mathematical Model of PET for FEA Drop-Test Analysis

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Summary
Attempting the use of finite element analysis (FEA) in aiding plastic – product design has yielded unacceptable results due to the limited knowledge of the material properties . this enigma even found greater pitfall when facing the drop – test requirement – which is a dynamic problem , where the mechanical property of the material is time dependent . this paper introduces a suitable mathematical model of material property for drop – test analysis . polyethylene terephthalate (PET) , as the popularly plastic used for beverage bottles was selected for this research . the proposed mathematical model of PET was tasted physically and digitally and compared with the FEA results . the MSC . Dytran , as a FEA solver , with our material model gave 1.303 % error of the peak impact force on ground.
Key words:
PET, FEA, Drop test,bottle.

1. Introduction

Every day consumers use millions of plastic bottles. the plastic bottles, not only help us to preserve the quality and freshness of fruits juice, soft drink, water and cooking, oil, they must be safe and durable enough during the transportation, storage and accidentally dropping. polyethylene terephthalate (PET or PETE) has become the material of choice for bottled beverages because it is lightweight and shatter resistant. PET is one of the most common consumer plastics used. The PET bottles are manufacturing by injection stretch blow molding process.
An important durability test of the bottle is the drop test {1}. the test is conducted by filling the bottle with water at full capacity, and dropping it from a certain height onto a hard steel or concrete floor. The bottle is then observed for the breakage {2}. This conventional drop test method is costly and time consumed. With the introduction of commercially available finite element analysis (FEA) software, the drop test for the PET bottle can be digitally simulated for strength without molding and testing {3, 4}. FEA enables prediction the performance of the bottle under any realistic loading situations. However the FEA solution for analysis the drop test has a disadvantage in the material data .
This paper describes the formulation for the mathematical model of the PET thermoplastic material for FEA drop – test simulation. The stress strain relation of PET under several strain rates is proven to be a suitable model .our introduction of the mathematical function of the material property in the nonlinear – plastic region by n-power function and Cowper – symond equation. The time function of impact force from experimental data and FEA result is compared to show the accuracy .

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2. Modeling the Material Property

2.1 Modeling Plastic Strain Hardening

Polyethylene terephthalate or PET exhibits the exhibits – plastic behavior. For an isotropic material , the stress strain relation of the elastic – plastic material is linear elastic at low strains or up to the yield stress ; follows by nonlinear plastic deformation for large strains. The linear elastic is characterized by the young's modulus (E) poisons ration (v) and proportional limit or yield stress. The nonlinear plastic deformation associates with the effects of stain hardening. The hardening behaviors is required, specially when the FEA is employed for studying the drop test where large deformation is commonly found.
Since PET is aviscoelastic material, then the hardening curve also depends on the strain rate associated with the applied tensile load {5}.
Hence the hardening behaviors is a plot of true tensile against the true tensile plastic strain. The true strain ЄT is obtained by

$$ЄT = \frac{\sigma e}{E} + Єp \quad (1)$$

Where Єt is true total strain
σ e is true elastic stress
Є p is true plastic strain

The true hardening stress σ h can be accurately modeled by the function of the form

$$\sigma h = \{ \sigma yl + (\sigma f - \sigma yl) \epsilon p \} (1 + \alpha \epsilon p) \quad (2)$$

where σ h is true hardening stress
σ yl is lower yield stress
σ f is flow stress
n and α are constants .

Equation (2) indicates that the stress σ h has the range between the lower yield stress σ yl and ultimate stress. As σ h = σ yl this corresponding to zero true plastic strain; and σ h = σ f indicates the stress after which is a sharp stress increasing is observed before the ultimate stress.
The parameter n influences the magnitude of the stress between σ yl and σf the constant α provides an amplification factor to the true plastic strain Єp.

2.2 Modeling Strain Rate Behaviour

The elastic – plastic material can be modeled, with sensitivity of the strain rate, by using the Cowper Symonds equation {6}. The equation relates the yield stress (σy) The initial yield stress (σ yo) and strain rate (Є&) the cowper– Symonds equation is descried by

$$\frac{\sigma y}{\sigma yo} = 1 + \{ \frac{\epsilon}{D} \}^{1/P} \quad (3)$$

Where σ y is the yield stress
σ yo is the initial yield stress
Є is the strain rate
D and P are constant .

The constants D and P need to be found experimentally to give the best fit to the experimental results .
The Cowper – Symonds equation can be written for law yield stress and flow stress respectively as

$$\frac{\sigma yl}{\sigma ylo} = 1 + \{ \frac{\epsilon}{D} \}^{1/P} \quad (4)$$

$$\frac{\sigma f}{\sigma fo} = 1 + \{ \frac{\epsilon}{D} \}^{1/P} \quad (5)$$

Where σ yl is lower yield stress
σ ylo is initial lower yield stress at minimum strain rate
σ f is flow stress
σ fo is initial flow stress at minimum strain rate.

The stresses σ ylo and σ fo are constants and can be obtained from tensile measurement and linear regression analysis.
Substituting σ yl and σ f in equation (2), the true hardening stress becomes

$$\sigma h = \{ \sigma ylo + (\sigma fo - \sigma ylo) \epsilon p \} (1 + \alpha \epsilon p) \{ 1 + (\frac{\epsilon}{D}) ^{1/p} \quad (6)$$

Equation (6) is found to be practical for describing the materials properties for PET as it is applicable to define the hardening function in most FEA packages.

2.3 Finite Element Modeling

The availability of the 3d finite element analysis software, at mechanical engineering department kasetsart university, is the MSC. Dytran it can perform transient non – linear analyses as for our case of the drop test simulation, the software is suitable MSC. Dytran bases on a numerical explicit time integration method. This method is appropriate for dynamic analyses MSC. Dytran permits the use of both lagrangian and eulerian processors for analyzing both structure and fluids. The lagrangain solver can use shell, beam plate and solid elements; while only 3D solid element type is available for the eulerian solver.
The mesh for the lagrangian solver, or lagrangain mesh, consists of grid points and constant – mass elements; they belong to the wall of the bottle. Grid points displace in 3D space, follow the element deformation, as the bottle deforms .
To allow the interaction between the bottle and the fluid the "general coupling" algorithm can be applied at the wall of the bottle. This coupling at the wall surface acts as a boundary condition for the flow of the fluid in the eulerian elements. using this " general coupling " the fluid surface can undergo arbitrary motions and shapes ; under the restriction that the lagrangian mesh is a closed volume .
Under a bottle motion , the fluid in the eulerian elements exerts a pressure load on the bottle surface or the lagrangian mesh . this load is converted to forces acting on the grid points of the surface .
The contact algorithm between the force plate and fluid – filled bottle can be selected to show the time record of the impact force { 7,8,9 }

3. Experimental Material and Equipment

The pet bottle used for this research is shown in figure 1a . the bottle has the capacity of 510 ml , 177 mm height , 67.7 mm diameter , and average thickness at the wall of the body is .56 mm . the mass of the bottle is 0.032 kg the bottle were obtained from TEDCO Thailand .
The stress – strain relation of PET was obtained with a tinius Olsen (HOUNSFIELD – h50ks Pennsylvania U.S.A) tensile testing machine .
The impact force on ground was measures and recorded by a force plate system , AMTI model OR6-7 , Massachusetts , U.S.A (figure 1 b) the signals from the force plate were taken to amplifier , A / D converter and storing the data in the hard disc of a personal computer .
A video recorder (SONY DCR – TRV310 E) was utilized for every dropping to ensure the proper bottle orientation at the instant of impact .
The pre – and post – processor for defining the finite element model (FEM) and model constrains was done by using MSC patran 2006. the finite element analysis of the drop test simulation was performed by a FEA software – MSC .Dytran 2006 . both MSC. patran 2006 and MSC .Dytran 2006 are the products of macneal – schwendler corporation California , U.S.A

4. Experimental Procedures

4.1 Stress – Strain Measurement of PET

The pet sample , for determining the strain rate , were obtained form the wall of the PET bottles . this type of bottle , from the same manufacturing lot , was used for the later drop test . each PET sample or specimen for the sensile testing according to ASTM D 882 standard { 10 } had the dimension of 8 X 110 mm . with grip separation length of 50 mm . the testing was done by follow the ASTM D638 standard { 11 } . the grip separation speeds were 5 , 100 , 300 and 500 mm / min . the strain rate was obtained form

$$\epsilon = \frac{V}{l_o} \qquad (7)$$

Where ϵ is strain rate (sec -1)
V is grip separation speed (mm / sec)
and l_o is grip separation length (mm)
hence , the strain rates were at 0.00167 , 0.03333, 0.1 and 0.16667 sec -1 respectively . figure 2 shown the experimental results of the true stress – strain curves at four strain rates . the obtained stress – strain measurement was used as the data for formulating the material model of the material

4.2 Developing A Mathematical Model of The Material

From the data of figure 2 , the young's modulus (E) at each strain tare were obtained at a strain less than 0.0807 , and has the value of E = 888.41 MPa . these were used for the relation , based on the assumption of equation 1 , of true elastic stress σ_e against true plastic strain ϵ_p .
Value pf parameters n and α can be obtained from the best fit curve of equation (2) to the measured stress – strain data of figure 2 . the resulting values of n and α are shown in table 1 . unlike σ_{yl} and σ_f the parameters n and α are independent of the strain rate , hence n = 2 and α = 0.25 .
The constants σ_{yo} , σ_{fo} , D and P can be obtained by the best fit curve of equation (4) and (5) and data in table 1 , the resulting values of σ_{yo} , σ_{fo} , D and P are shown in table 2 , that is σ_{yo} = 7.5865 , σ_{fo} = 15.1730 , D = 0.1005 and P = .46 respectively .
Figure 3 shown the comparison between equation (6) and the measured data (figure 2) by using parameters in table 2. it can be seen that the proposed function comparing well to the measured data specially for strain less than 0.6 for all strain rates , and up to the strain of 1.2 for strain tare of 0.1667 sec -1 .

4.3 Drop Test and Impact Force Measurement

One way to investigate the drop – test results . the measuring impact force on ground was obtained by conducting the drop test on the force plate platform . the drop test was performed at the dropping height of 0.5, 1.0 and 1.5 m .each experiment was performed with water filled to full capacity . the time records of the impact force were obtained from the output signal of the force plate . figure 4 shown four time frames of the water – filled bottle dropping form 0.5 meters .
Due to the unrepeatability of the orientation of the bottle at the instant of the plate impact , a video recorder was employed to examine the deviation angle of the bottle
Away from the gravitational axis at the initial contact on the plate . for this paper , this angle is referred as the " impact angle " it is measured from the vertical axis . an acceptable impact force signal was selected form the impact angle within + - 3 degrees . figure 5 to 7 shown the impact force functions at the drop height of 0.5 , 1.0 and 1.5 meters respectively . each drop height contained five records .
The records of the impact force measurement were used to compare with FEA results .

4.4 Finite Element Modeling for Drop Test

The MSC . patran is used for creating the finite element model of bottle , force plate and water . the bottle had 998 quadratic shell elements , and the force plate had 121 quadratic shell elements . material model of the bottle was the piecewise linear plasticity (DMAT24) definable in MSC patran and MSC .Dytran this DMAT24 material model was suitable for an elastic material . the mathematical function of PET according to equation (6) was Applicable to DMAT24 .

The elements of the force plate were set to be rigid material .all contact elements of the force plate (121 elements) are selected by contact algorithm to show the time record of the impact force .

Eulerian elements for the water were modeled by using 1.300 rectangular 3D solid elements material model of the water was the linfluid (DMAT) definable in MSC .patron and MSC. Dytran . this linfluid material model was for an incompressible fluid material . figures 8a and 8b are the finite element models of lagrangain mesh (bottle and force plate) and eulerian mesh (water) respectively .

The initial condition setting . for FEA simulating the dropping of the bottle , depends on the drop height . instead of using the actual drop height for the analysis , a simulating initial velocity and a simulating initial position were calculated from the required actual drop height . the simulating initial velocity was applied as it shortening the drop height . this practice greatly reduced the computation time and error simulating initial velocity can be calculated from .

$$V1 = 2g\ (\ h - x) \qquad (8)$$

Where v1 is the simulating initial velocity
G is acceleration of gravity
H is the required actual drop height
And X is the simulating initial height

The initial position x can be viewed as the simulating height from the force plate to the bottle base (figure 9) notice that , unlike the physical bottle , the impact angle of zero is always obtained in FEA.

Figure 10 illustrated the simulated results of the bottle after impact the plate by 0.319 sec ; the drop height is 0.5 m . the figures shown deformation (figure 10 a) stress (figure 10b) and stain (figure 10 c) .

Averaged of the experimental data at 0.5 , 1.0 meters respectively.

5. Results and Discussion

The mathematical function described the material property of PET in the plastic region is

$$\sigma h = (\ 7.5865 + 7.5865\ \epsilon\ 2)\ (\ 1 + 1.25\ \epsilon p)\ [\ 1 + (\ \frac{\epsilon}{0.1005}\ )\ 2.17\ \} \qquad (9)$$

This proposed mathematical model of PET illustrated good correlation with the measured stress – stress relation for all four strain rates – as shown in figure 3 . for all strain rates the function mapped well up to the strain of 0.6 ; and up to 1.2 for strain of 0.1667 sec -1 .

By applying the proposed material model of equation (9) to the finite element model for simulating the drop test , the results are shown in figure 11 to 13 . each figure also compares the FEA result with the ensemble average of the time records of the impact force on the plate at the drop height of 0.5 , 1.0 and 1.5 meters.

Plot of figure 14 is the maximum impact force of the FEA and ensemble averaged of the measured data at the three drop height . the average error of the peak impact force is 1.303 %

6. Conclusions

The objective of this research is to develop a suitable mathematical model for popularly used plastic – PET with the purpose for applying to FEA drop – test analysis . the parameters of the mathematical model rely on tensile testing data at several strain rates . the accuracy of the PET material model was evaluated by comparing the impact force of a 510 ml PET bottle dropping from 0.5 , 1.0 and 1.5 meters above the ground the water was filled to full level of the bottle . and a force plate measuring system measured the impact force on the ground .

The MSC .Dytran 2006 was applied for the drop – test FEA simulation using the material model of equation (9) . the obtained PET material model shown good agreement in term of the park impact force with average error of 1.303 % .

Since there are variation in material manufacturing processes and chemical composition of commercial Pets' material model of equation (9) is not ensured the accuracy to most Pets' available in the market . hence the fit – curve procedure to the tensile results is recommended for evaluating the suitable values of

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