A compact lens-less optical image encoding system using diffraction grating

A.S. Samra, S.S. Kishk, and S.S. Elnaggar
Department of Communications Engineering and Electronics, Mansoura University, Mansoura 35516, Egypt.

Summary
A method for a compact lens-less image encoding system using diffraction grating and random phase masks is presented. Lens-less double phase encoding is implemented using diffraction grating. The Beam Propagation Method (BPM) is used to analyze optical waveguide grating. The phase keys functions and grating parameters are the encryption keys. Simulation results are presented in support of the proposed idea. Results are compared to the conventional Double random phase encoding (DRPE) method. The robustness and performance of the proposed technique is analyzed using the mean-square-error (MSE)
Keywords:

Data processing by optical means; Diffraction gratings; Optical security and encryption

1. Introduction

Optical image encryption is gaining interest due to its distinct advantages such as parallel processing ability, and providing many degrees of freedom for coding information.
Several techniques and applications are proposed for optical image encryption [1-7]. Double random phase encoding (DRPE), proposed by Refregier and Javidi [1], is considered among the widely used optical encryption techniques. Fractional Fourier transform (FRFT) [2,8] is proposed as a generalization of the conventional plane encodings. Optical Fourier Transform (OFT) plays an important role in optical image processing and double phase encoding systems. For the Fraunhofer integral to be valid, the observation distance should be very large compared to the aperture dimensions. So, lenses are used to perform optical Fourier transform (OFT) of an object in the double random phase encoding technique. But lenses suffer from aberration which produces errors in the calculated Fourier transform. Recently other optical encryption techniques using diffraction gratings are proposed [9],[10].
In this work, a compact lens-less method for images encryption and decryption is proposed. This method relies on the use of diffraction grating instead of the optical lens. So, the encryption process is performed by using two random phase masks and a 2-D phase transmission grating of certain periodicity. The use of the phase grating enhances the security of the encoding system as the grating parameters must be correct to successfully retrieve the original image, so the system become more secure. Also, the system is compact due to removal of the lenses and the need of a propagation distance that equals the lens focal length.
Simulation results of encryption and decryption of 2-D images by the conventional DRPE method and by using the diffraction grating waveguide are presented to evaluate the reliability of the technique, MSE between the decrypted and original image has been calculated. The paper is organized as follows. Section 2 is a theoretical analysis, Simulations and discussions are presented in section 3. Finally section 4 is a conclusion.

Manuscript received June 5, 2010
Manuscript revised June 20, 2010

2. Theoretical analysis

2.1. Double random phase encoding (DRPE)

DRPE uses two statistically independent random phase masks in the input and the Fourier planes to encrypt the input image into a stationary white like noise. During the encryption process, the image to be encrypted, *I(x,y)*, where *x, y* are the spatial coordinates, is immediately followed by a first random phase mask R_1(x,y). Both the image and the mask are located in the object focal plane of a first lens (L1). FT of the product $R_1(x,y)I(x,y)$ is obtained at the image focal plane of this lens. The second random key $R_2(u,v)$ is multiplied by the calculated FT, where *u,v* are the Fourier plane coordinates. Lastly, another FT is performed by a second lens (L2) to transform back to the spatial domain. Fig. 1 shows an optical implementation of the DRPE. The encoded image Ψ(x,y) is given by:

$$\Psi(x,y) = IFT\left[R_2(u,v)FT\left[I(x,y)R_1(x,y)\right]\right] \qquad (1)$$

For decryption, the encrypted image is Fourier transformed and multiplied with the complex conjugate of R2(u, v). Then, inverse Fourier transformed to get the decrypted image. The conjugate of the used two random phase masks used during the process of encryption act as the keys for data security during decryption [1].

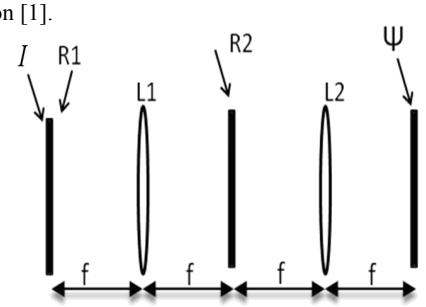


Fig.1 Optical implementation of DRPE

2.2. Diffraction Grating

Diffraction gratings are optical elements that are superimposed with a precise pattern of microscopic periodic structures, producing periodic alternations in the phase and/or amplitude of an emergent wave [11]. Gratings are used in several applications

include, multiplexing, demultiplexing, holographic optical elements, interferometry, lenses, etc. [12]. Gratings can be classified as either amplitude gratings or phase gratings. Another classification is as reflection gratings or transmission gratings.
In this work, a two-dimensional phase transmission grating and double-random phase masks are used to perform image encryption. That is to say that the lenses are replaced by the phase grating. During encryption, the original image is multiplied by the first phase mask, then propagates along a diffraction grating of interaction length *d* and periodicity Λ. then the output is multiplied by the second phase mask. Figure 2(a) shows the equivalent of what is called a ruled grating in conventional three dimensional optics. Figure2(b) shows a phase grating : two planar optical waveguides are separated by a zone where the effective index of the guide is periodically modulated. Here we will be concerned with integrated Bragg phase transmission grating. Several methods have already been used for inducing the refractive index modulation[13],[14].

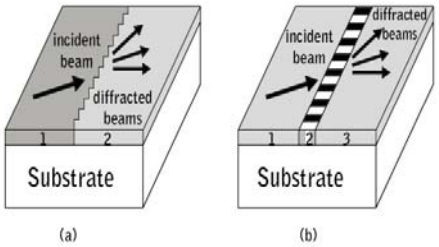


Fig.2 Integrated optical gratings(a)Equivalent of ruled grating (b) Phase diffraction grating

Figure3 is the top view of a transmission phase grating, where the spatial periodicity is Λ , the interaction length of the grating is d, n_l and n_h are the refractive indices in the slots and in the interslots areas respectively. When guided light beam of wavelength λ is incident with angle θ_i on the grating,the periodic change in the refractive index produces a phase transmission-type grating with square profile.

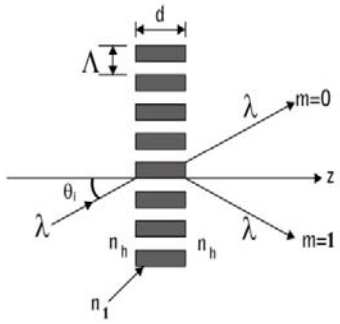


Fig.3 Top view of a phase diffraction grating illuminated by beam with wavelength λ

The diffraction efficiency "η" of the first diffracted order is given by [15],[16]];

$$\eta = \frac{\sin^2\sqrt{\zeta^2+v^2}}{1+(\zeta^2/v^2)}\ , \qquad (2)$$

where

$$v=K_c d, \qquad \zeta=Ed/2\cos\theta_i$$

E is a dephasing factor that is introduced if the angle of incident θ_i is not equal to the Bragg angle θ_B:

$$E= 2\,\pi\,(\sin\theta_i - \sin\theta_B)\ /\ \Lambda\ . \qquad (3)$$

Finally the parameter K_c is called the coupling coefficient and is given as:

$$K_c= 2\,\Delta n / \lambda \cos\theta_i\ . \qquad (4)$$

where Δn is the amplitude of the refractive index modulation which is assumed to be sinusoidal. In our case the modulation is nearer to a rectangular one, varying from n_h to n_l over a period Λ; so, for a given wavelength λ, Δn will be approximated by the fundamental sinusoidal component of a rectangular refractive index modulation:

$$\Delta n =2[\ n_h(\lambda) - n_l(\lambda)\ ] / \pi\ . \qquad (5)$$

It is clear that when $\theta_i = \theta_B = \sin^{-1}\left[\frac{\lambda}{2n_h(\lambda)\Lambda}\right]$, the phase mismatch equals zero, and hence the efficiency for the first order of diffraction at wavelength λ is

$$\eta\ (\lambda)\ = \sin^2\left[\frac{2\Delta n\, d}{\lambda\cos\theta_B}\right]. \qquad (6)$$

From the previous grating equations, we note that a random phase and amplitude errors in transmission interaction length (d), and index modulation (Δn), will degrade the grating performance. They reduce the strength of the peak transmission, broaden the spectrum, and produce fluctuations in the phase response as shown in figure 4.

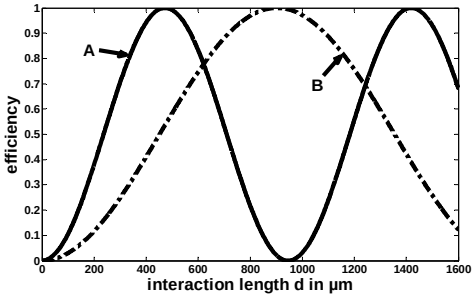


Fig.4 Illustration of a grating subjected to phase error, (A) correct grating (Δn =1.01e-03) and (B) grating with phase error (Δn=0.55e-03).

However, for image encryption purposes, these disadvantagees can be turned into desired characteristics.In this paper we will study the deviation effects for the grating parameters on the image encryption and measure its MSE.
The fields inside the grating could be obtained by applying the beam propagation method (BPM) [17]. The beam propagation method is used to simulate the propagation of an optical beam excitation along a waveguide structure. The BPM method decomposes a mode into a superposition of plane waves traveling in different directions. These individual plane waves are propagated through a finite predetermined distance through the

waveguide until the end is arrived. At this point, all the individual plane waves are added to get the spatial mode back.

There are various kinds of BPMs [9], in this work fast Fourier transformation (FFT-BPM) is adapted. In this method one propagates an input field *u(x,y;z)* over a small distance *Δz* to obtain the field at *z+Δz* in the grating. The FFT-BPM calculation procedure for a distance *Δz* can be summarized as follows [17], where step i-*v* corresponds to the labels shown in Fig.5:

i. At propagation position z, the spectral domain wave function $U_{mn}(z)$ is calculated by taking the Fourier transform of the space-domain wave function *U(x,y,z).*

ii. To get the spectral domain wave function $U_{mn}(z + \Delta z/2)$, the spectral domain wave function $U_{mn}(z)$ is multiplied by $\exp(j\frac{(2\pi)^2}{4\beta}[(\frac{m}{X})^2+(\frac{n}{Y})^2]\Delta z)$ .This multiplication corresponds to the propagation over the distance Δz/2 in free space, where $\beta = k_0 n_o$, n_o is the refractive index, $k_0 = 2\pi/\lambda$, λ is the wavelength of the used beam. X and Y are the widths in the x and y directions.

iii. Taking the inverse Fourier transform of the spectral domain wave function $U_{mn}(z+\Delta z/2)$ obtained in step *ii*, one obtains the space-domain wave function *u(x,y;z+Δz/2)* just in front of the phase-shift lens. Then, multiplying the phase-shift term *exp[-χ]* due to the phase-shift lens by the space-domain wave function *u(x,y;z+Δz/2),*one obtains the space-domain wave function just after the phase-shift lens given by *exp[-jχ]u(x,y;z+Δz/2),* where:

$$\chi = \frac{k_0^2}{2k_0 n_{eff}}\{(n_{eff}+\Delta n)^2 - n_{eff}^2\}\Delta z \cdot$$

iv. Taking the Fourier transform of the space-domain wave function just after the phase-shift lens and multiplying it by

$$\exp\left[j\frac{(2\pi)^2}{4\beta}[(\frac{m}{X})^2+(\frac{n}{Y})^2]\Delta z\right]$$

corresponding to the propagation over Δz/2 in free space to obtain the spectral domain wave function $U_{mn}(z + \Delta z)$ at $z + \Delta z$.

v. To get the wave function *u(x, y; z + Δz),* the inverse Fourier transform of the spectral domain wave function $U_{mn}(z + \Delta z)$ obtained in step *iv* is calculated.

The space-domain wave function at the target propagation position is calculated by repeating steps *i-v*. The accuracy of the method depends on the smallness of the step size Δz and the grid size Δx, Δy.

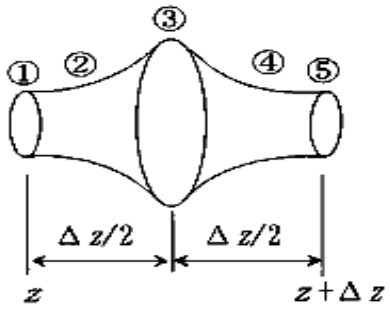


Fig.5 Calculation of one period in the FFT-BPM

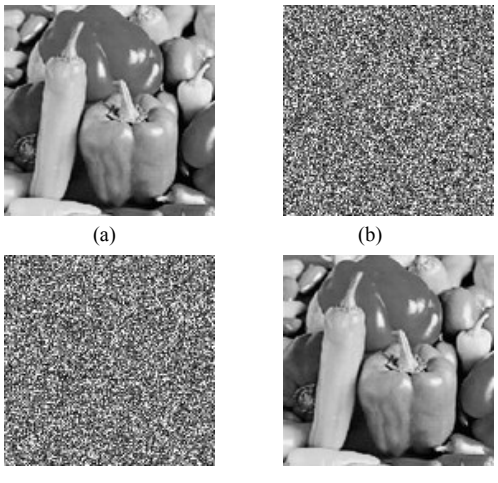
3. Numerical Simulation Results & Discussion

A set of numerical experiments are performed to test the proposed method performance. A grayscale and binary text images, shown in Figures 6.a and 7.a are used in the experiments as input images. Numerical simulations are applied to compare the performance of the proposed method using planar diffraction grating waveguide and the conventional DRPE system. The quality of the decoding image is measured using the Mean Square Error (MSE) between the original (input) image and the decrypted image. Mathematically [5],

$$MSE = \frac{1}{M \times N}\sum_{y=0}^{N-1}\sum_{x=0}^{M-1}\left|f_d(x,y) - f(x,y)\right|^2 \qquad (7)$$

where ($N \times M$) is the size of the image in pixels. $f_d(x,y)$ and $f(x,y)$ are respectively the decrypted image and the primary image.

The first experiment shows the results of using lens based conventional to encrypt and decrypt a grayscale and binary text images using ideal lenses as shown in figures 6,7. The MSE between the input image and the corresponding decrypted image with correct RPM2 for grayscale and binary text images is negligible (1.7828e-31 and 5.0528 e-32 respectively), and with wrong RPM2 the MSE is 0.1234 and 0.0814.



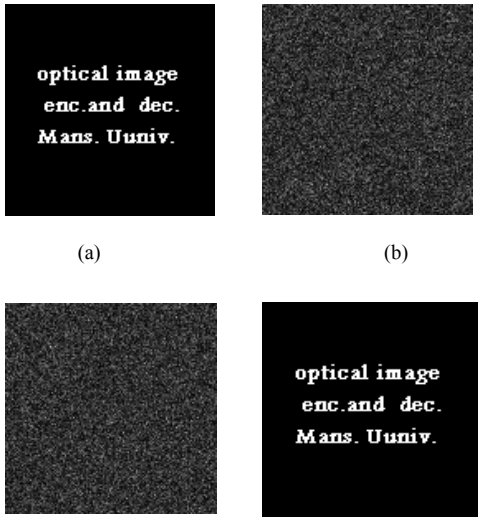
(a) (b)

(c) (d)

Fig. 6. Simulation results of amplitude based grayscale image using lens based DRPE (a) Input image (b) Encrypted image (c) Decrypted image with wrong key (RPM2) (d) Decrypted image with correct key.

In the second experiment planar diffraction grating waveguide are used to perform the DRPE. A diffraction gratings illuminated with plane wave monochromatic light with normal incidence and wavelength λ are used to replace the lenses. The effective refractive index of the guided wave mode (n_{el} and n_{eh}) have been done with a He-Ne laser (λ=0.6328μm) through prism couplers equal to n_{el}= 1.512689, n_{eh} = 1.513739. The periodicity of the grating is Λ=10μm.

The computational window used for the simulation is 50 μm. The accuracy of the results depends on the number of grid points *N* in the transverse *x* and *y* directions, and the size of the propagation steps, Δz, in the direction of propagation. During encryption, first, the input image *I(x,y)* is multiplied by the first random phase mask $R_1(x,y)$ at the input plane, then the FFT-BPM is applied to simulate the propagation in the grating waveguide until the point where the field needs to be determined, then multiplied by the second random phase maskR2(u,v)as shown in figure 8. A value of N=128, and Δz=2 μm are used.



(a) (b) (c) (d)

Fig.7 Simulation results of amplitude based binary text image using lens based DRPE (a) Input image (b) Encrypted image (c) Decrypted image with wrong key (RPM2) (d) Decrypted image with correct key.

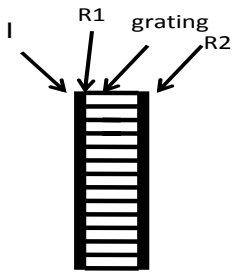
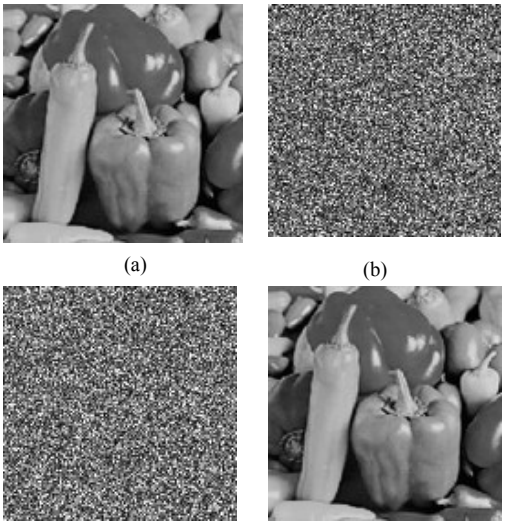


Fig.8 Proposed compact image encoding system using double phase masks and diffraction grating.

Figures 9 and 10 show the decoded image using the proposed system with the correct and wrong key (RPM2) at interaction length *d*=474μm (where maximum efficiency) and index modulation Δn = 1.05e-03 for grayscale and binary text images respectively.

The MSE between the original image and the corresponding decrypted image is calculated with correct RPM2 for grayscale and binary text images and its value is also negligible (2.3688e-028 and 7.8219e-029 respectively), and with wrong RPM2 the MSE is 0.1226 and 0.0809. These results are very near to those of conventional DRPE, which clear that the proposed system using diffraction grating is very suitable for image encryption and decryption.

Simulation has also been performed to test the system robustness against the error in the grating interaction length (*d*) and the index modulation (Δn) for grayscale and binary text images as shown in figures 11,12 respectively. It is noted that small deviation from the correct interaction length d (Fig.11) and also very small deviation from the correct index modulation Δn (Fig.12) produces sufficiently high MSE, so the distance d and the index modulation Δn can be used as a good keys to protect the image due to sensitivity.



(a) (b) (c) (d)

Fig.9 Simulation results of amplitude based grayscale image using grating: (a) Input image (b) Encrypted image (c) Decrypted image with wrong key (RPM2) (d) Decrypted image with correct key.

4. Conclusion

In this paper, we have presented the simulation results of encryption and decryption of 2-D images by using two-dimensional phase transmission grating. The FFT-BPM used to simulate the propagation of an optical beam along the grating. The successful retrieval of the correct image is possible only if both the RPM2 and grating parameters occupied be them during the encryption. When using grating the system become more secure as compared to use a simple RPM at the Fourier transform plane due to larger key size. To evaluate the reliability of the technique, MSE between the decrypted and original images has been calculated, it is found that small deviation from the correct grating interaction length d and index modulation Δn produces sufficiently high MSE.

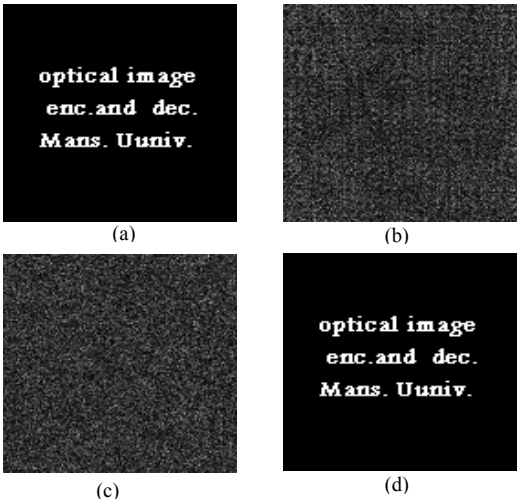


Fig.10 Simulation results of amplitude based binary text image using grating: (a) Input image (b) Encrypted image, (c) Decrypted image with wrong key(RPM2) and (d) Decrypted image with correct key

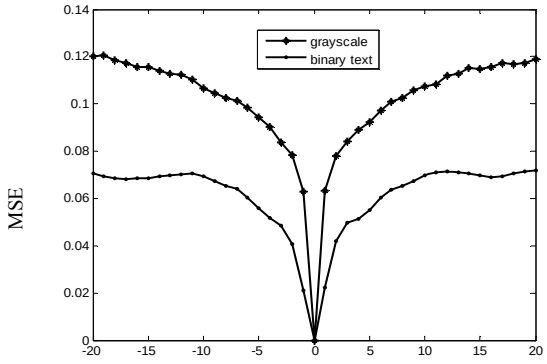


Fig.11 MSE between the decrypted and the input images with error in interaction length *d* for grayscale and binary text images.

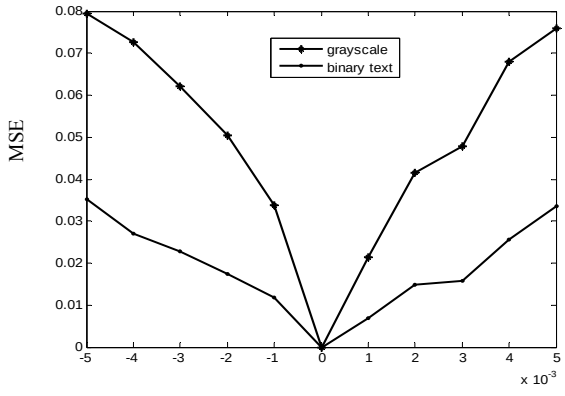


Fig.12 MSE between the decrypted and the input images with error in index modulation Δn for grayscale and binary text images.

References

[1] P. Refrégiér and B. Javidi, "Optical image encryption based on input plane and Fourier plane random encoding", *Opt. Let*. **20**, 767–769 (1995).

[2] B. Hennelly and J. T. Sheridan, "Image encryption and the fractional Fourier transform", *Optik* (Stuttgart) **114**, 251–265 (2003).

[3] D. S. Monaghan, U. Gopinathan, T. J. Naughton,2 and J. T. Sheridan1, " Key-space analysis of double random phase encryption technique" , *App. Opt*. **46**, (2007).

[4] S. Kishk and B. Javidi , "Information hiding technique with double phase encoding", *Applied Optics* **41**, 5462-5470 (2002)

[5] R. Tao, Y. Xin, and Y. Wang, "Double image encryption based on random phase encoding in the fractional Fourier domain" , *Opt. Express* **15**, 16067-16077 (2007).

[6] Y. Frauel, A. Castro, T.J. Naughton, and B. Javidi, " Resistance of the double random phase encryption against various attacks" , *Opt.Express* **15**, 10253 (2007).

[7] B.M. Hennelly and J.T. Sheridan, "Image encryption and the fractional Fourier fransform, " , *Optik* **114**, 251-265 (2003).

[8] M. Joshi, Chandrashakher, and K. Singh, "Color image encryption and decryption for twin images in fractional Fourier domain", *Opt. Commun*. **281**, 5713–5720 (2008).

[9] Jose M.Castro,Ivan B.Djordjevic,and David F.Geraghty, "Novel Super Structure Bragg Gratings for Optical Encryption," *„J.Lightwave Tech*. **24**, 1875-1885(2006).

[10] Madan Singh, Arvind Kumar, and Kehar Singh, "Encryption and decryption using a phase mask set consisting of a random phase mask and sinusoidal phase grating in the fourier plane ", *ICOP 2009- international conference on optics and photonics.CSIO*, 30 Oct.-1 Nov. Chandigarh,India, (2009).

[11] C. Palmer , *Diffraction Grating Handbook*, 4th edition, (2000).

[12] M. G. Moharam and T. K. Gaylord, "Analysis and Applications of Optical Diffraction by Gratings", *J.Opt.Soc.Am.***72**, 1383-1392.

[13] Delavaux JMP , and Chang WSC, "Design and fabrication of on efficient diffraction transmission gratings On Step-index waveguide", *Appl.opt*. **23**, 3004-3009 (1984).

[14] Chartier G.H.,Laybourn PJR and Girod A, " Masking process for double ion-exchanged glass optical Waveguides", *Electronic lett*. **22**, 925-926 (1986).

[15] H.Kogelnik, " Coupled wave theory for thick hologram grating, " **,** *Bell syst. Tech.J*. **48**, 2909-2915 (1969).

[16] M..Moharm, T.K.Gaylord, and R.Magnusson, " Criteria for Bragg regime diffraction by phase grating", *J.Opt.Comm*. **23**, p.14, (1980).

[17] Katsunari Okamoto, *Fundamentals of Optical Waveguides*, Chapter 7, 273- 281,(2000).