A Mathematical Model for Verification of Iris Uniqueness

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Summary
Iris recognition is a method of biometric authentication that uses pattern-recognition techniques based on high-resolution images of the irises of an individual's eyes. Uniqueness is one of the most important salient features of iris systems. Most of iris systems assume that the pupil and outer iris contours to be pure circle. In this paper, we have found that the inner and outer contours of the human iris are not pure circles. In addition, we have proved that the mathematical formula describing the left side of an eye differs from that describing the right part of the same eye; of the same person.
Key words:
Iris recognition; pupil detection;

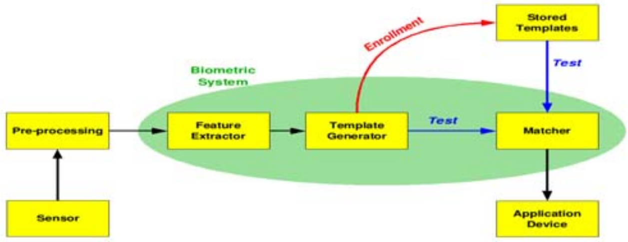
1. Introduction

With an increasing emphasis on security, automated personal identification and verification based on biometrics has been receiving extensive attention over the past two decades. Searching for a definition of biometrics; Fig.1, in both specialized and general information sources, leads to several variants, among which are: (i) Biometrics is a term that encompasses the application of modern statistical methods to the measurements of biological objects; (ii) However, by language misuse, the term biometrics usually refers to automatic technologies for measuring and analyzing biological and anthropological characteristics such as fingerprints; eye retinas and irises; voice patterns; facial patterns, and hand measurements, especially for identity prove, and (iii) Biometrics refer to identify an individual based on his/her distinguishing characteristics [1,2].

A good biometric is characterized by using a feature vector that is: (i) highly unique: the chance of any two people having the same characteristic will be minimal; (ii) stable: the feature vector does not change over time, and (iii) be easily captured: in order to provide convenience to the user, and prevent misrepresentation of the feature vector. The use of biometric systems has been increasingly encouraged by both government and private entities in order to replace or improve traditional security systems [3].

The iris is commonly recognized as one of the most reliable biometric measures. It has a random morphogenesis and no genetic penetrance. In 1987, Flom and Safir studied the problem and concluded that iris morphology remains stable throughout human life and also estimated the probability of the existence of two similar irises on distinct persons at 1 in 10^{72} [4-7].

This paper is intended to prove the uniqueness of iris information. In addition to, derive a mathematical model for the inner iris (pupil) and the outer iris contours. This paper is organized as follows: (2) gives an overview of iris system; (3) introduces the iris database; (4) provides the required preprocessing algorithms; (5) presents the detections of pupil and outer iris contours; (6) discusses the mathematical description of pupil and iris; (7) gives the experiments results, and (8) produces the conclusion.



2. Iris System

1.1 2.1 Iris System Challenges

One of the major challenges of automated iris recognition systems is to capture a high quality image of iris while remaining noninvasive to the human operator. Moreover, capturing the rich details of iris patterns, an imaging system should resolve a minimum of 70 pixels in iris radius. In the field trials to date, a resolved iris radius of 80–130 pixels has been more typical. Monochrome CCD cameras (480×640) have been widely used because near infrared (NIR) illumination in the 700–900-nm band was required for imaging to be nonintrusive to humans. Some imaging platforms deployed a wide-angle camera for coarse localization of eyes in faces; to steer the optics of a narrow-angle pan/tilt camera that acquired higher resolution images of eyes [8]. Given that iris is a relatively

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small (1 cm in diameter), dark object and that human operators are very sensitive about their eyes; this matter required careful engineering. Some points should be taken into account: (i) acquiring images of sufficient resolution and sharpness; (ii) good contrast in the interior iris pattern without resorting to a level of illumination that annoys the operator; (iii) the images should be well framed (i.e. centered), and (iv) noises in the acquired images should be eliminated as much as possible.

2.2 Advantages of iris systems

Iris recognition is especially attractive due to high degree of entropy per unit area of iris; as well as, the stability of iris texture patterns with age and health conditions. Moreover, there are several advantages of iris: (i) an internal organ; (ii) mostly flat with muscles; which control the diameter of the pupil, (iii) no need for a person to be identified to touch any equipment that has recently been touched by strangers; (iv) surgical procedures do not change the texture of the iris; (v) immensely reliable, and (vi) it has responsive nature [9- 11].

2.3 Disadvantages of iris systems

However, there are some disadvantages of using iris as a biometric measurement are: (i) small target (1-cm) to acquire from a distance (about 1-m) therefore it is hard to detect from a distance; (ii) illumination should not be visible or bright; (iii) the detection of iris is difficult when the target is moving; (iv) the cornea layer is curved; (v) eyelashes, corrective lens and reflections may blur iris pattern, it also Partially occluded by eyelids, often drooping; (vi) iris will deform non-elastically when the pupil changes its size, and (vii) iris scanning devices are very expensive.

3. IRIS DATABASE

Despite the fact that many of the iris recognition approaches obtain almost optimal results, they do it under particularly favorable conditions, having few image noise factors. These conditions are not easy to obtain and require a high degree of collaboration from the subject, The aim of University of Beira iris (UBIRIS) database is related with this point: it provides images with different types of artifacts, simulating image captured without or with minimal collaboration from the subjects, pretending to become an effective resource for the evaluation and development of robust iris identification methodologies. UBIRIS database has two distinct versions: The first version is composed of 1877 images collected from 241 persons during September, 2004 in two distinct sessions. It constitutes the world´s largest public and free available iris database at present date [4]. Its main characteristic result from the fact that, in opposition to the existing public and free databases CASIA (the Chinese Academy of Sciences Institute of Automation) and UPOL (University of Palack´eho and Olomouc); it incorporates images with several noise factors, thus permitting the evaluation of robustness iris recognition methods.

Based on the strong acceptance of the first version of the UBIRIS database and in the observed lack of more realistic noise factors, the second version of the UBIRIS database; UBIRIS.v2, was built. The second version of the UBIRIS database has over 11000 images (and continuously growing) The goal was to more realistically simulate less constrained image capturing conditions, either at-a-distance, on-the-move and with minimal subjects cooperation. When compared to its predecessor, this database contains more images and with new noise factors [12]. Fig.2 shows some image of used data base for three different persons.

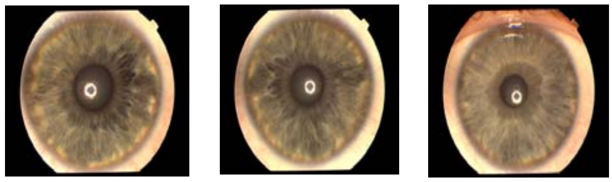


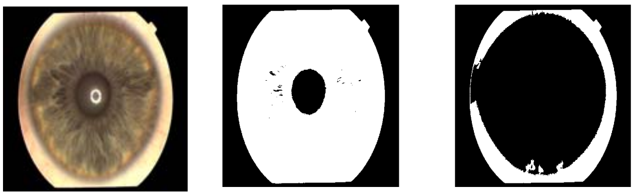
Fig.2 an example of UBIRIS

4. Preprocessing

The first step in iris detection is to enhance the quality of iris information; which is called iris preprocessing. Preprocessing may include: (i) noise removal; (ii) intensity adjustment; (iii) histogram equalization; (iv) converting color images into grayscale images, and (v) centering: forcing the iris data to a zero mean. These processes make the iris database ready to any further processing steps; including pupil and iris detection.

5. Pupil and Iris Detection

Since the iris data was acquired using an infrared camera the pupil is a very distinct black region. The pupil is in fact so black relative to everything else in the image, so simple edge detection should be able to find its outside edge very easily. Furthermore, the thresholding on the edge detection can be set very high as to ignore smaller less contrasting edges while still being able to retrieve the entire perimeter of the pupil. Canny edge detector was used for outlining the pupil contour. According to the thresholding value, the image pixels will take values 0 or 1; i.e. binary image. Then; the objects that contain one of the specified pixels can easily be selected. In this work we repeat the same approach used in pupil detection to detect the iris but with different threshold values. Fig.3 shows the original image; the detected pupil, and iris after using detection.



(a) original image (b) the pupil (c) the iris

Fig.3 The original image with detected pupil and iris

6. Mathematical Description of Pupil and Iris Contour

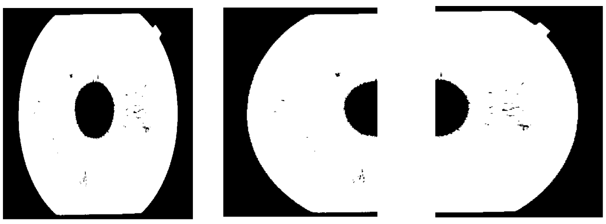
Most of iris processing algorithms assume that the inner iris (pupil) and the outer iris contours are complete circles of slightly different centers. Actually, the complete circle assumption is fairly good for simple systems and moderate accuracy. In order to, get highly accurate systems, one should estimate an accurate mathematical model of the inner and outer iris contours. These mathematical models should also help in verifying the uniqueness of the human iris.

6.1 nth Order Polynomial

Once the outer iris counters (points) were detected, one attempts to estimate or interpolate a model for the outer iris. First of all, the outer iris image is divided into two vertical parts; left and right; as shown in Fig.4. After this division process, each contour has to be expressed in terms of an n[th] order polynomial. The suggested polynomial model can be represented in the form:

$$y = P_0 x^n + P_1 x^{n-1} + P_2 x^{n-2} + \ldots + P_{n-1} x + P_n$$

(a) The detected iris (b) the left half (c) the right half



(d) The detected iris (e) the left half (f) the right half

Fig.4 the detected iris; pupil and their two halves

Actually, this polynomial has been applied four times: (i) left part of the inner iris; (ii) right part of the inner iris; (iii) left part of the outer iris, and (iv) right part of the outer iris. Hence, if the polynomial order and/or coefficients are different for each of these four parts; one can prove the iris uniqueness. We not only stop at this, but also we calculate the correlations between the coefficients in each side of iris, and the results obtained also stresses that there is a nonsymmetrical not only in the two eyes, but also in the two vertical parts of one eye. The results will be discussed in the following section. There are still a number of active research topics within iris biometrics. Many of these are related to represent the iris by a circle. But in this article we prove that there is no symmetrical between the two vertical halves of the iris, so we can say that the iris is not circle.

Besides of expressing each vertical part in the form of an n[th] order polynomial, one attempts to estimate or interpolate another models for the outer iris. First of all, the outer iris image in this case is divided into four quarters. After this division process, each contour has to be expressed in terms of an n[th] order polynomial, Gaussian form, Sum of sine functions, Exponential function, and the form of Fourier representation . The suggested models can be represented in the following different forms.

6.2 Gaussian Model

$$y = a_1 \exp^{(-((x-b_1)/a_1))^\wedge 2)} + a_2 \exp^{(-((x-b_2)/a_2))^\wedge 2)} + a_3 \exp^{(-((x-b_3)/a_3))^\wedge 2)} + a_4 \exp^{(-((x-b_4)/a_4))^\wedge 2)}$$

6.3 Sum of Sin Functions

$$y = a_1 \sin(b_1 x + c_1) + a_2 \sin(b_2 x + c_2) + \ldots + a_n \sin(b_n x + c_n)$$

6.4 Exponential Model

$$y = a\,\mathrm{e}^{(b*x)} + c\,\mathrm{e}^{(d*x)}$$

6.5 Fourier Model

$$y = a_0 + a_1 \cos(\omega x) + b_1 \sin(\omega x) + \ldots + a_4 \cos(4\omega x) + b_4 \sin(4\omega x)$$

7. Experimental Results

The experiment was applied on two iris images of two different persons. Each person has two iris images; left and right. **Tables 1 and 2** provide the interpolation results of the inner iris polynomial with the correlation coefficients. **Tables 3 and 4** provide the interpolation results of the outer iris polynomials of 3[rd], 4[th], and 5[th] order respectively with the correlation coefficients of the same iris data. **Tables 5 and 6** provide the interpolation results for the 1st and the 2nd quarter's outer iris of one eye. RMSE represent root mean square error between the exact and approximated data.

Different forms of correlation coefficient have been computed: (i) $\mathbf{R_1}$**;** represents the correlation between the coefficients in the right and the left halves of the same eye; (ii) $\mathbf{R_2}$ represents the correlation between the coefficients in the two left halves of the two eyes; (iii) $\mathbf{R_3}$ represents the correlation between the coefficients in the two right halves of the two eyes, and (v) $\mathbf{R_4}$ represents the correlation between the coefficients in the two quarters of the same iris. Finally, root mean square error (RMSE) between the exact and calculated values was calculated.

8. Conclusion

Due to the advantages of iris technology, it has gained a very high degree of importance in biometrics. In this paper, three main items have been discussed and proved: (i) the uniqueness of the human iris; (ii) the asymmetry of any eye if divided vertically, and (iii) the inner and outer contours of the human iris are not pure circles, it has been proved that, they are a general order polynomial functions and different mathematical forms. The practical test results showed that; the mathematical description of the uniqueness of iris information. There are still a number of active research topics within iris biometrics; many of these are related to the desire to make iris in the form of pure circle, but here we suggest polynomials of degree five at most to represent each vertical half of the iris. The difference between the coefficients of the polynomials proves the uniqueness not only in two different irises, but also in each half of on.

9. References

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Table 1: Coefficients for left pupil

			Image1			***Image 2***		
			3rd	***4th***	***5th***	***3rd***	***4th***	***5th***
Left pupil	**Left Half**	P_0	-3.35E-02	5.41E-04	-9.04E-06	8.37E-03	-2.10E-04	4.52E-06
		P_1	-7.55E+00	1.23E-01	-2.08E-03	2.15E+00	-5.03E-02	1.07E-03
		P_2	-5.56E+02	9.19E+00	-1.56E-01	1.76E+02	-3.94E+00	8.22E-02
		P_3	-1.33E+04	2.22E+02	-3.81E+00	4.50E+03	-9.81E+01	2.02E+00
		P_4		0.00E+00	0.00E+00		0.00E+00	0.00E+00
		P_5			0.00E+00			0.00E+00
	RMSE		6.68E-12	8.68E-12	4.63E-11	2.08E-12	9.11E-12	7.07E-12
	Right Half	P_0	2.05E-02	5.27E-04	1.45E-05	1.59E-02	5.23E-04	2.17E-05
		P_1	-3.19E+00	-8.38E-02	-2.35E-03	-1.92E+00	-6.73E-02	-2.93E-03
		P_2	1.57E+02	4.24E+00	1.21E-01	7.08E+01	2.67E+00	1.23E-01
		P_3	-2.42E+03	-6.68E+01	-1.94E+00	-7.47E+02	-3.10E+01	-1.55E+00
		P_4		0.00E+00	0.00E+00		0.00E+00	0.00E+00
		P_5			0.00E+00			0.00E+00
	RMSE		1.60E-12	2.05E-12	5.83E-12	3.11E-13	2.68E-12	2.42E-12
	$\mathbf{R_1}$		0.90	-0.90	0.9	-0.9	0.9	-0.9

Table 2: Coefficients for right pupil

			Image1			***Image 2***		
			3rd	***4th***	***5th***	***3rd***	***4th***	***5th***
Right pupil	**Left Half**	P_0	-1.81E-02	-4.00E-04	-1.25E-05	-1.60E-02	3.00E-04	-5.91E-06
		P_1	2.32E+00	5.42E-02	1.85E-03	-3.02E+00	5.74E-02	-1.15E-03
		P_2	-8.80E+01	-2.25E+00	-8.80E-02	-1.82E+02	3.51E+00	-7.14E-02
		P_3	8.73E+02	2.72E+01	1.34E+00	-3.44E+03	6.78E+01	-1.41E+00
		P_4		0.00E+00	0.00E+00		0.00E+00	0.00E+00
		P_5			0.00E+00			0.00E+00
	RMSE		7.62E-13	2.10E-12	5.70E-12	1.52E-12	4.36E-12	2.78E-12
	Right Half	P_0	2.86E-03	-4.64E-05	7.33E-07	-2.37E-03	-9.27E-05	-2.72E-06
		P_1	3.68E-01	-5.81E-03	9.00E-05	6.87E-01	2.32E-02	6.57E-04
		P_2	1.01E+01	-1.52E-01	2.28E-03	-5.91E+01	-1.81E+00	-5.00E-02
		P_3	-5.03E+01	0.00E+00	0.00E+00	1.44E+03	4.23E+01	1.16E+00
		P_4		-5.99E+01	0.00E+00		0.00E+00	0.00E+00
		P_5			-6.00E+01			0.00E+00
	RMSE		3.04E-13	2.86E-13	3.68E-13	1.52E-12	4.60E-12	1.00E-11
	$\mathbf{R_1}$		-1.00	0.23	0.18	-1.00	1.00	-1.00
	$\mathbf{R_2}$		-0.99	0.99	-1.00	1.00	-1.00	1.00
	$\mathbf{R_3}$		0.99	-0.23	0.19	-1.00	-1.00	-1.00

Table 3: Coefficients for left iris

			Image1			***Image 2***		
			3rd	***4th***	***5th***	***3rd***	***4th***	***5th***
Left iris	**Left Half**	P_0	-3.08E-04	2.98E-06	-3.57E-08	5.50E-04	-6.43E-06	5.23E-08
		P_1	-1.95E-01	2.00E-03	-2.48E-05	4.94E-01	-4.83E-03	3.73E-05
		P_2	-3.69E+01	4.07E-01	-5.23E-03	1.33E+02	-1.16E+00	8.61E-03
		P_3	-1.87E+03	2.25E+01	-3.06E-01	1.08E+04	-8.79E+01	6.35E-01
		P_4		0.00E+00	0.00E+00		0.00E+00	0.00E+00
		P_5			0.00E+00			0.00E+00
	RMSE		2.29E-12	6.96E-12	1.15E-11	8.86E-12	3.25E-11	7.46E-12
	Right Half	P_0	-2.30E-03	-1.20E-05	-6.46E-08	-3.38E-03	-1.62E-05	-1.82E-08
		P_1	1.22E+00	6.38E-03	3.48E-05	1.50E+00	7.56E-03	8.30E-06
		P_2	-1.91E+02	-1.01E+00	-5.61E-03	-1.92E+02	-1.06E+00	-9.60E-04
		P_3	7.87E+03	4.24E+01	2.49E-01	7.12E+03	4.39E+01	6.13E-03
		P_4		0.00E+00	0.00E+00		0.00E+00	0.00E+00
		P_5			0.00E+00			0.00E+00
	RMSE		1.71E-11	2.28E-11	6.17E-12	1.44E-11	1.64E-11	3.90E-11
	$\mathbf{R_1}$		-0.9	0.9	-0.9	0.9	-0.9	0.9

Table 4: Coefficients for right iris

			Image1			***Image 2***		
			3rd	***4th***	***5th***	***3rd***	***4th***	***5th***
Right iris	**Left Half**	P_0	-5.20E-04	2.14E-06	-4.03E-09	-2.89E-03	1.46E-05	-7.83E-08
		P_1	-2.85E-01	1.09E-03	-9.04E-07	-1.91E+00	9.82E-03	-5.38E-05
		P_2	-4.57E+01	1.50E-01	2.69E-04	-4.09E+02	2.16E+00	-1.20E-02
		P_3	-1.98E+03	3.73E+00	5.69E-02	-2.85E+04	1.53E+02	-8.68E-01
		P_4		0.00E+00	0.00E+00		0.00E+00	0.00E+00
		P_5			0.00E+00			0.00E+00
	RMSE		5.05E-12	5.49E-12	2.88E-13	2.74E-11	3.17E-11	7.92E-11
	Right Half	P_0	-4.81E-05	-4.98E-07	-9.23E-09	9.77E-04	6.18E-06	-9.97E-08
		P_1	2.04E-02	3.22E-04	6.37E-06	-5.70E-01	-3.75E-03	5.13E-05
		P_2	-3.30E+00	-7.81E-02	-1.50E-03	1.05E+02	7.19E-01	-8.21E-03
		P_3	4.10E+02	7.60E+00	1.24E-01	-5.93E+03	-4.27E+01	4.00E-01
		P_4		0.00E+00	0.00E+00		0.00E+00	0.00E+00
		P_5			0.00E+00			0.00E+00
	RMSE		2.63E-13	8.53E-13	2.84E-12	5.39E-12	8.56E-12	1.77E-11
	$\mathbf{R_1}$		-0.90	0.90	-0.90	0.90	-0.90	0.90
	$\mathbf{R_2}$		1.00	0.90	-0.90	-0.90	-0.90	-0.90
	$\mathbf{R_3}$		0.90	0.90	0.90	-0.90	-0.90	0.90

Table 5: Coefficients for right iris

			Polynomials							
			2nd	***3rd***	***4th***	***5th***	***6th***	***7th***	***8th***	***9th***
Right Iris	Q1	P_0	-3.1E-03	-9.2E-06	-6.8E-08	-3.8E-10	3.3E-12	-8.0E-14	4.7E-16	1.6E-17
		P_1	1.9E-01	6.1E-04	2.7E-05	1.9E-07	-2.9E-09	7.8E-11	-5.8E-13	-2.1E-14
		P_2	2.6E+02	-1.9E-01	-5.5E-03	-3.2E-05	9.58E-07	-3.0E-08	2.95E-10	1.01E-11
		P_3		2.7E+02	1.5E-01	2.0E-04	-1.4E-04	6.00E-06	-7.9E-08	-2.6E-09
		P_4			2.6E+02	-4.9E-02	6.9E-03	-6.2E-04	1.23E-05	3.9E-07
		P_5				2.64E+02	-2.1E-01	2.9E-02	-1.1E-03	-3.4E-05
		P_6					2.6E+02	-5.9E-01	4.5E-02	1.51E-03
		P_7						2.6E+02	-8.1E-01	-2.9E-02
		P_8							2.6E+02	5.5E-02
		P_9								2.6E+02
	RMSE		4.591	2.395	1.355	1.143	1.11	0.8919	0.9064	0.683
	Q2	P0	-2.8E-03	6.70E-06	-2.5E-09	-2.7E-10	-9.2E-12	3.03E-14	2.70E-15	-1.3E-16
		P1	-2.10E-01	5.32E-04	5.03E-06	-2.3E-07	-9.4E-09	2.5E-11	3.6E-12	-1.9E-13
		P2	2.52E+02	2.84E-01	1.4E-04	-6.5E-05	-3.9E-06	7.14E-09	2.01E-09	-1.2E-10
		P3		2.73E+02	2.47E-01	-1.0E-02	-8.0E-04	3.2E-07	6.18E-07	-4.3E-08
		P4			2.72E+02	-4.50E-01	-8.88E-02	-1.93E-04	1.13E-04	-9.5E-06
		P5				2.55E+02	-4.63E+00	-3.84E-02	1.27E-02	-1.4E-03
		P6					1.69E+02	-2.44E+00	8.34E-01	-1.3E-01
		P7						2.07E+02	2.98E+01	-7.1E+00
		P8							7.04E+02	-2.3E+02
		P9								-2.8E+03
	R4		1.0	1.0	1.0	1.0	0.99	1.0	0.99	0.99
	RMSE		1.942	1.013	1.057	1.074	1.015	1.073	1.08	0.7541

Table 6: Coefficients for right iris

		Gaussian		***Sum of sine function***		***Exponential function***		***Fourier function***	
Right Iris	Q1	a_1	67.47	a_1	3.7E+02	a	-1.23E+01	a_0	188.1
		b_1	-17.1	b_1	7.9E-03	b	1.04E-02	a_1	58.37
		c_1	35.21	c_1	1.1E+00	c	2.78E+02	b_1	74.34
		a_2	249.9	a_2	1.2E+02	d	-1.23E-05	a2	32.76
		b_2	44.09	b_2	1.3E-02			b_2	-23.95
		c_2	105.7	c_2	3.7E+00			a_3	-9.09
		a_3	132.9	a_3	3.5E+00			b_3	-15.06
		b_3	170.3	b_3	5.8E-02			a_4	-5.724
		c_3	725.9	c_3	2.4E+00			b_4	1.251
		a_4	67.55	a_4	3.6E+00			w	0.014
		b_4	240.2	b_4	6.6E-02				
		c_4	52.23	c_4	4.2E+00				
	RMSE	0.6966		06823		1.824		0.5936	
	Q2	a_1	2.3E+02	a_1	3.2E+02	a	3.03E+02	a_0	188.2
		b_1	-1.6E+01	b_1	9.4E-03	b	-3.46E-04	a_1	-36.45
		c_1	1.3E+02	c_1	-3.6E+00	c	-3.50E+01	b_1	-61.02
		a_2	1.5E+02	a_2	9.9E+01	d	-6.97E-03	a2	-13.51
		b_2	-1.9E+02	b_2	1.8E-02			b_2	-25.4
		c_2	1.2E+02	c_2	1.0E+00			a_3	-6.729
		a_3	5.1E+00	a_3	1.0E+01			b_3	-9.478
		b_3	-2.5E+02	b_3	3.2E-02			a_4	-2.123
		c_3	1.8E+01	c_3	-5.8E+00			b_4	-3.663
		a_4	1.8E+01	a_4	6.1E-01			w	0.01998
		b_4	-9.2E+01	b_4	7.3E-02				
		c_4	2.3E+02	c_4	3.2E+02				
	R_4	-0.09		-0.4		0.67		0.72	
	RMSE	1.206		1.451		1.028		0.6982	