Fidelity and Robust Digital Watermarking Adaptively Pixel based on Medial Pyramid of Embedding Error Gray Scale Images

Mehemed Bashir Aliwa†, Tarek El-Ahmady El-Tobely†, Mahmood M. Fahmy†, Mohamed EL Said Nasr† and Mohamed Hashem Abd El-Aziz††

†Faculty of Engineering-TANTA University, Egypt.
††Faculty of Computers & Information Sciences-Ain Shams University, Egypt.

Summary
Digital image watermarking is one such technology that has been developed to protect digital images from illegal manipulations and prevent malicious and non-malicious attacks to detect hidden information. In particular, digital watermarking techniques in frequency domain are proliferation of digitized media due to the rapid growth of watermarking system and have been widely recognized to be more prevalent than others, but in recent years the techniques in spatial domain technologies they are becoming generally abandoned. One of the problems in digital watermarking is that the three requirements of imperceptibility, capacity, and robustness that are must be satisfied but they almost conflict with each other, accordingly there are trade-off between fidelity and robustness. *In this paper, w*e proposed a novel fidelity and robust watermark embedding method that satisfies the requirements of imperceptibility, capacity, and robustness, called adaptively pixel adjustment process based on medial pyramid of embedding error applying in the falling-off-boundary in corners board of the cover image set-of-the Most-Significant-Bit-6 blind in spatial domain (APAP-MPOEE-FOBCB$_{MSB6}$). *In addition*, the paper provides a comprehensive overview and analysis of previous methods**.** *Theoretically* analysis of the proposed technique proves the effectiveness of the technique in the average of worst case and minimizing the number of embedding error to the half. *Experimental results* of the proposed technique was applied on the different benchmark of six gray scale images and two quantum of watermark bit embedded are compared with previous works and was found better. Moreover in all different benchmark of test-images the watermarks were extracted from watermark degrading, removal and geometric transformations attacks to an acceptable degree of similarity function and normalized cross correlation.

Key words:
Fidelity, digital watermarking, imperceptible, spatial domain, LSB & MSB, benchmark.

1. Introduction

Digital watermarking is a technique which allows an individual to add hidden copyright notices or other verification messages to digital audio, video, or image signals and documents. Such a message is a group of bits describing information pertaining to the signal or to the author of the signal (name, place, etc.). The technique takes its name from watermarking of paper or money as a security measure[35]. Digital watermarking can be a form of steganography[14][38][40], in which data is hidden in the message without the end user's knowledge. In the term of hiding can refer to either for information imperceptibility (watermarking) or information secrecy (steganography) means that the existence of a message is secret, thus the steganography is the art of concealed communication[14][29][35]. Digital watermarking refers to techniques that are used to protect digital data by imperceptibly embedding watermark into the original data in such a way that always remains present[26][33]. Watermarking and steganography are two important sub disciplines of information hiding that are closely related to each other and may be coincide but with different underlying properties, requirements and designs, thus result in different technical solutions[14][35]. Moreover the digital watermarking differs from Cryptography, where cryptography is the art of sending a message by converting it into a secret code called as cipher text. The conversion is done using an algorithm and a secret key. Once the receiver receives the cipher text, he can decode it and convert it into plaintext using his private key. Here, the very existence of the message is not being kept secret but only the contents are. This rouses suspicion and curiosity[31]. On the other hand, the digital watermarking, unlike cryptography, leaves the original medium or data almost unaltered even after embedding it with the copyright information. The naked eye cannot tell the difference in the alteration. The main purpose of using watermarks is to convey ownership, protect copyrighted materials from being illegally distributed, and to prevent various other kinds of fraud. In certain instances it is also used in security applications like the ID cards or covert communication (Defense and Intelligence applications). On the other hand in digital watermarking has the additional concept of resilience against attempts to remove the hidden data. This is because the information hidden by watermarking systems is always associated to the digital object to be protected its owner, while steganographic systems just hide any information. Robustness criteria are also different since steganography mainly concerns with detection of hidden message while watermarking concerns potential removal by a pirate. Besides, steganography typically relates to covert point-to-point communication while watermarking is usually one-to-many[14][34].

1.1 History

Although paper was invented in China over a thousand years ago, the Europeans only began to manufacture it in the 11th and 12th centuries, after Muslims had established the first paper mills in Spain. Soon after its invention, Chinese merchants and missionaries transmitted paper, and knowledge of papermaking, to neighboring lands such as Japan, Korea, and Central Asia. It was there that Muslims first encountered it in the 8th century. Islamic civilization spread knowledge of paper and papermaking to Iraq, Syria, Egypt, North Africa and finally, Spain. Most accounts of the history of paper focus either on its origins in China or its development in Europe. This explains why the oldest watermarked paper found in archives dates back to 1292, in Fabriano, Italy [34]. The marks were made by adding thin wire patterns to the paper molds. The paper would be slightly thinner where the wire was and hence more transparent. At the end of 13th century about 40 paper mills were sharing the paper market in Fabriano and producing paper with different format, quality and price[14]. The digitization of today's world has expanded the watermarking concept to include digital approaches for use in authenticating ownership claims and protecting proprietary interests. Digital Watermarking became famous only in the early of 1990 the idea of digital watermarking, embedding imperceptible information using digital images[39]. This was due to the growth of the Internet. The Internet was a big factor in propelling the growth because illegal distribution of copyrighted material became very easy. File sharing technology grew and companies made it easy for users to share for example music and other copyrighted materials like video. This cost the entertainment industry in the millions if not in billions of dollars of lost revenue. This was one of the primary reasons for the rapid development of digital watermarking[31]. The first publication in 1993, when Tirkel et al 1993[28] presented technique to hide data in image. The method based on modification to the least significant bit (LSB) of the pixel values[38]. Since then worldwide research activities have been increasing dramatically and the industrial interest in digital watermarking methods keeps growing.

1.2 General Framework for Watermarking

In general, any watermarking scheme consists of three parts. The watermark, encoder (insertion algorithm), and decoder with comparator (verification or extraction or detection algorithm). All watermarking methods share the same generic building blocks a watermark embedding system also called (Encoder process or insertion algorithm) and a watermark recovery system (also called watermark extraction or watermark decoder) [11] [14] [19][34][38] shown in Fig.1.

❖ **Encoding process:** The input to the scheme is the watermark, the cover-original image F and an optional public or secret key. The watermark can be of any nature such as a number, text, or an image. The key used to enforce security that is the prevention of unauthorized parties from recovering and manipulating the watermark. All practical systems employ at least one key, or even a combination of several keys. In combination with a secret or a public key the watermarking techniques are usually referred to as secret and public watermarking techniques, respectively. The output of the watermarking scheme is the watermarked image f(x,y). Mathematically, $E[F(x,y),k\times W(x,y)]=f(x,y)$, Where, F(x,y) denotes the actual cover image. W(x,y) denotes the watermark image. K denotes the public or secret key. f(x,y) denotes the watermarked image. From the above equation, if the watermark insertion process is designed correctly, the result is media that appears identical to the original when perceived by a human, but which yields the encoded watermark information when processed by a watermark detector.

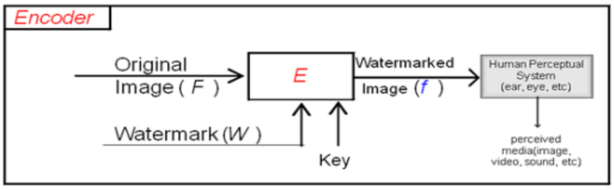

Fig.1: Encoder process of watermarking.

❖ **Decoding process:** The decoding process is depicted in Figure. Inputs to the scheme are the watermarked data, the secret or public key and, depending on the method, the original data and/or the original watermark. The output is either the recovered watermark W. Three types of watermarking systems can be identified. Their difference is in the nature and combination of inputs and outputs. A decoder function D takes an image (can be a watermarked or un-watermarked image, and possibly corrupted) whose ownership is to be determined and recovers a watermark W' from the image. In this process an additional image

F(x,y) can also be included which is often the original and un-watermarked version of f(x,y), this information can be referred to as a 'key" that is the level of availability of the key in turn determines who is able to read the watermark. Mathematically, D[f(x,y) , F(x,y)]=W'(x,y). A watermark must be detectable or extractable W(x,y) to be useful. Depending on the way the watermark is inserted and depending on the nature of the watermarking algorithm.

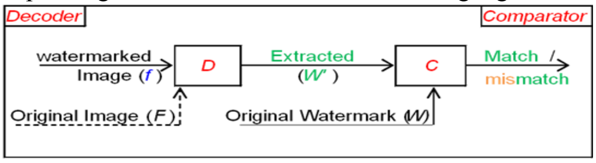

Fig.2: Decoder process of watermarking.

❖ **Comparison process:** The comparison process is depicted in Fig.2, the extracted payload W'(x,y) is compared with the original payload W(x,y) (i.e. the payload that was initially embedded) by a comparator function and a binary output decision is generated. The comparator is basically a correlator depending on the comparator output it can be determined if the data is authentic or not, for e.g. Using a normalized cross correlation (NCC) or similarity function (SM), whereas the similarity values NCC and SM of about 0.75 or above is considered acceptable[5][16][19].

1.3 Types of digital watermarking

Watermarks and watermarking techniques can be divided into various categories in various ways. Watermarking techniques can be divided into five categories according to the type of document to be watermarked as follows: image, video, text and audio watermarking[38]. In other way, the digital watermarks can be divided into four different types according to human perception as follows, visible watermark, invisible robust watermark, invisible fragile watermark and Dual watermark. ***Visible watermarking***: The idea behind the visible watermark is very simple; a visible watermark makes slight modifications to an image. The transformation is such that the image can still be seen, but the watermark is effectively laid over the top of it. One of the advantages of visible watermarks is that even if an image is printed and scanned the watermark is still visible[1]. It is equivalent to stamping a watermark on paper, and for this reason is sometimes said to be digitally stamped. An example of visible watermarking is provided by television channels, like BBC, whose logo is visibly superimposed on the corner of the TV picture[38]. ***Invisible watermarking***: a pattern is applied to a file or image so that it is undetectable by the human eye. With an invisible watermark you can change certain pixels in an image so his human eye cannot tell the difference from the original image the strength of invisible watermarks is that the image quality is not degraded or changed according to the user or consumer. Invisible watermarks are effective, though, only while the image is in digital form. If a digital image that has an invisible watermark is printed out, and then rescanned, the watermark is effectively removed[1]. On the other hand, it is a far more complex concept? It is most often used to identify copyright data, like author, distributor, and so forth[38]. ***Invisible fragile watermarking:*** fragile are embedded with very low robustness[39]. Invisible fragile watermarks are ready to be destroyed by random image processing methods. The change in watermark is easy to be detected[14]. The fragile watermarks are used to detect any corruption of an image. In some application, we want exactly the opposite of robust[19]. The main application of fragile watermarking is data authentication, where watermark loss or alteration is taken as evidence that data has been tampered with[33]. ***Semi-fragile Watermarking:*** the idea is to insert a watermark in the original image in such a way that the protected image can undergo some specific image processing operations while it is still possible to detect malevolent alterations and to locate and restore image regions that have been altered[12]. Furthermore watermarks can help localize the exact location where the tampering of the cover work occurred[38]. Watermark is semi-fragile if it survives a limited well specified, set of manipulations, leaving the quality of the host document virtually intact. ***Dual watermarking:*** is a combination of a visible and an invisible watermark. In this type of watermark an invisible watermark is used as a backup for the visible watermark as clear from the following diagram.

Fig.3: Schematic representation of dual watermarking.[24]

From application point of view digital watermark could be source based or destination based[14]. Source-based watermark are desirable for ownership identification or authentication where a unique watermark identifying the owner is introduced to all the copies of a particular image being distributed. A source-based watermark could be used for authentication and to determine whether a received image or other electronic data has been tampered with. The watermark could also be destination-based where each distributed copy gets a unique watermark identifying the particular buyer. The destination-based watermark could be used to trace the buyer in the case of illegal reselling.

1.4 Characteristics of a watermarking

There are a number of important characteristics that watermarks exhibit. Imperceptibility: means that the perceived quality of the host image should not be distorted by the presence of the watermark[5]. Imperceptibility due to the particular nature of the authentication task, it is usually necessary that watermark imperceptibility is guaranteed. Nevertheless, some applications may exist in which a slightly perceptible watermark is allowed[33]. Furthermore the imperceptibility for hidden information[17]. Moreover the modifications caused by watermark embedding should be below the perceptible threshold, which means that some sort of perceptibility criterion should be used not only to design the watermark, but also quantify the distortion. As a consequence of the required imperceptibility used for watermark embedding are only modified by a small amount[34]. Fidelity: This refers to the term imperceptible as it is referred in the literature of watermarks[9][14][13]. The watermark should not be noticeable to the viewer nor should the watermark degrade the quality of the content[19][35]. The fidelity of a watermarked signal depends on the amount of embedded information, the strength of the mark, and the characteristics of the host signal[38]. Perceptual Transparency: Refers to the property of the watermark of being imperceptible in the sense that humans can not able distinguish the watermarked images from the original ones by simple inspection[1][6], where the embed data without affecting the perceptual quality of the host signal[4]. Undetectability: The aim of the undetectability as well as the removal attacks is to render the embedded watermark undetectable[39]. Additionally, we say that a watermark is wide-sense reversible if once it has been decoded/detected it can be made undecodable/undetectable without producing any perceptible distortion of the host asset[33]. It should not be possible for an attacker to find any significant statistical differences between an unmarked signal and a marked signal[4]. Data payload: refers to the amount of information stored in the watermark, which in general depends on the application[6][13]. For a photograph, the data payload would refer to the number of bits encoded within the image. For audio, data payload refers to the number of embedded bits per second that are transmitted. For video, the data payload may refer to either the number of bits per field (or frame) or the number of bits per second[35]. Capacity: knowing how much information can reliably be hidden in the signal is very important to users especially when the scheme gives them the ability to change this amount. Moreover refers to the bit size of a payload that a watermark access unit can carry[4]. How many marks can be added simultaneously[21]?. Security: The security of a watermark refers to its ability to resist hostile attacks[35]. The embedded watermark cannot be removed beyond reliable detection by targeted attacks based on a full knowledge of the embedding algorithm and the detector (except a secret key). Computational cost: The time that it takes for a watermark to be embedded and detected can be a crucial factor in a watermarking system[38]. On the other hand speed be sides fidelity, where the content owner might be interested in the time it takes for an algorithm to embed a mark. Although speed is dependent on the type of implementation (hardware or software), some applications require real time embedding and/or detection[4][38]. Moreover the efficiency of computing time in storage requirements, and software or hardware size of the mark writing and reading processes? Are they real-time, so that they can be incorporated into playback or display mechanisms in an on-line setting[21]. Data secrecy (secret keys): What information needs to be retained, or kept secret, about the marks, their meaning, and the marked material? Depending upon the watermarking method, such information can include encryption and decryption keys for computing and interpreting marks[21]. A watermark should usually be secret and only accessible by authorized parties. Knowledge of a watermark inserter or detector can make a method more vulnerable to attack. For more protection to the watermark bits a secret-Key has been used to permute the watermark bits before embedding it to achieve cryptographic security[3]. In general, watermarking systems should use one or more cryptographically secure keys (called watermark keys) to ensure that the watermark cannot be manipulated[38]. Robustness: The ability of the watermark to survive normal processing of content[35]. Moreover refers to the capacity of the watermark to remain detectable after alterations due to processing techniques or intentional attacks[6]. The watermark should be resistant to distortion introduced during either normal use (unintentional attack), or a deliberate attempt to disable or remove the watermark present (intentional, or malicious attack). Unintentional attacks involve transforms that are commonly applied to images during normal use, such as cropping, noise, scaling and compression…etc[17]. Accuracy of detection: How accurately can the mark be read? What is the chance of a false positive (unmarked content appearing to have a mark)[4], a false negative (marked data appearing to be unmarked), or a false reading (a mark misread as another mark)[21]. Redundancy: To ensure robustness, the watermark information is embedded in multiple places on the cover data file. This means that the watermark can usually be recovered from just a small portion of the watermarked file[38]. The watermark information is usually redundantly distributed over many samples (or pixels, features, etc.) of the cover data[34]. Furthermore redundancy in distribution of the hidden information inside the cover image to satisfy robustness in watermark extraction process even from the truncated (cropped) watermarked image[17]. The redundancy of the data helps to hide the existence of a secret message[1].

1.5 Watermarking system

Private watermarking systems (or called non-blind watermarking)[15][34] require at least the original data in the reading process[39]. It guarantees better robustness but may lead to multiple claims of ownerships[32]. **Public watermarking systems** (or called blind or oblivious watermarking)[15][34] means watermark detection and extraction do not depend on the availability of original image. It is the biggest challenge to the development of a watermarking system [27]. The drawback is when the watermarked image is seriously destroyed; watermark detection will become very difficult[32]. **Semi-blind watermarking systems** (or called semiprivateor semi blind watermarking) [15][34], as a subclass of blind system[32], is capable of detecting only the presence of the embedded symbol with the help of secret key and the watermark symbol but without the cover image[17].

In recent years, watermarking has become an attractive topic and many watermarking schemes have been proposed. The current watermarking techniques can be grouped into categories[15] in spatial domain[1][2][3][7][18][26][27][28], in frequency domain[5][6][8][10][14][16][19]and feature domain[15]. Among these schemes, the ones which require the original information and secret keys for the watermarking extraction are called private watermark schemes[8][10][16][19]. Schemes which require the watermark information and secret keys are called semi-private or semi-blind schemes. Schemes which need secret keys rather than the original information are called public or blind watermark schemes[3][5][9][18].

The paper is organized as follows. Section 2 describes the problem definition. In section 3 describes the principle of previous works. In section 4 describes the performance evaluation of watermarking system. Section 5 describes the study with analysis and modified previous works. In Section 6 describes the proposed method insertion, extraction and analysis of watermarking scheme. Experimental of performance results computed in two parts (i)-Theoretically analysis, and (ii)-by applied on the different benchmark of six gray scale images and two quantum of watermark bit embedded are compared with an previous works and modified algorithms are given in section 7. Finally, Section 8 conclusion and future work.

2. Problem Definition:

The digital watermarking technology is a way to apply digital information hiding techniques, including the ability to hide digital information inside digital images (gray scale images), to prevent malicious and non-malicious attacks to detect hidden information. In particular, digital watermarking techniques in frequency domain have been widely recognized to be more prevalent than others[5], but in recent years the techniques in spatial domain technologies they are becoming generally abandoned[32]. The problem in digital watermarking is that there are three requirements of imperceptibility, capacity, and robustness which must be satisfied but they almost always conflict with each other, in the same case there are trade-off between fidelity and robustness. Accordingly, the proposed solution is to embed a watermark image within the pixels of the cover image in spatial domain technologies, but still there is another problem, (i): when an image is being embedded, it shouldn't cause any visual change to the cover image, whereas almost techniques using a Least-significant-bit (LSB) insertion in spatial domain to hide a watermark image or (massage)[2][3][7][18][25][26][27] within a low embedding errors, where the authors are avoiding to use the Most-Significant-Bit (MSB). While the statement problem there are a trade-off between the embedding error in the LSB and MSB. Furthermore the embedding process in the LSB do not introduce any perceptible into the cover image, as well as the embedding errors in the LSB growth up from (1Min to 8Max). While in the MSB growth up from (16Min to 128Max), with introducing higher perceptible into the cover image. On the other hand the authors investigated into the use of the LSB substitution technique in digital watermarking[1] described in section.4 the LSB embedded watermark bits can easily be removed using techniques, that do not affect the image visually to the point of being noticeable and if the watermark is hidden in the LSB, all the individual has to do is flip one LSB, thus the information cannot be recovered, by the way in recent years the techniques in spatial domain technologies they are becoming generally abandoned. (ii): Another problem appears with this since the image is limited by its dimensions, the number of bits that are usable for embedding is also limited and the watermark image should be chosen in such that it could fit in the cover image. From these problems we aim at introducing to development an enhanced approach for digital watermarking for hiding information that is satisfies these requirements and problems at the same time in an acceptable manner.

3. Previous Methods

The principles of related works in spatial domain:

Wang-Lin-Lin[20] proposed hiding data in images by optimal moderately-significant-bit replacement scheme using a genetic algorithm. Instead of embedding the data in the LSB of the cover image, they proposed embedding the data in the moderately-significant-bit (**LSB_4**) ***the fifth bit accounted from left to right hand*** as shown in fig.4 of the cover image. Here, the LSB_4 is called the first bit, while the LSB_1 is called the eighth bit. With the use of the optimal substitution process by local pixel adjustment process (LPAP), thus the proposed algorithm: let p and p' be the corresponding (8 bit) grey values of a pixel of cover image and resulting of embedding image, respectively, and δ be the value of the last three bits (bits 6-8) ($LSB_{1,2,3}$) in p' as shown in Fig.4. Notice that the max-embedding error in the $LSB_4 = 2^{4-1} = 8$. If $p \neq p'$, then either (i) $p' = p - 8$ or (ii) $p' = p + 8$ (because the only difference between cover image and resulting of embedding image is the 5 bit $_{\text{plane}}$).

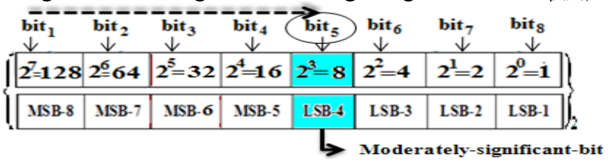

Fig.4: The one pixel of cover image is converted to the binary bits.

Case 1: *when* $p' = p - 8$. If $\delta \geq 4$, then the value $(8-\delta-1)$ is added *to* p'. If $\delta < 4$ and if the fourth bit of p' is 0, then the fourth bit of p' is changed to 1, and the value δ is subtracted from p'. Do nothing otherwise.

Case 2: when $p' = p + 8$. If $\delta < 4$, then the value δ is subtracted from p'. If $\delta \geq 4$ and if the fourth bit of p' is 1, then the fourth bit of p' is changed to 0, and the value $(8-\delta-1)$ is added to p'. Do nothing otherwise. The image quality of the resulting embedding-image is much better than that of the simple replacement method.

❖ **Chi-Kwong et al**.[2] proposed hiding data in images by simple LSB substitution scheme by applying an optimal pixel adjustment process (OPAP) to the embedding image obtained by the simple LSB substitution method, then derived the worst case mean-square-error between the embedding image and the cover image. The authors using to embedding the data bits in the k means capacity of the embedding data bits in the (**k-LSB**) of the cover image, where k given the high capacity of the embedding date bits. They proposed embedding the data in the LSB_1 when k=1, $LSB_{1,2}$ when k=2, $LSB_{1,2,3}$ when k=3 and $LSB_{1,2,3,4}$ when k=4, to at chive high capacity of embedding data in the (LSB_4) that a fifth bit accounted from left to right hand as shown in Fig.4 of the cover image. The authors proposed OPAP: Let P_i , P'_i and P''_i be the corresponding pixel values of the ith pixel in the cover-image C, the embedding-image C' obtained by the simple LSB substitution method and the refined embedding-image obtained after the OPAP. Let $\delta_i = P'_i - P_i$ be the embedding error between P_i and P'_i. P'_i is obtained by the direct replacement of the k-LSB of P_i with k data bits, therefore, $-2^k < \delta_i < 2^k$, the value of δ_i can be further segmented into three intervals, such that:

Interval 1: $2^{k-1} < \delta_i < 2^k$. **Interval 2:** $-2^{k-1} \leq \delta_i \leq 2^{k-1}$. **Interval 3:** $-2^k < \delta_i < -2^{k-1}$.

Based on the three intervals, the OPAP, which modifies P'_i to form the embedding pixel P''_i, can be described as follows:

Case 1: ($2^{k-1} < \delta_i < 2^k$): If $P'_i \geq 2^k$, then $P''_i = P'_i - 2^k$; otherwise $P''_i = P'_i$;

Case 2: ($-2^{k-1} \leq \delta_i \leq 2^{k-1}$): $P''_i = P'_i$;

Case 3: ($-2^k < \delta_i < -2^{k-1}$): If $P'_i < 256 - 2^k$, then $P''_i = P'_i + 2^k$; otherwise $P''_i = P'_i$.

Where the embedding error between P_i and P''_i computed by $\delta'_i = P''_i - P_i$. The authors obtained the embedding error after the proposed OPAP is limited to $0 \leq |\delta'_i| \leq 2^{k-1}$ and computing the Worst PSNR for the capacity of k-data bits by:

$$PSNR_{worst} = 10 \times \log_{10} \frac{255^2}{WMSE} = 10 \times \log_{10} \frac{255^2}{\left(2^k - 1\right)^2} dB$$

Where the worst mean square error (WMSE)= 1 when the k-of-capacity data bits=1, WMSE = 3 when k-of-capacity data bits=2, WMSE = 7 when k-of-capacity data bits=3 and WMSE = 15 when k-of-capacity data bits=4. The authors obtained the $WMSE^* = (2^{k-1})^2$ after applying OPAP and by combining the WMSE with the $WMSE^*$ after applying OPAP , reveals that:

$$\frac{WMSE^*}{WMSE} = \frac{(2^{k-1})^2}{(2^k - 1)^2} \text{ , then } WMSE^* = \frac{(2^{k-1})^2}{(2^k - 1)^2} WMSE$$

Thus the $WMSE^* = 0.2844 WMSE$, when k=4.

❖ **Aiad Abdul-Sada et al.**[3] proposed hiding data using LSB-3 in the cover image. The $\mathbf{LSB_3}$ has been used to increase the robustness of the system and protect the data against the external influences such as noise, compression …etc. The authors using the LPAP by $LSB_{1,2}$ to modified according to the bit of the data embedded , to minimize the difference between the cover image and the embedding image. Let's have the data bits set P={ P_0, P_1, P_2, …, P_{L-1}}, where L is the length of the data that is embedded, and P_i={0,1}, for i=0,..,L-1. Let's have the cover image={$pixel_{0,1}$, $pixel_{1,1}$,…, $pixel_{(N,M)}$}. Suppose that LSB3 of the cover image is LSB_3={c_0, c_1, c_2, …,c_L}, where c_j={0,1} for each i=0,..,L. The embedding process is very easy, which is only replace the permutated bits of the data(P_i) by the LSB_3 set of the cover image to obtain the new embedding image Z={$newpixel_{(0,1)}$, $newpixel_{(1,1)}$, …, $newpixel_{(N,M)}$}. To minimize the difference between the old value (pixel) in the cover image and the new value (newpixel) in the embedding image, the authors propose the following embedding algorithm:

Step 1: Extract LSB_1 set of the cover image, LSB_1={a_0, a_1,…,a_L}. //first plane

Step 2: Extract LSB_2 set of the cover image, LSB_2={b_0, b_1,…, b_L}.// second plane

Step 3: For i=1 to L do

```
if  pi==ci , Then do nothing
else
   if  pi==1 and ci==0, Then
   ai=0;  bi=0;
```

else if p_i==0 and c_i==1, Then
a_i=1; b_i=1;
};
};c_i=p_i ; embed data bit in the LSB_3 of the cover
}

The authors explained the above algorithm; let's have the following pixel in the cover image, pixel=$(3)_{10}$=$(00000011)_2$. Suppose we need to embed **p=1** in the LSB-3, so the new pixel will be, newpixel=$(00000111)_2$=$(7)_{10}$. Notice that the difference is 7-3=4. In embedding algorithm, The authors say will set $LSB_{1,2}$ to **0** when **p=1** and **c=0**. So newpixel=$(00000100)_2$=$(4)_{10}$. Where the deference is becomes 4-3=1. On the other hand, suppose that pixel=$(4)_{10}$=$(00000100)_2$, and p=0. The newpixel=$(00000000)_2$=$(0)_{10}$. The difference is 4-0=4. The embedding algorithm, in this case will be set $LSB_{1,2}$ to '**1**', so newpixel=$(00000011)_2$=$(3)_{10}$. Where the difference is becomes 4-3=1. Thus the differences in the LSB_3 replacement are less or equal one as in the LSB_1 but in more robust.

❖ **Kevin Curran-Xuelong**[1] proposed an investigation into the use of the least significant bit substitution technique in digital watermarking, study presents the results of implementing a LSB in digital watermarking system to investigation the digital watermarking is used by those who wish to prevent others from stealing their material. The authors say the LSB substitution is not a very good candidate for digital watermarking, but it is very useful in the art of steganography, due to its lack of robustness. The LSB embedded watermarks can be easily recovered and even altered by an attacker. Otherwise if the watermark is hidden in the LSB, all the individual has to do is flip one LSB and the information cannot be recovered. It would appear that LSB will remain in the domain of steganography due to its useful nature and its overall capacity of information. Where image Steganography, in the LSB substitution, the least significant bit is changed because this has little effect to the appearance of the carrier message. This shows that the gray scale image would change significantly if there were any other bit changed than the LSB. It changes more and more the closer you get to the *Most-Significant-Bit.* When the LSB is changed, the pixel bit value changes from 128 to 129, which is undetectable with the human eye. With the MSB changed, the pixel bit value changes from 128 to 0, which makes *high a significant change to the gray scale view*. The theory is that if you take two gray scale images, and change the LSB of image one to the LSB of image two for each coordinate or pixel, image two will be hidden in image as (changes from 128 to 129), then the embedded of massage bit in image pixel (129) in the first LSB, there should be no detectable change or alteration to the appearance of the first image pixel (128). Otherwise there are a variety of digital carriers or places where data can be hidden. Data may be embedded in files at *imperceptible levels* of noise and properties of images can be changed and used in a way useful to your aim. The authors study focuses on bit values of pixel in the gray scale range which can be altered to embed hidden images inside other images, without changing the actual appearance of the carrier image. While the watermarking is the process of hiding information in a carrier in order to protect the ownership of image, text, music, films and art, where watermarking can be used to hide or embed visible or hidden copyright information. Watermarking does not impair the image. This is a main concern with visible watermarking. Even though the watermark can be seen, it must be inserted in such a way that it does not interfere with the original image with an invisible watermark you can change certain pixels in an image so the human eye cannot tell the difference from the original image. The important properties of watermarking are perceptual transparency, robustness**,** security and payload. Finally the authors concluded the LSB substitution is not a very good candidate for digital watermarking techniques.

❖ **Recently,** a new digital watermarking technique robust and oblivious digital watermarking image in spatial domain[9] capable of embedding a totally indistinguishable in original image by the human eye by applying falling-off-boundary in corners board of cover image (FOBCB) with the random pixel manipulation set of the Most-Significant-Bit-6 (**MSB_6**) (*the bits are accounted from right to left hand*) as shown in Fig.5 is developed to improve the quality of embedding results, *imperceptibility,* undetectability and robustness. Whereas the binary watermark insertion process needs the secret Ke_{y1} to determine the number of frames per row in watermark and secret Ke_{y2} to changing the pixels of watermark depending on the number of frames per row determined by secret Ke_{y1}**.** **<u>Setp1</u>:** One watermark pixel is inserted in each of FOBCB of cover image with random pixel manipulation set of the MSB_6. Before insertion will be using secret key for spatial dispersion of the watermark to rearranging pixels as the following below: <u>First</u>: Reads the indexed of watermark *W* into X. From the indexed identify the size of matrix X[W, L]. <u>Second</u>: Secret Key_1 using to determine the number of frames per row, where the Key_1 chosen the dimension number divided by the frame number without remainder as Eq(3.1) and Eq(3.2).

m = width (w) / number of frames (Key_1) →(3.1)
n = length (l) / number of frames (Key_1) →(3.2)

<u>Third</u>: Define the indexed identify the size of new matrix Y[ ] for the rearranging pixels of the watermark and then using Key_2 to generate the random permutation of the integers depending on the number of Secret Key_1:

Key_2 = (1: number of frames per row (Key_1)) →(3.3)

Four: Generate two loops [i, j] to selecting a frames by secret Key_1 from indexed identify the size of matrix X[W, L] and by defined the indexed identify the size of new matrix Y[ ] to changing pixels of watermark depending on the secret Key_2 as the algorithm below:

Algorithm:

For i=1 to Secret Key_1 do

For j=1 to Secret Key_1 do

Selecting the frames from indexed identify the size of matrix X[W, L] by Secret Key_1 in to matrix WK.

$WK= (X [(i-1) \times m+1:i \times m,(j-1) \times n+1: j \times n]) \rightarrow (3.4)$

From Eq(3.4) by arranging the frames of pixels by using Secret Key_2 to changing the selecting frames in the new image Y[ ] as Eq(3.5):

$Y[(Key_2(i) - 1) \times m+1 : Key_2(i) \times m, (Key_2(j) - 1) \times n+1: Key_2(j) \times n] = WK \quad \rightarrow (3.5)$

}; }.

Five: For more robustness in digital watermarking applying drawbacks of the payload of watermark in the FOBCB are placed in more than one place in the cover image to prevent the blurring attacks to alter it and cannot defeat the purpose, as the algorithm below:

For ii= 1 to T do

For jj= 1 to U do

Drawback(ii,jj)=payload(mod(ii,T)+1,mod(jj,U)+1);

}; } where is the size of drawbacks [T, U].

Setp2: In this published method, the cover image is of size [M, N] 512*512 gray scale image has been used. In this scheme hide a payload of watermark up to 2025 bits. Embedding payload of watermark in FOBCB of cover image with random pixel manipulation between boundary corners board set of the MSB_6. Let's have the drawbacks payload bits set of the $WL_{(ii,jj)}$, the max-bits can be embedded $1 \leq T \times U \leq 2025$bits, whereas the size of WL=[T,U] and T equal U. Let's have the cover image $F=\{pixel_0, pixel_1, \ldots, pixel_{262144}\}$. So, has been determine the pixels of FOBCB of cover image employed as a sequence number k_1, k_2, k_3, k_4 where $k_1=1,2,\ldots,N$, $k_2=1, 2,\ldots,N$, $k_3=2, 3,\ldots,M-1$, and $k_4=2, 3,\ldots,M-1$, then employed sequence number G to manipulation of pixel between boundary corners board in cover image where $1 \leq G \leq 4$, as the following embedding algorithm:

Embedding algorithm:

For ii = 1 to size of drawback

For jj = 1 to size of drawback

if G==1, then do

if $k_1 <= N$, then do get the corner pixel in FOBCB when $F(1, k_1)$ and set bit of the MSB_6, then $f(1, k_1)$ = embedded the payload of watermark bit $WL_{(ii,jj)}$ to MSB_6 of the pixel $F(1, k_1)$

$k=k_1+1;$

}; }

if G==2

if $k_2 <= N$, then get the corner pixel in FOBCB when $F(M, k_2)$ and set bit of the MSB_6 , then $f(M, k_2)$ = embedded the payload of watermark bit $WL_{(ii,jj)}$ to MSB_6 of the pixel $F(M, k_2)$

$k_2=k_2 +1;$

}; }

if G==3

if $k_3 \sim= M$, then get the corner pixel in FOBCB when $F(k_3, 2)$ and set bit of MSB_6 , then $f(k_3, 2)$ = embedded the payload of watermark bit $WL_{(ii,jj)}$ to MSB_6 of the pixel $F(k_3, 2)$

$k_3=k_3+1;$

}; }

if G==4

G=0;

if $k_4 \sim= M$, then get the corner pixel in FOBCB when $F(k_4, N)$ and set bit of the MSB_6, then $f(k_4, N)$ = embedded the payload of watermark bit $WL_{(ii,jj)}$ to MSB_6 of the pixel $F(k_4, N)$

$k_4= k_4+1;$

}; }

G=G+1;

}; }.

The algorithm protects the payload bits by sequence number? This sequence of indexes used to permute the payload bits. The embedding process is very easy to a achieve the low complexity time, which is only replace the permutated bits of the payload by the MSB_6 set of the FOBCB in cover image with random pixel manipulation between boundary corners board obtain the new digital watermarking $f_{(M,N)}=\{newpixel_0, newpixel_1, \ldots, newpixel_{262144}\}$. **Setp3:** Reconstruct the watermark using to extracted watermark bits from drawbacks in FOBCB of digital watermarking $f_{(M,N)}$ by using inverse the same procedure of embedded algorithm with sequence number *G* to know the manipulation pixel between boundary corners board in digital watermarking $f_{(M,N)}$, then select one of drawbacks set of MSB_6, after extracted watermark required the secret $Key_{1,2}$ to rearranging the frames per row, then watermark in original form is thus obtained. This is completes watermark extraction process.

4. Benchmark of Watermarking System

❖ **Imperceptibility:** Means that the perceived quality of the host image should not be distorted by the presence of the watermark[5]. Developers and implementers of watermarking image need a standard metric to measure the quality of watermarked images compared with the original image. In this section, will be lists the most popular difference distortion measures of pixel based metrics[38]. These measures metrics are all based on the difference between the original images and undistorted or the modified image (watermarked image). The mathematical formulae for the list of pixel based metrics are:

a) ***Average absolute difference:*** The AD is used as the dissimilarity measurement between original image $F_{(M,N)}$ and watermarked image $f_{(M,N)}$ to enhancement the watermarked image. Whereas a lower value of AD signifies lesser error in the watermarked image.

$$AD = \frac{1}{M \times N} \sum_{i=1}^{M} \sum_{j=1}^{N} \left| F_{(i,j)} - f_{(i,j)} \right| \quad \text{---------} (4.1)$$

b) ***Normalized average absolute difference:*** The NAD is quantization error for any single pixel in the image. This distance measure is normalized to a range between 0 and 1.

$$NAD = \frac{\sum_{i=1}^{M} \sum_{j=1}^{N} \left| F_{(i,j)} - f_{(i,j)} \right|}{\sum_{i=1}^{M} \sum_{j=1}^{N} \left| F_{(i,j)} \right|} \quad \text{----------} (4.2)$$

c) ***Mean square error:*** The MSE is the cumulative squared error between the watermarked image $f_{(M,N)}$ and the original image $F_{(M,N)}$[42]. Moreover the MSE measures the error with respect to the centre of the image values, i.e. the mean of the pixel values of the image, and by averaging the sum of squares of the error between the two images[17]. A lower value of MSE signifies lesser error in the watermarked image[42][17].

$$MSE = \frac{1}{M \times N} \sum_{i=1}^{M} \sum_{j=1}^{N} \left(F_{(i,j)} - f_{(i,j)} \right)^2 \quad \text{-----------} (4.3)$$

d) ***Normalized mean square error:*** The normalized mean quantization square error for any single pixel in the image. This distance measure is normalized to a range between 0 and 1. It is independent of the range of gray scale values in the image[43].

$$NMSE = \frac{\sum_{i=1}^{M} \sum_{j=1}^{N} \left(F_{(i,j)} - f_{(i,j)} \right)^2}{\sum_{i=1}^{M} \sum_{j=1}^{N} F_{(i,j)}^2} \quad \text{-----------} (4.4)$$

e) ***Signal to noise ratio:*** The SNR is a measure used to quantify how much a signal has been corrupted by noise[41]. It is defined as the ratio of signal power (original image) to the noise power corrupting the signal (embedding errors between original image and watermarked image). A ratio higher than 1:1 indicates more signal than noise[45].

$$\text{The singnal power} = \sum_{i=1}^{M} \sum_{j=1}^{N} F_{(i,j)}^2$$

$$\text{The noise power corrupting the signal} = \sum_{i=1}^{M} \sum_{j=1}^{N} \left(F_{(i,j)} - f_{(i,j)} \right)^2$$

$$SNR = \frac{\sum_{i=1}^{M} \sum_{j=1}^{N} F_{(i,j)}^2}{\sum_{i=1}^{M} \sum_{j=1}^{N} \left(F_{(i,j)} - f_{(i,j)} \right)^2} \quad \text{-----------} (4.5)$$

They are usually measured in decibels (dB)[16][19]: SNR (dB) = 10×log10 (SNR).

$$SNR = 10 \times \log_{10} \left(\frac{\sum_{i=1}^{M} \sum_{j=1}^{N} F_{(i,j)}^2}{\sum_{i=1}^{M} \sum_{j=1}^{N} \left(F_{(i,j)} - f_{(i,j)} \right)^2} \right) dB \quad \text{--------} (4.6)$$

Notice that the higher ratio means the less obtrusive of the embedding errors (noise) in the watermarked image[41][45]. Whereas the SNR is a technical term used to characterize the quality of the watermarked image detected of a measuring watermark system[46].

f) ***Peak signal to noise ratio:*** The PSNR computes, in decibels, between two images[5]. This ratio is often used as a quality measurement between the original and a watermarked image[29]. The higher the PSNR is the better the quality of the watermarked image[17][29] and *is a standard way to measure image fidelity*[17]. The PSNR is derived by setting the MSE in relation to the maximum possible value of the luminance (for a typical 8-bit value this is 2^8-1= 255) [29] as follows:

$$PSNR = 10 \log_{10} \frac{255^2}{MSE} dB \quad \text{-------------} (4.7)$$

Notice that the MSE and the PSNR are the two error metrics used to compare watermarking image quality[44]. Also the PSNR can be computed with the root mean squared error (RMSE)[16][19]. The RMSE is the square root of mean square error. It quantifies the average sum of distortion in each pixel of the reconstructed image (watermarking image)[17].

$$PSNR = 20 \log_{10} \frac{255}{RMSE} dB \quad \text{-------------} (4.8)$$

Whereas a lower value for MSE means lesser error, and as seen from the inverse relation between the MSE and PSNR, this translates to a high value of PSNR. Logically, a higher value of PSNR is good because it means that the ratio of signal to noise is higher. Here, the 'signal' is the original image, and the 'noise' is the embedding errors of embedded watermark bits in cover image. So, if you find an embedding watermark scheme having a lower MSE (and a high PSNR), you can recognize that it is a better one[42].

❖ **Image fidelity (IF):** refers to the closeness of a watermarked image to a reference of original image[38]. On the other hand the image fidelity how closely the image represents the real source distribution depends not only on random noise but also on errors in the data, sampling, and image artifacts. Thus is a comparative measure of the distance between a pair of images:

$$IF = 1 - \frac{\sum_{i=1}^{M}\sum_{j=1}^{N}\left(F_{(i,j)} - f_{(i,j)}\right)^2}{\sum_{i=1}^{M}\sum_{j=1}^{N} F_{(i,j)}^{\;2}} \quad \text{--------- (4.9)}$$

However, that *Image fidelity* in Eq.(4.9) does not provide an adequate measure of perceived image fidelity, *Thus that an image fidelity measure is also commonly referred to as an image metric, the traditional image fidelity standard is the MSE, SNR, and the PSNR for the original image with watermarked image*[17]. Nowadays, the most popular distortion measures in the field of image or a common measure used of the quality of a watermarked image $f_{(i,j)}$ and compression are the *SNR*, MSE and PSNR is typically used. It is familiar to workers in the field, it is also simple to calculate, but it has only a limited, approximate relationship with the perceived errors noticed by the human visual system. This is why higher PSNR values imply closer resemblance between the watermarking image $f_{(i,j)}$ and the original images $F_{(i,j)}$. Denoting the pixels of the original image by $F_{(i,j)}$ that contains (M by N) pixels represent the size of image (M,N are the dimensions of the images) and the pixels of the image watermarking image by $f_{(i,j)}$, where f is reconstructed watermarking image by decoding the encoded version of original image $F_{(i,j)}$. Error metrics are computed on the luminance value only so the pixel values $f_{(i,j)}$ in the range between black (0) and white (255)[34][37]. For a gray scale image with eight bits per pixel, the numerator is 255. For colour images, only the luminance component is used. The typical PSNR values range between 20 and 40[36][37][41]. Some definitions of PSNR use Eq.(4.7) rather than Eq.(4.8). Either formulation will work because we are interested in the relative comparison, not the absolute values. For our proposal we will use the definition given above in Eq.(4.7).

❖ **Robustness:** it is a measure of the immunity of the watermark against attempts to remove or degrade it, intentionally or unintentionally, by different types of digital signal processing attacks[32]. We will report on robustness results which we obtained of major attacks:

a) ***<u>Watermark degrading attacks</u>:*** Gaussian noise, Salt & Pepper noise and Speckle noise[16][19].

b) ***<u>Watermark removal attacks</u>:*** Changing in lower order bit manipulation of gray scale values $LSB_{1,2,3,4}$, Altered image, and Drawing image[18].

c) ***<u>Geometric transformations attack</u>***[17][11][22][23]***:*** *Image cropping[32]:* In some cases, infringers are just interested by the "central" part of the copyrighted material. *Scaling*[18][21]: it can be divided into two groups, uniform and non-uniform scaling. Under uniform scaling we understand scaling which is the same in horizontal and vertical direction. Non-uniform scaling uses different scaling factors in horizontal and vertical direction. *Lossy data compression like JPEG*[18][32]: JPEG is currently one of the most widely used compression algorithms for images and any watermarking system should be resilient to some degree of compression. *Rotation:* Small angle rotation[11][22]. *Finally: Horizontal flip*[11] [17][22].

They are good representatives of the more general attacks. We measured the similarity between the original watermark W(i,j) and the watermark extracted W'(i,j) from the attacked image, whereas the similarity values NCC and SM of about 0.75 or above is considered acceptable[5][16][19].

i) **<u>Normalized Cross Correlation</u>:** The quantitative estimation for the quality of extracted watermark image $W'_{(i,j)}$ with reference to the original watermark $W_{(i,j)}$ can be expressed as normalized cross correlation gives maximum value of (NCC) as unity defined as[34][38]:

$$NCC = \frac{\sum_{i=1}^{M}\sum_{j=1}^{N}\left(W_{(i,j)} \times W'_{(i,j)}\right)}{\sum_{i=1}^{M}\sum_{j=1}^{N} W_{(i,j)}^{\;2}} \quad \text{--------------- (4.10)}$$

ii) **<u>Similarity Function</u>:**

Function similarity estimation between extracted watermark $W'_{(i,j)}$ and original watermark $W_{(i,j)}$ is computed by the following formula[32]:

$$SM = \frac{\sum_{i=1}^{M}\sum_{j=1}^{N}\left(W_{(i,j)} \times W'_{(i,j)}\right)}{\sqrt{\sum_{i=1}^{M}\sum_{j=1}^{N} W_{(i,j)}^{\;2} \times \sum_{i=1}^{M}\sum_{j=1}^{N} W'^{\;2}_{(i,j)}}} \quad \text{------------ (4.11)}$$

If the result is larger than some determined threshold, we consider the extracted watermark $W'_{(i,j)}$ are equal original watermark $W_{(i,j)}$.

5. Study with Analysis and Modified the Previous Methods

To study the performance and comparisons between the state-of-the-art algorithms will be modified the algorithms of pixel adjustment process (PAP) are based in the LSB techniques proposed in Ref.[1][2][3][20], after that will be applying the modification algorithms of PAP by the our embedding algorithm of the FOBCB set-of-the Most-Significant-Bit-6 (MSB6)[9], reviewed in section.4. Notice that the maximum of embedding errors in the MSB6 by directly replacement of embedding watermark bits = 2n-1=32 as shown in Fig.5.

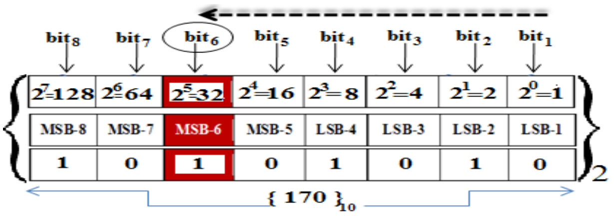

Fig.5: One pixel of cover image is converted to the binary bits (LSB(1,2,3,4) and MSB(5,6,7,8)).

Moreover there is a trade-off between the embedding errors in the LSB and MSB, where the embedding errors are growth up as shown in Fig.5. Let's have the binary watermark image WL(ii,jj), with the size of WL=[T,U], WL={0,1} and the cover image F={pixel1, pixel2,…, pixel(M×N)}= P(i,j), after extracted the pixels from cover image, will be converted the cover image pixels P(i,j) in to the binary numbers (8 bits grey values par pixel), then set of the Most-Significant-Bit (MSB6) in each pixel of the cover image P(i,j) as shown in Fig.5 accounted from right to the left hand, the following modified algorithms in the MSBn, where is n equal 6:

❖ **PAP-algorithm-1:**

We modified the scheme of Wang-Lin-Lin[20] using a local pixel adjustment process (LPAP) the proposed algorithm used LSB_4 for embedding data bits, thus will be modified the algorithm of LPAP on the most-significant-bit-6 (MSB_6). However the embedding error in MSB_6 equal 32 was trade-off with the embedding error in LSB_4 equal 8. Let $\mathbf{P_{(i,j)}}$ and $\boldsymbol{p'_{(i,j)}}$ be the corresponding 8 bit grey values of a pixel in the cover image as shown in Fig.5 and $\boldsymbol{p'_{(i,j)}}$ watermarked image obtained by the FOBCB set of most-significant-bit-6 (MSB_6) scheme, respectively, and $\boldsymbol{\delta}$ be the value of the ($LSB_{1,2,3,4}$ & MSB_5) as well as from {bit_1 to bit_5} in $\boldsymbol{p'_{(i,j)}}$ as shown in Fig.5.

If $P_{(i,j)} \neq p'_{(i,j)}$, then either (i): $p'_{(i,j)} = P_{(i,j)} - 2^{n-1}$ or (ii): $p'_{(i,j)} = P_{(i,j)} + 2^{n-1}$ (because the only difference between cover image and resulting of embedding image is the six bit plane (MSB_6)).

Case 1: *when* $p'_{(i,j)} = P_{(i,j)} - 2^{n-1}$. If $\delta \geq 2^{n-2}$, then the value $(2^{n-1} - \delta - 1)$ is added *to* $p'_{(i,j)}$. If $\delta < 2^{n-2}$ and if the seven bit of $p'_{(i,j)}$ is zero, then the seven bit of $p'_{(i,j)}$ is changed to one, and the value δ is subtracted from $p'_{(i,j)}$. Do nothing otherwise.

Case 2: *when* $p'_{(i,j)} = P_{(i,j)} + 2^{n-1}$. If $\delta < 2^{n-2}$, then the value δ is subtracted from $p'_{(i,j)}$. If $\delta \geq 2^{n-2}$ and if the seven bit of $p'_{(i,j)}$ is one, then the seven bit of $p'_{(i,j)}$ is changed to zero, and the value $(2^{n-1} - \delta - 1)$ is added to $p'_{(i,j)}$. Do nothing otherwise.

- **Analysis:**

Notice that from the PAP of the Wang-Lin-Lin scheme, we know that only the first three bits (bits 1-3) and the five bit (MSB_5) as shown in Fig.5 are modified to improve the image quality. It is obvious that the algorithm is not optimal by analysis the possibility values of gray scale image as shown in Fig.6.

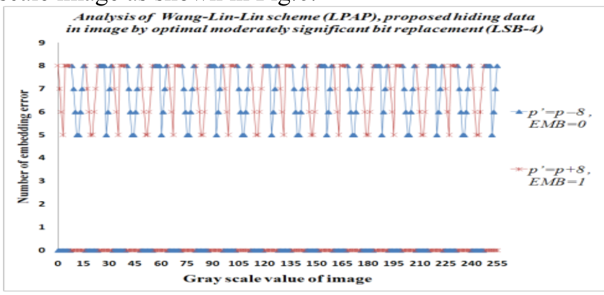

Fig.6: The embedding error of proposed Wang-Lin-Lin scheme.

While if $P_{(i,j)}$= 8, 15, 24, 25, 26, 27, 31, 40, 248, 249, 250 and 251 when the embedded watermark bit equal zero and when $P_{(i,j)}$= 0, 4, 5, 6, 7, 16, 23, 31, 32, 228, 229, 230, 231 and 240 with the embedded watermark bit equal one, whereas the embedding error go to level of maximum error in the $LSB_4=2^{n-1}=8$, for example if $P_{(i,j)} = (31)_{10}$ $(00011111)_2$, then the LSB_4 in cover image is '1'. If the embedded watermark bit is '0' using the Wang-Lin-Lin scheme, we have $p'=(23)_{10}$ $(00010111)_2$ as well as the embedding error for the modification is = |31 – 23| = 8 as the same of listed pixel before as shown in Fig.6. However, it can be seen that the embedding error go to the minimum error if $P_{(i,j)}$= 11, 12, 28, 60, 75, 76, 92, 107, 108, 124, 235, 236 and 252 when the embedded watermark bit equal zero and $P_{(i,j)}$ = 3, 19, 20, 35, 51, 52, 67, 83, 84, 99, 115, 116, 243 and 244 when the embedded watermark bit equal one; as well as the embedding error go to the minimum level error in $LSB_4=(2^{n-2}+1)=5$, as shown in the analysis result in Fig.6. ***Also the same problem of modified scheme (PAP-algorithm-1),*** when applying the algorithm in MSB_6 as showed the analysis result in Fig.7. Also it is obvious that the modification is not optimal where the embedding error are confined between the maximum level of the embedding error in $MSB_6=(2^{n-1})=32$ and in the minimum level of the embedding error in $MSB_6=(2^{n-2}+1)=17$. Thus the observation of the analysis result in the both algorithms (LSB_4 and MSB_6); where the embedding error are growth up one by one start from $(2^{n-2}+1)$ to the maximum of embedding error (2^{n-1}) and then go down to the minimum of the embedding error $(2^{n-2}+1)$ and growth up again ..etc, when $P_{(i,j)} \neq p''_{(i,j)}$ and the embedding error equal zero when $P_{(i,j)} = p''_{(i,j)}$ as shown in Fig.(6&7).

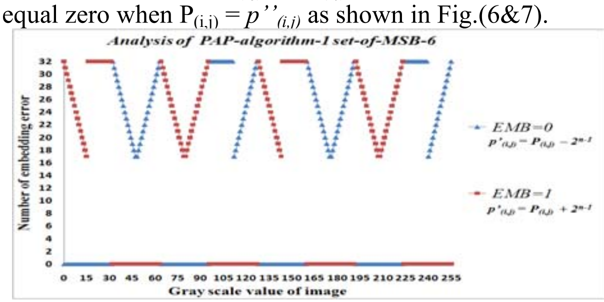

Fig.7: The embedding errors of modified scheme (PAP-algorithm-1)

On the other hand from the analysis result of the embedding errors obtained in the Fig.(6&7). Theoretically, can be calculated the average of embedding errors between the maximum and minimum number in both algorithms. Here the following formula will be useful to calculate the average of embedding errors: thus the embedding error is growth one by one start from $\mathbf{2^{n-2}+1}$ to the maximum embedding error $\mathbf{2^{n-1}}$ and then go down to the $\mathbf{2^{n-2}+1}$ and growth again ...etc. However when the number ***'i'*** of embed errors are grow up one by one start from $\mathbf{2^{n-2}+1}$ to the maximum embedding error $\mathbf{2^{n-1}}$, thus the formula will be useful to calculate as:

$$\sum_{i=1}^{2^{n-2}} i = 1 + 2 + ... + 2^{n-2} = \frac{2^{n-2}\left(2^{n-2}+1\right)}{2} = 2^{n-3}\left(2^{n-2}+1\right) \quad \rightarrow (5.1)$$

$$\sum_{i=1}^{2^{n-1}} i = 1 + 2 + ... + 2^{n-1} = \frac{2^{n-1}\left(2^{n-1}+1\right)}{2} = 2^{n-2}\left(2^{n-1}+1\right) \quad \rightarrow (5.2)$$

Subtracting Eqs.(5.1) and (5.2), then we get the summation of the embedding errors 'i':

$$2^{2n-3} + 2^{n-2} - 2^{2n-5} - 2^{n-3} = 2^{n-2}\left(2^{n-1} - 2^{n-3} + 1\right) - 2^{n-3} \quad \rightarrow (5.3)$$

From Eqs.(5.3) the average of embedding errors in both algorithms between the cover image and watermarked image can be derived by Eqs.(5.4):

$$\text{The average of embedding error} = \frac{2^{n-2}\left(2^{n-1} - 2^{n-3} + 1\right) - 2^{n-3}}{2^{n-2}} = \frac{2\left(2^{n-1} - 2^{n-3} + 1\right) - 1}{2} \quad \rightarrow (5.4)$$

When the $P_{(i,j)} \neq p''_{(i,j)}$. Suppose that all the pixels in the cover image are used for the embedding of watermark bit by simple (LSB or MSB_6) substitution method. Theoretically, in the average of worst mean square error in both algorithms are derived from Eqs.(5.4)&(4.3) as:

$$\text{Averg.WMSE}^* = \frac{1}{M \times N}\sum_{i=1}^{M}\sum_{j=1}^{N}\left[\frac{2\left(2^{n-1} - 2^{n-3} + 1\right) - 1}{2}\right]^2 \quad (5.5)$$

$$\text{Averg.WMSE}^* = \left[\frac{2\left(2^{n-1} - 2^{n-3} + 1\right) - 1}{2}\right]^2 \quad \rightarrow (5.6)$$

From the above Eq.(5.6). Theoretically, can be derived by Eq.(4.3) as well as the maximum of worst mean square error are $\text{Max.WMSE}^*=(2^{n-1})^2$ and the minimum are $\text{Min.WMSE}^*=(2^{n-2}+1)^2$ are obtained after applying modified algorithm.

❖ **PAP-algorithm-2:**

We modified the scheme of Chi-Kwong-L.M.Cheng[2] using optimal pixel adjustment process (OPAP) the proposed embedding algorithm in the **k**, means capacity of embedding data bit in least-significant-bit-n (k-LSB_n) of the cover image, where k given the high capacity of the embedding data bit. while the embedding data bit in the LSB_1 when k=1, $LSB_{1,2}$ when k=2, $LSB_{1,2,3}$ when k=3 and $LSB_{1,2,3,4}$ when k=4, where the maximum embedding errors grow up respectively from {1,3,7and 15} depending in the value of **k**. Notice that in the directly replacement of embedding data bit the embedding errors grow depending on the **k**, where the maximum embedding error equal **15** when k=4. *So that will be applying the OPAP algorithm in the Most-Significant-Bit-n (MSB_6) without using k of capacity. However the embedding errors in MSB_6 equal=32 are greater than with compared by using capacity of k-LSB when embedding data bit in ($LSB_{1,2,3,4}$) are equal* ***15***. Let is $P_{(i,j)}$, $P'_{(i,j)}$ and $P''_{(i,j)}$ be the corresponding pixel values of a pixel in the cover image, the embedding image $P'_{(i,j)}$ obtained by the embedding algorithm FOBCB set of most-significant-bit-n (MSB_6) scheme and the refined embedding image obtained after the modified *PAP-algorihm2* $P''_{(i,j)}$. Let absolute $\boldsymbol{\delta_{(i,j)}=|P'_{(i,j)}-P_{(i,j)}|}$ be the embedding error between $P_{(i,j)}$ and $P'_{(i,j)}$, therefore, $\boldsymbol{-2^n<\delta_{(i,j)}<2^n}$, the value of $\boldsymbol{\delta_{(i,j)}}$ can be further segmented into three intervals, such that: **Interval-1**: $2^{n-1}< \delta_{(i,j)} <2^n$. **Interval-2**: $-2^{n-1}\leq \delta_{(i,j)} \leq 2^{n-1}$. **Interval-3**: $-2^n< \delta_{(i,j)} <-2^{n-1}$.

The PAP-algorithm-2 based on the three intervals, which modifies $\boldsymbol{P'_{(i,j)}}$ to form the embedding pixel $\boldsymbol{P''_{(i,j)}}$, can be described as follows:

Case 1: ($2^{n-1}< \delta_{(i,j)} <2^n$): If $P'_{(i,j)}\geq 2^n$, then $P''_{(i,j)}=P'_{(i,j)} -2^n$; otherwise $P''_{(i,j)}=P'_{(i,j)}$;

Case 2: ($-2^{n-1}\leq \delta_{(i,j)} \leq 2^{n-1}$): $P''_{(i,j)} = P'_{(i,j)}$;

Case 3: ($-2^n< \delta_{(i,j)} <-2^{n-1}$): If $P'_{(i,j)}<256 - 2^n$, then $P''_{(i,j)}=P'_{(i,j)} +2^n$; otherwise $P''_{(i,j)}=P'_{(i,j)}$.

Where the $P''_{(i,j)}$ are obtained by the FOBCB set-of-MSB_6 with applying PAP-algorithm2 and the embedding error occurred between $P_{(i,j)}$ and $P''_{(i,j)}$ computed by $\boldsymbol{\delta'_{(i,j)}=|P''_{(i,j)}-P_{(i,j)}|}$.

- **Analysis:**

Notice that from the optimal pixel adjustment process of the Chi-Kwong-L.M.Cheng scheme, we know that the scheme using 'k' capacity of embed watermark bits, where the algorithm minimized the embedding error from (2^k-1) to 2^{k-1} as shown in Fig.(8 and 9) are sketching the embedding error as a spring shape, our results obtained as the same analysis of the authors. On the other hand from the our analysis results of the embedding errors obtained in the Fig.(8 and 9). Theoretically, can be calculated the number of embedding errors *'i'* are start from one to=2^{k-1} with are growth up one by one. Moreover can be calculated the average of embedding errors, the formula will be useful to calculate the summation of embedding errors 'i' can be derived by:

$$\sum_{i=1}^{2^{k-1}} i = 1 + 2 + ... + 2^{k-1} = \frac{2^{k-1}\left(2^{k-1}+1\right)}{2} = 2^{k-2}\left(2^{k-1}+1\right) \quad \rightarrow (5.7)$$

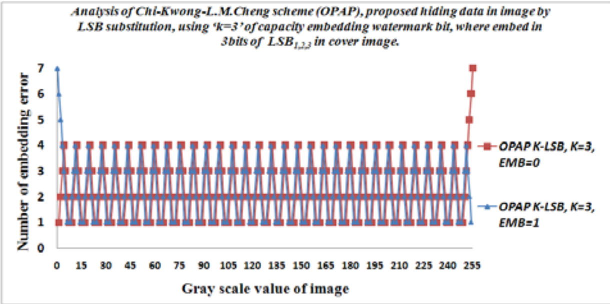

Fig.8: The embedding error of proposed Chi-Kwong scheme k=3.

Hence from the above Eq.(5.7) of the summation of embedding errors *'i'*. Thus the average of embedding errors with 'k' capacity of embeds watermark bits in k-LSB are derived by Eq.(5.8):

$$\text{The average of embedding error} = \frac{2^{k-2}\left(2^{k-1}+1\right)}{2^{k-1}} = \frac{2^{k-1}+1}{2} \rightarrow (5.8)$$

Thus theoretically from Eq.(4.3), the average worst mean square error can be derived as:

$$\text{Averg.WMSE}^{*} = \frac{1}{M \times N}\sum_{i=1}^{M}\sum_{j=1}^{N}\left[\frac{\left(2^{k-1}+1\right)}{2}\right]^2 = \left[\frac{\left(2^{k-1}+1\right)}{2}\right]^2 \rightarrow (5.9)$$

For example if $P_{(i,j)}$= 255 & k=2, when the embeds watermark bit equal zero and when $P_{(i,j)}$= 0, when the embeds watermark bit equal one, thus the embedding error go to the level of max-error in $LSB_{1,2} = (2^k-1)=3$. On the other hand if $P_{(i,j)}$= 2,6, 10, 14, 18,.....,254 & k=2,when the embeds watermark bit equal zero and when $P_{(i,j)}$=1, 5, 9, 13,17,..,253, when the embedding watermark bit equal one

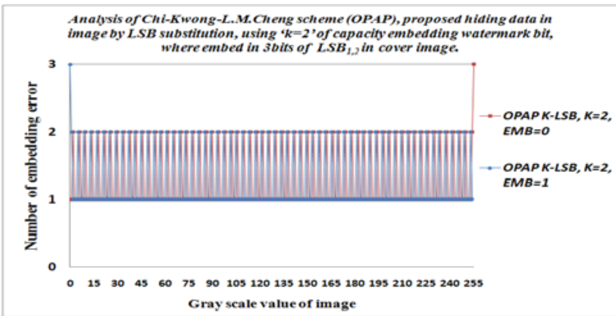

Fig.9: The embedding error of proposed Chi-Kwong scheme k=2.

Since that our observations, where the embedding error goes to half (½) of the max-embedding errors added to half (½), then the max-embed errors of proposed Chi-Kwong-L.M.Cheng scheme are $= \frac{2^k - 1 + 1}{2} = 2^{k-1}$ and the average worst mean square error are obtained by Eq.(5.9). On the other hand by using the same algorithm of proposed Chi-Kwong-L.M.Cheng scheme, where modified to the algorithm called PAP-algorithm-3 set of MSBn where n=k=6. Notice that here in this modification the embeds watermark bit adjust only embedded one bit only in each pixel of the cover image, thus that the maximum of embedding errors are =2n-1=32 by the direct replacement embedding process of the simple MSBn substitution method when n=6. Otherwise by applying the modified PAP-algorithm-3 set of MSB6 the our analysis results in MSB6 as shown in Fig.10, where the embedding errors are always constant great to the maximum level in all cases of embed watermark bit tested in LSB1, LSB2, LSB3, LSB4, MSB5 and MSB6 =2n-1 as shown in Fig.10. Thus theoretically the worst mean square error are constant equal WMSE* = (2n-1)2, after Appling the modified PAP-algorithm-3 set of MSB6.

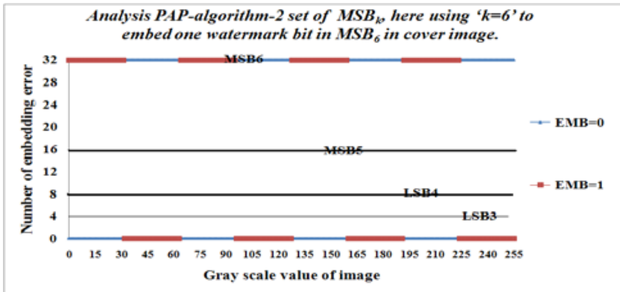

Fig.10: The embedding error of modified scheme (PAP-algorithm-2).

❖ PAP-algorithm-3:

We modified the algorithm of Aiad and Abdul[3] using local pixel adjustment process (LPAP) the proposed algorithm used least-significant-bit (LSB3) to embedded message bit with modified LSB1,2 according to the embedding data bit in LSB3, to minimize the difference between the cover image and the embedding image. So that will be modified the algorithm of LPAP in to most significant bit-6 (MSB6) by applying on the embedding algorithm FOBCB set of most-significant-bit-6 (MSB6) scheme[9], Let's have the cover image P(i,j). Suppose that MSB6 of the cover image is MSB6={MSB1, MSB2, MSB3,…,MSB(N×M)}, where MSB6={0,1}. The embedding process of the watermark bit (EMB) by applying the embedding algorithm FOBCB set of the MSB6 of the cover image to obtain the new embedding image={newpixel(1,1),newpixel(2,1), …, newpixel(N,M)}. To minimize the difference between the old value (pixel) P(i,j) in the cover image and the new value (newpixel) in the embedding image, the following embedding algorithm of LPAP set of MSB6, and the size of cover image (N×M)=L:

Step 1: Extract LSB_1 set of the cover image, LSB_1={ $LSB\text{-}1_1$, $LSB\text{-}1_2$,…, $LSB\text{-}1_L$}./first plane
Step2: Extract LSB_2 set of the cover image, LSB_2={ $LSB\text{-}2_1$, $LSB\text{-}2_2$,…, $LSB\text{-}2_L$}./second plane
Step 3: Extract LSB_3 set of the cover image, LSB_3={$LSB\text{-}3_1$, $LSB\text{-}3_2$,…,$LSB\text{-}3_L$}. /third plane
Step 4: Extract LSB_4 set of the cover image, LSB_4={$LSB\text{-}4_1$, $LSB\text{-}4_2$,…, $LSB\text{-}4_L$}./fourth plane
Step 5: Extract MSB_5 set of the cover image, MSB_5={$MSB\text{-}5_1$, $MSB\text{-}5_2$,…,$MSB\text{-}5_L$}./five plane
Step 6: Extract MSB_6 set of the cover image, MSB_6={$MSB\text{-}6_1$, $MSB\text{-}6_2$,…,$MSB\text{-}6_L$}./five plane
Step 7: Set binary watermark image WL={EMB_1, EMB_2, ..., $EMB_{(T\times U)}$}.
Step 8: For ii = 1 to T

```
For jj = 1 to U
  if MSB6(ii,jj)== EMB(ii,jj), Then do nothing
  else
    if MSB6(ii,jj)==0 and EMB(ii,jj)==1, then
      LSB1(ii,jj)=0; LSB2(ii,jj)=0; LSB3(ii,jj)=0;
      LSB4(ii,jj)=0;MSB5(ii,jj)=0; MSB6(ii,jj)= EMB(ii,jj);
    else
      if MSB6(ii,jj)==1 and EMB(ii,jj)==0, Then
        LSB1(ii,jj)=1; LSB2(ii,jj)=1; LSB3(ii,jj)=1;
        LSB4(ii,jj)=1; MSB5(ii,jj)=1; MSB6(ii,jj)= EMB(ii,jj);
      }; }
    MSB6(ii,jj)= EMB(ii,jj);
  }; }; }.
```

- **Analysis:**

Notice that from the algorithm LPAP of the Aiad and Abdul scheme, we know that only the first two bits (bits 1-2) as shown in Fig.5 are modified to improve the image quality. It is obvious that the modification are minimized the embedding errors as shown in Fig.11 of analysis the possibility gray scale values of image, where are sketching as a spring shape of the embedding error restricted between a minimum '1' and maximum '4' of embedding errors. If P(i,j)= 7, 15,23,31,39 and 47,...etc when the embedded watermark bit equal zero and if P(i,j)= 0, 8, 16, 24, and 32,.....etc when the embedded watermark bit equal one, hence that the embedding error grow up to the maximum error in LSB3=2n-1=4, for example when P(i,j)=(31)10 (00011111)2, then the LSB3 in cover image is '1'. Suppose we need to embed watermark bit is EMB=0 in the LSB3 using the Aiad and Abdul scheme, so that the new-pixel will be have p'=(27)10 (00011011)2 ,thus the embedding error become=|31–27| = 4, then the algorithm of Aiad and Abdul scheme, will be set LSB1,2 to '1' when LSB3=1 and EMB=0. So that the new-pixel=(27)10 (00011011)2 as the same of our analysis results as shown in Fig.11. It is obvious that the modification is not decrease the embedding error in this case. Otherwise if the cover image pixel=(3)10=(00000011)2. Suppose we need to embed EMB=1 in the LSB3, so that the new pixel will be, newpixel=(00000111)2=(7)10 . Notice that the difference is |7-3|=4. Then the algorithm will set LSB1,2 to '0' when EMB=1 and LSB3 of cover image =0. So that newpixel=(00000100)2=(4)10 . As you see the deference becomes 4-3=1 as the same of analysis results as shown in Fig.11.

On the other hand the same procedures applied on the modification algorithm (PAP-algorithm-3) set of MSB6. Thus our analysis results of the embedding errors in MSB6 as shown in Fig.12. we see that the embedding errors grow up to the high in P(i,j)= 63,191and 255 as a pyramid shape of embedding error, when the embeds watermark bit equal zero and if P(i,j)= 0, 64, 128 and 192 when the embeds watermark bit equal one, as shown the embedding error greats to the maximum error in MSB6=2n-1=32 as a pyramid shape of embedding error.

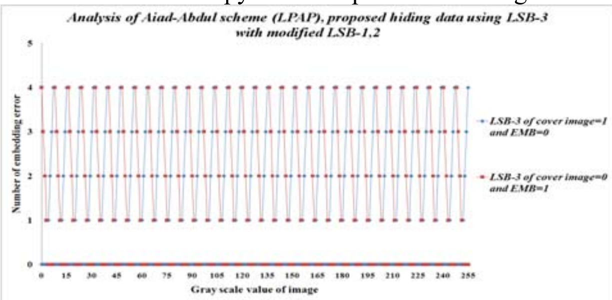

Fig.11: The embedding error of proposed Aiad and Abdul scheme.

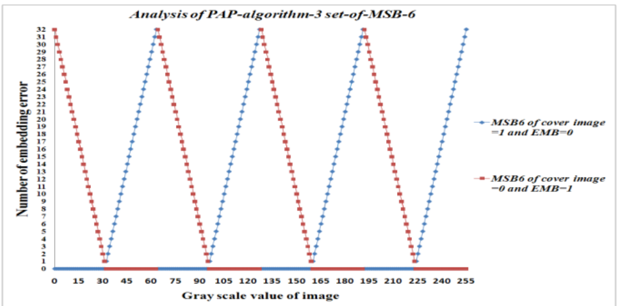

Fig.12: The embedding error of modified scheme (PAP-algorithm-3).

On the other hand the embedding errors decreasing to the low level in the $P_{(i,j)}$=32, 96,160 and 224 when the embedded watermark bit equal zero and if $P_{(i,j)}$= 31, 95, 159 and 223 when the embedded watermark bit equal one, as shown the embedding error go to the minimum error in MSB_6=1. Our observation from the analysis result of both algorithms between the LSB_3 and MSB_6, here the embedding error are grow up one by one start from '1' to the maximum embedding error= 2^{n-1} and then go down to the '1' and grow up again,...etc, it are sketching as pyramid shape of embedding error, when $P_{(i,j)} \neq p'_{(i,j)}$ and the embedding error equal zero when $P_{(i,j)} = p'_{(i,j)}$ as shown in Fig.(7&8) and the maximum embedding error =2^{n-1} and the minimum embedding error=1, where the embed error in the range from 1 to 2^{n-1} increased one by one. Since that from the analysis result of the embedding error obtained Fig.(11&12). Theoretically, can be calculated the average of embedding errors between the maximum and minimum in both algorithms. The following formula will be useful to calculate the average of embedding errors:

$$\sum_{i=1}^{2^{n-1}} i = 1 + 2 + \ldots + 2^{n-1} \qquad (5.10)$$

Hence that 'i' is the number of embedding errors as Eq.(5.10), then.

$$\sum_{i=1}^{2^{n-1}} i = \frac{2^{n-1}\left(2^{n-1}+1\right)}{2} \qquad (5.11)$$

$$\sum_{i=1}^{2^{n-1}} i = 2^{n-2}\left(2^{n-1}+1\right) \qquad (5.12)$$

So that from Eq.(5.12) the average of embedding errors in both algorithms between the cover image and watermarked image, when the $P(i,j) \neq p'(i,j)$, Theoretically can be derived by:

$$\text{The average of embedding errors} = \frac{\left(2^{n-1}+1\right)}{2} \qquad (5.13)$$

Suppose that all the pixels in the cover image are used for the embedding of watermark bit by simple (LSB or MSB6) substitution method, theoretically, from the Eq.(5.13) the average of worst mean square error in both algorithms can be derived by Eq(4.3) as:

$$\text{Averg.WMSE}^{*} = \frac{1}{M \times N}\sum_{i=1}^{M}\sum_{j=1}^{N}\left[\frac{\left(2^{n-1}+1\right)}{2}\right]^2 = \left[\frac{\left(2^{n-1}+1\right)}{2}\right]^2 \qquad (5.14)$$

Whereas the maximum and minimum of worst mean square error are equal Max.WMSE*=(2n-1)2 and Min.WMSE*=1 after applying both algorithm.

6. The Proposed Method

In this section, have been propose a novel method of an adaptively pixel adjustment process based on medial pyramid of embedding error set of the Most-Significant Bit-n (APAP-MPOEEMSBn) is proposed to maybe enhance the gray scale image quality of the watermarked image obtained by a new digital watermarking technique in spatial domain by applying falling-off-boundary in corners board (FOBCB) of gray scale images with the random pixel manipulation set of the Most-Significant-Bit-6, (MSBn), where $5 \leq n \leq 8$ as shown in Fig.5. The basic concept of the pixel adjustment process of the LSBn, when $1 \leq n \leq 4$ based on the technique proposed in Ref[1][2][3][20]. Hence that the ideas are derive from the our analysis of previous works and modified algorithms as shown in Fig.(6 to12) are described in section.5, however the embedding errors are sketched as a pyramid or spring shape, furthermore the embedding error are restricted between the maximum and minimum of embedding errors. Here in the proposed method we used to embed watermark bits and to trying to minimize the embedding error in to the medial pyramid of embedding error. Let's P(i,j) , 'P(i,j) and "P(i,j) be the corresponding pixel values of the cover image F(i,j) that contains (M×N) pixels represent the size of image, 'P(i,j) the watermarked image f(i,j) obtained by a new digital watermarking technique in the spatial domain by applying falling-off-boundary in corners board of cover image with the random pixel manipulation set of the most-significant-bit-6 (MSB6) and "P(i,j) the refined watermarked image obtained after the applying proposed method of an adaptively pixel adjustment process based on medial pyramid of embedding error (APAP-MPOEE) by applying falling-off-boundary in corners board (FOBCB) of gray scale images set of the Most-Significant-Bit-6 (MSB6) with the random pixel manipulation (APAP-MPOEE-FOBCBMSB6). Let's Ω'=|'P(i,j)−P(i,j)| be the embedding error between P(i,j) and 'P(i,j) according to the embedding process of the falling-off-boundary in corners board of cover image with the random pixel manipulation set of the MSB6 described in Section.3. where 'P(i,j) is obtained by the direct replacement of embedded watermark bit (EMB) W(i,j) equal zero EMB='0' or equal one EMB ='1' in the MSB6 of the cover image pixel with a constant of embedding error. In this paper we shall propose a novel method of APAP-MPOEE set of the MSB6 by applying falling-off-boundary in corners board of cover image with the random pixel manipulation blind in spatial domain (APAP-MPOEE-FOBCBMSB6) maybe to enhance the image quality of the watermarked image to great fidelity and imperceptibility under three steps:-

- **Step 1:**

Extract pixel from the cover image P(i,j) and converted in to the binary bits (LSB(1,2,3,4) and MSB(5,6,7,8)) as shown in Fig.5, then set of the most-significant-bit-6 (MSB6) in each pixel within the boundary of corners board, as well as when the MSB6 equal zero MSB='0' and the embedded watermark bit (EMB) of the binary log image W(i,j) equal zero EMB='0' or when the MSB6 of cover image pixel equal one MSB6='1' and when the EMB of the binary log image equal one EMB='1', if MSB6=EMB, then do nothing, whereas the APAP-MPOEE of watermarked pixel "P(i,j) = cover image pixel P(i,j).Where the embedded bit (EMB) of the binary log image is the same as bit value of MSB6 in cover image. Otherwise when the MSB6 in cover image not equal the embedded watermark bit (EMB), MSB6≠EMB thus the pixel value of cover image P(i,j) can be further segmented into intervals, whereas the maximum pixel value of cover image in interval with 8 bit at in the range $0 \leq p_{(i,j)} < 256$, theoretically can be derived the intervals depending on the error value (2n-1) in each bit as:

$$\text{The number of intervals in MSB}_n = \frac{256}{2^{n-1}} = 8 \quad \rightarrow (6.1)$$

From Eqs.(6.1) the all number of intervals in MSBn=8, when n=6. Furthermore will be divided the eight intervals depending on the step of the embedding Step2: when MSB6 equal zero (MSB6='0') and the embedded watermark bit (EMB) equal one (EMB='1'). Step3: when MSB6 equal one (MSB6='1') and the embedded watermark bit (EMB) equal zero (EMB='0'). From step2 and step3, theoretically can be further segmented into four intervals in step 2 and step3 derived by:

$$\text{The number of intervals in each step}_{2,3} = \frac{256}{2^n} = 4 \quad \rightarrow (6.2)$$

Hence that the embedding process in the MSB6 of the cover image pixel in the boundary of corners board to form the watermarked pixel "P(i,j) that required eight intervals as Eq.(6.1), thus each interval will be divided in to two intervals to minimizing the embedding error in to the medial pyramid of embedding error, then will be get sixteen intervals, can be described as: First: will be divided each interval in to two intervals as shown in Fig.(13 & 14). Second: Added (2n-2) in each start interval to get the end of a new interval as shown in Fig.(13 & 14), where are from the interval-1 will be get two intervals as shown in Fig.13 and in case.1. Hence that each interval from (1 to 8) is divided in to the half (½) in each interval to obtained a sixteen intervals as in case(1 to 8) in step:(2&3) shown in Fig.(13&14), can be derived in step:(2&3). Notice that n in MSBn within interval $6 \leq n \leq 8$:

- **Step 2:**

In this step when the MSB_6='0' in the pixel of cover image and EMB='1' o f the embedded watermark bit, then the value pixels of cover image $P_{(i,j)}$ can be further segmented into four intervals as Eqs.(6.2) from the total intervals as Eqs.(6.1), such that:

Interval 1: $0 \leq p_{(i,j)} < 2^{n-1}$

Interval 2: $2^n \leq p_{(i,j)} < 3\times 2^{n-1}$

Interval 3: $2^{n+1} \leq p_{(i,j)} < 5\times 2^{n-1}$

Interval 4: $3\times 2^n \leq p_{(i,j)} < 7\times 2^{n-1}$

In this step based on four intervals from (1 to 4), the APAP-MPOEE, which the algorithm requires a checking between the MSB_6='0' in the pixel of cover image and EMB='1' of the embedded watermark bit before embedding the watermark bit depending on the nearest of adaptively pixel in the medial pyramid of embedding error to inform the watermarked image $"P_{(i,j)}$ as shown in Fig.13, can be described as follows:

Case 1: ($0 \leq P_{(i,j)} < 2^{n-1}$), then
  if ($0 \leq P_{(i,j)} < 2^{n-2}$), then
    $"P_{(i,j)} = 2^{n-1}$;
  else
    $"P_{(i,j)} = 2^{n-1}$;
  end

Case 2: ($2^n \leq P_{(i,j)} < 3\times 2^{n-1}$), then
  if ($2^n \leq P_{(i,j)} < 5\times 2^{n-2}$), then
    $"P_{(i,j)} = 2^n - 1$;
  else
    $"P_{(i,j)} = 3\times 2^{n-1}$;
  end

Case 3: ($2^{n+1} \leq P_{(i,j)} < 5\times 2^{n-1}$), then
  if ($2^{n+1} \leq P_{(i,j)} < 9\times 2^{n-2}$), then
    $"P_{(i,j)} = 2^{n+1} - 1$;
  else
    $"P_{(i,j)} = 5\times 2^{n-1}$;
  end

Case 4: ($3\times 2^n \leq P_{(i,j)} < 7\times 2^{n-1}$), then
  if ($3\times 2^n \leq P_{(i,j)} < 13\times 2^{n-2}$), then
    $"P_{(i,j)} = 3\times 2^n - 1$;
  else
    $"P_{(i,j)} = 7\times 2^{n-1}$;
  end

- **Step 3:**

In this step when the MSB_6='1' in the pixel of cover image and EMB='0' o f the embedded watermark bit, then the value pixels of cover image $P_{(i,j)}$ can be further segmented into four intervals as Eq.(6.2) from the total intervals as Eq.(6.1), such that:

Interval 5: $2^{n-1} \leq p_{(i,j)} < 2^n$

Interval 6: $3\times 2^{n-1} \leq p_{(i,j)} < 2^{n+1}$

Interval 7: $5\times 2^{n-1} \leq p_{(i,j)} < 3\times 2^n$

Interval 8: $7\times 2^{n-1} \leq p_{(i,j)} < 2^{n+2}$

In this step based on four intervals from (5 to 8), the APAP-MPOEE, which the algorithm requires a checking between the MSB_6='1' in the pixel of cover image and EMB='0' of the embedded watermark bit before embedding the watermark bit depending on the nearest of adaptively pixel in the medial pyramid of embedding error to inform the watermarked image $"P_{(i,j)}$ as shown in Fig.14, can be described as follows:

Case 5: ($2^{n-1} \leq P_{(i,j)} < 2^n$), then
  if ($2^{n-1} \leq P_{(i,j)} < 3\times 2^{n-2}$), then
    $"P_{(i,j)} = 2^{n-1} - 1$;
  else
    if ($6 \leq n \leq 7$), then
      $"P_{(i,j)} = 2^n$;
    else
      $"P_{(i,j)} = 2^{n-1} - 1$;
    end
  end

Case 6: ($3\times 2^{n-1} \leq P_{(i,j)} < 2^{n+1}$), then
  if ($3\times 2^{n-1} \leq P_{(i,j)} < 7\times 2^{n-2}$), then

$''P_{(i,j)} = 3\times 2^{n-1} - 1;$
else
if (n==6) , then
$''P_{(i,j)} = 2^{n+1};$
else
$''P_{(i,j)} = 3\times 2^{n-1} - 1;$
end
end

Case 7: ($5\times 2^{n-1} \le P_{(i,j)} < 3\times 2^{n}$), then
if ($5\times 2^{n-1} \le P_{(i,j)} < 11\times 2^{n-2}$), then
$''P_{(i,j)} = 5\times 2^{n-1} - 1;$
else
$''P_{(i,j)} = 3\times 2^{n};$
end

Case 8: ($7\times 2^{n-1} \le P_{(i,j)} < 2^{n+2}$), then
if ($7\times 2^{n-1} \le P_{(i,j)} < 15\times 2^{n-2}$), then
$''P_{(i,j)} = 7\times 2^{n-1} - 1;$
else
$''P_{(i,j)} = 7\times 2^{n-1} - 1;$
end

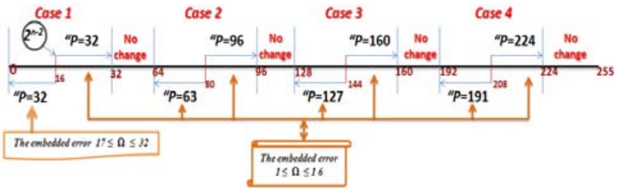

Fig.13: Interval of embedding process when MSB_6=0 and EMB=1.

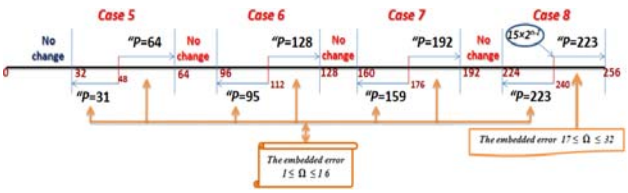

Fig.14: Interval of embedding process when MSB_6=1 and EMB=0.

❖ Encoding process of APAP-MPOEE:

The proposed algorithm of APAP-MPOEE-$_{MSBn}$ developed to the most-significant-bit$_n$ MSB_n in spatial domain, whereas $5 < n \le 8$. Moreover the proposed algorithm of APAP-MPOEE-$_{MSBn}$ used for embedding watermark bits in the cover image and before embedding requires a checking between the MSB_6 in the pixel of cover image and EMB of the embedded watermark bit depending on the nearest of the adaptively pixel in the medial pyramid of embedding error to inform the watermarked image $''P_{(i,j)}$. **<u>First:</u>** Have been permute the pixel of watermark image before inserted to protect the watermark bit by rearranged pixel according to the security key, to avoid possible attack as we used in the[9] reviewed in section.3 within step.1. **<u>Second:</u>** After permute the pixel of watermark image, then will be insertion with redundantly distributed the watermark bits over many pixels of the cover image, using a small watermark image 16×16, are added simultaneously to improve the capacity and to ensure robustness. **<u>Third:</u>** The following algorithm of APAP-MPOEE set of the MSB_6: P=imread(num2str(Cover gray scale image)); then extracted pixel from the cover image, F={pixel$_1$, pixel$_2$,…, pixel$_{(M\times N)}$}= $P_{(i,j)}$, and convert the cover image pixel $P_{(i,j)}$ to the binary bits (8 bit grey values) as a $LSB_{(1,2,3,4)}$ and $MSB_{(5,6,7,8)}$ from right to left as shown in Fig.5, then set of the most-significant bit-n (MSB_6) in each pixel of the cover image $P_{(i,j)}$. Let's have a binary watermark image $WL_{(i,j)}$, whereas the size of WL=[T,U] and WL={0,1}, then extracted binary bits from the watermark image as EMB={EMB_1, EMB_2,…, $EMB_{(T\times U)}$}= $EMB_{(i,j)}$, where is $n = 6$; then can be described as follows:

```
For i = 1 to M
For j = 1 to N
 MSB6 = bitget (P(i,j), n );
 if(MSB6==0&EMB==0)|(MSB6==1&EMB==1), then
   ″P(i,j) = P(i,j) ;  No change.
 else
  if ( MSB6 == 0 and EMB == 1 ), then
     if ( P(i,j) > = 0 and P(i,j) < 2^n-1 ) then   0 0 1 0 0 0 0 0
       ″P(i,j) = 2^n-1;
     else %
       if ( P(i,j) > = 2^n and P(i,j) < 3×2^n-1 ), then
         if ( P(i,j) > = 2^n and P(i,j) < 5×2^n-2 ), then
           ″P(i,j) = (2^n) -1;   0 0 1 1 1 1 1 1
         else
                 ″P(i,j)=3×2^n-1;   0 1 1 0 0 0 0 0
         end
       else %
        if(P(i,j) >= 2^n+1 and P(i,j)< 5×2^n-1 ), then
          if(P(i,j)>=2^n+1 and P(i,j) < 9×2^n-2 ), then
            ″P(i,j)=(2^n+1)-1;   1 1 1 1 1 1 1 1
          else
            ″P(i,j)=5×2^n-1;   1 0 1 0 0 0 0 0
          end
        else %
          if(P(i,j)>=3×2^n and P(i,j)<7×2^n-1 ), then
            if(P(i,j)>=3^n and P(i,j)<13×2^n-2), then
              ″P(i,j)=(3×2^n)-1;   1 0 1 1 1 1 1 1
            else
      ″P(i,j) = 7×2^n-1;   1 1 1 0 0 0 0 0
     end
   end %
  end
 end
end
else
  if (MSB6 ==1 & EMB == 0 ), then
   if ( P(i,j) > = 2^n-1 and P(i,j) < 2^n ), then
     if (P(i,j) > = 2^n-1 and P(i,j) < 3×2^n ), then
       ″P(i,j)= (2^n-1) - 1;   0 0 0 1 1 1 1 1
     else
       ″P(i,j) = 2^n;   0 1 0 0 0 0 0 0
     end
```

```
    else %
      if ( P(i,j) >= 3×2^n-1 and P(i,j) < 2^n+1), then
        if (P(i,j) >= 3×2^n-1 and P(i,j) < 7×2^n-2), then
          ″P(i,j)= (3×2^n-1) - 1;   [0|1|0|1|1|1|1|1]
        else
          ″P(i,j) = 2^n+1;   [1|0|0|0|0|0|0|0]
        end
      else %
        if ( P(i,j) >= 5×2^n-1 and P(i,j) < 3×2^n), then
          if (P(i,j) >= 5×2^n-1 and P(i,j) < 11×2^n-2), then
            ″P(i,j) = (5×2^n-1) - 1;   [1|0|0|1|1|1|1|1]
          else
            ″P(i,j) = 3×2^n;   [1|1|0|0|0|0|0|0]
          end
        else %
          if ( P(i,j) >= 7×2^n-1 and P(i,j) < 2^n+2 ), then
″P(i,j)=(7×2^n-1)-1;   [1|1|0|1|1|1|1|1]
          end
        end
      end
   end   % if MSB6==1 & EMB==0
end   % if MSB6==0 & EMB==1
end % if(MSB6=0&EMB=0)|(MSB6=1& EMB=1)
end      % For
end        % For
```

Four: From the above algorithm of proposed method (APAP-MPOEE set of the MSB_6) used by applying a falling-off-boundary in corners board of the cover image with the random pixel manipulation in the spatial domain (APAP-MPOEE-$FOBCB_{MSB6}$). The proposed **APAP-MPOEE-$FOBCB_{MSB6}$** method using as a embeds watermark bits in a boundary in corners board of the cover image and before embedding requires a checking between the MSB_6 in the boundary in corners board pixel of the cover image and EMB of the embedded watermark bit, depending on the nearest of the adaptively pixel in the medial pyramid of embedding error to inform the watermarked image "$P_{(i,j)}$ obtained by a APAP-MPOEE-$FOBCB_{MSB6}$ scheme as shown in the flowchart of embedding process in Fig.15.

zero EMB=0 in the MSB_6, where the MSB_6 of the pixel in the cover image MSB_6 equal one MSB_6=1, then the $Ƥ_{(i,j)}=(00011111)_2=(31)_{10}$. Notice that the difference error is $\Omega'=|Ƥ_{(i,j)}-P_{(i,j)}|=|31-63|=32$, then the max-embedding error = 32 for all the embedding process, only the embedding error equal zero $\Omega'=|Ƥ_{(i,j)}-P_{(i,j)}|=0$ when the embed watermark bit equal the same of the MSB_6 in the cover image pixel. Hence that the embedding errors are constant Ω'=32.

Second: From the above analysis have been proposed a novel algorithm of APAP-MPOEE developed to the Most-Significant-bit MSB_n in spatial domain, where is $5 < n \le 8$, for trying to enhance the image quality of the watermarked image. Hence that when applying proposed method of APAP-MPOEE set of the MSB_6 in spatial domain. For example let's have the following pixel in the cover image, $P_{(i,j)}=(31)_{10}=(00\mathbf{0}11111)_2$. Suppose the embed watermark bit equal one EMB=1 in the MSB_6, where the MSB_6 of the cover image pixel equal zero MSB_6=0, then have been apply case.1 in the proposed method (APAP-$MPOEE_{MSB6}$)$=''Ƥ_{(i,j)}=2^{n-1}=(00100000)_2=(32)_{10}$ as shown in Fig.13. Notice that the difference error is $\mathbf{\Omega=|''Ƥ_{(i,j)}-P_{(i,j)}|}$ be the embedding error between $P_{(i,j)}$ and $''Ƥ_{(i,j)}$ according to the embedding process of the watermark bit equal one EMB=1 in the $MSB_6=|32-31|=1$. Another example let's have $P_{(i,j)}=(16)_{10}=(00010000)_2$. Suppose the embed equal one in the MSB_6, where the MSB_6 of the cover image pixel equal zero, then have been apply case.1 in the proposed method (APAP-$MPOEE_{MSB6}$)$=''Ƥ_{(i,j)}=2^{n-1}=(00100000)_2=(32)_{10}$ is obtained by the direct replacement of APAP-$MPOEE_{MSB6}$ as shown in Fig.13, so that the difference error is $\Omega=|''Ƥ_{(i,j)}-P_{(i,j)}|=|32-16|=16$, notices that the embedded error in the case.1 are become in proposed method as in to a interval $2^{n-2}\le P_{(i,j)}<2^{n-1}$, where are in the range from $1\le\Omega\le 16$. It has the same embedding errors in case.2,3,4,5,6 and 7 as shown in the intervals of embedding process in Fig.(13&14), where are the embedding errors are minimized to the half (½) with compared by the direct replacement of the embedded watermark bit.

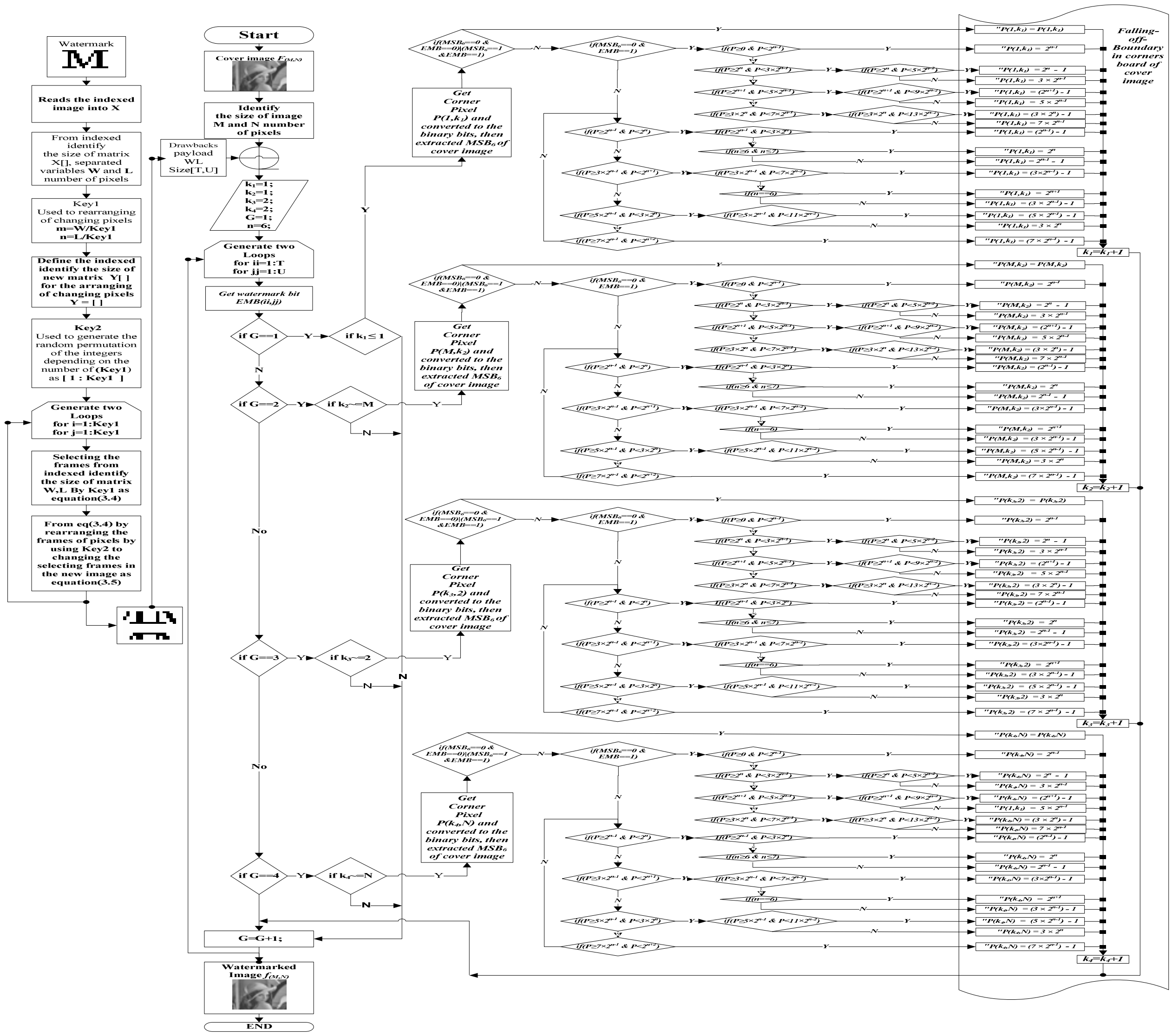

Fig.15: The flowchart embedding process of proposed method (APAP-MPOEE-FOBCBMSB6)

❖ **Decoding process of APAP-MPOEE:-**

A decoder using to extracted watermark bits from drawbacks in falling-off-boundary in corners board of the watermarked image $f_{(M,N)}$ by using inverse the same procedure of the embedded algorithm without using the steps of embedding process in proposed method adjust recovery the watermark bits from the falling-off-boundary in corners board in watermarked image depending on the sequence number ***G*** to know the manipulation pixel between boundary corners board in the watermarked image $f_{(M,N)}$ and then select one of drawbacks in the MSB_6, after extracted watermark required a secret $Key_{1,2}$ to rearranging the change of the frames per row, then the watermark in original form is thus obtained. This is completes watermark extraction process. A quantitative estimation for the quality of extracted watermark image $W'_{(i,j)}$ under inspection with or without external attacks by compared with the original watermark $W(i,j)$ as reference can be expressed as a normalized cross correlation, psnr and similarity function.

❖ **Analysis the scheme of APAP-MPOEE$_{MSB6}$:**

To analysis the above algorithm of APAP-MPOEE$_{MSB6}$ according to the cases from (1 to 8) are generated a ***sixteen intervals***, where are two interval in each pyramid case of embedding error as shown in Fig.16.

First: Let's have the following pixel in the cover image, $P_{(i,j)} =(31)_{10} =(00011111)_2$. Suppose the embed binary watermark bit

equal one EMB=1 in the MSB_6, where the MSB_6 of the pixel in the cover image MSB_6 equal zero MSB_6=0, so the

$Ƥ_{(i,j)}$ is obtained by the direct replacement of embedded watermark bit (EMB) of binary image $W(i,j)$, when MSB_6='0'and EMB='1' in the most-significant-bit-6 (MSB_6) of the cover image pixel to produce watermarked pixel $Ƥ_{(i,j)}=(00111111)_2=(63)_{10}$. Notice that the difference is $\Omega' =|Ƥ_{(i,j)} - P_{(i,j)}|$ be the embedding error between $P_{(i,j)}$ and $Ƥ_{(i,j)}$ according to the embedding process of the binary watermark bit equal one EMB=1 in the MSB_6 =|63–31|=32, otherwise $P_{(i,j)} =(0)_{10} =(00000000)_2$. Suppose the embed equal one EMB=1 in the MSB_6, where the MSB_6 of the pixel in the cover image MSB_6 equal zero MSB_6=0, so the $Ƥ_{(i,j)} = (00100000)_2 = (32)_{10}$. Notice that the difference error is $\Omega'=|Ƥ_{(i,j)}-P_{(i,j)}|= 32 - 0 = 32$, then the max-embedding error = 32. Otherwise let's have the following pixel in the cover image, $P_{(i,j)} = (32)_{10} = (00010000)_2$. Suppose the embed binary watermark bit equal zero EMB=0 in the MSB_6, where the MSB_6 of the pixel in the cover image MSB_6 equal one MSB_6=1, then the $Ƥ_{(i,j)}$ is obtained by the direct replacement of embedded watermark bit (EMB) when MSB_6='1'and EMB='0' in the MSB_6 of the cover image pixel to produce watermarked pixel $Ƥ_{(i,j)}=(00000000)_2=(0)_{10}$. Notice that the difference is $\Omega'=|Ƥ_{(i,j)}-P_{(i,j)}|$ be the embedding error between $P_{(i,j)}$ and $Ƥ_{(i,j)}$ according to the embedding process of the binary watermark bit equal zero in the MSB_6=|0–32|=32, also when $P_{(i,j)}=(63)_{10}=(00111111)_2$. Suppose the embed equal zero EMB=0 in the MSB_6, where the MSB_6 of the pixel in the cover image MSB_6 equal one MSB_6=1, then the $Ƥ_{(i,j)}=(00011111)_2 =(31)_{10}$. Notice that the difference error is $\Omega' =|Ƥ_{(i,j)}-P_{(i,j)}| =|31-63|=32$, then the max-embedding error = 32 for all the embedding process, only the embedding error equal zero $\Omega' = |Ƥ_{(i,j)}-P_{(i,j)}| = 0$ when the embed watermark bit equal the same of the MSB_6 in the cover image pixel. Hence that the embedding errors are constant Ω'=32.

Second: From the above analysis have been proposed a novel algorithm of APAP-MPOEE developed to the Most-Significant-bit MSB_n in spatial domain, where is $5 < n \le 8$, for trying to enhance the image quality of the watermarked image. Hence that when applying proposed method of APAP-MPOEE set of the MSB_6 in spatial domain. For example let's have the following pixel in the cover image, $P_{(i,j)}=(31)_{10}=(00\mathbf{0}11111)_2$. Suppose the embed watermark bit equal one EMB=1 in the MSB_6, where the MSB_6 of the cover image pixel equal zero MSB_6=0, then have been apply case.1 in the proposed method ($APAP\text{-}MPOEE_{MSB6}$)$={}''Ƥ_{(i,j)} = 2^{n-1} =(00100000)_2 = (32)_{10}$ as shown in Fig.13. Notice that the difference error is $\boldsymbol{\Omega=|''Ƥ_{(i,j)} - P_{(i,j)}|}$ be the embedding error between $P_{(i,j)}$ and $''Ƥ_{(i,j)}$ according to the embedding process of the watermark bit equal one EMB=1 in the MSB_6 =|32 – 31| = 1. Another example let's have $P_{(i,j)} =(16)_{10} =(00010000)_2$. Suppose the embed equal one in the MSB_6, where the MSB_6 of the cover image pixel equal zero, then have been apply case.1 in the proposed method ($APAP\text{-}MPOEE_{MSB6}$)$={}''Ƥ_{(i,j)} = 2^{n-1} = (00100000)_2 = (32)_{10}$ is obtained by the direct replacement of $APAP\text{-}MPOEE_{MSB6}$ as shown in Fig.13, so that the difference error is $\Omega=|''Ƥ_{(i,j)}-P_{(i,j)}| = |32 - 16| = 16$, notices that the embedded error in the case.1 are become in proposed method as in to a interval $2^{n-2} \le P_{(i,j)} < 2^{n-1}$, where are in the range from $1 \le \Omega \le 16$. It has the same embedding errors in case.2,3,4,5,6 and 7 as shown in the intervals of embedding process in Fig.(13&14), where are the embedding errors are minimized to the half (½) with compared by the direct replacement of the embedded watermark bit.

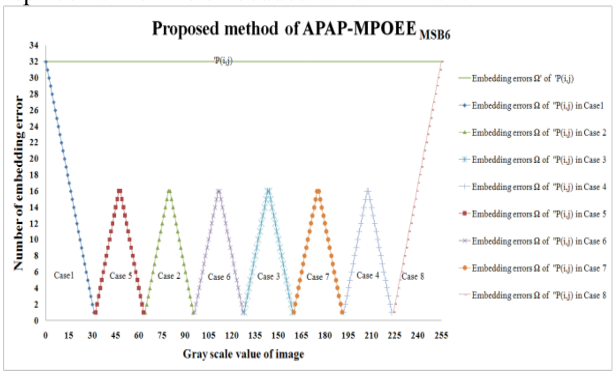

Fig16: Comparisons proposed method of $APAP\text{-}MPOEE_{MSB6}$ with the $Ƥ_{(i,j)}$ are obtained by the direct replacement of embeds watermark bits.

Finally, from the above analysis of proposed method of $APAP\text{-}MPOEE_{MSB6}$ according to the cases from (1 to 8) the embedding errors Ω increased one by one according to the values of cover image pixel $P_{(i,j)}$ in the range from $2^{n-2} \le P_{(i,j)} < 2^{n-1}$, where are become when $n=6$ in the range of $1 \le \Omega \le 16$ as shown in Fig.16, where is each case minimizing the embedding errors to the medial pyramid of embedding error to inform the watermarked pixel? By the way shown in the sketched of the Fig.16, each case sketched the pyramid of embedding error are minimized to the half (2^{n-2}) of maximum embedding error(2^{n-1}), when the values of gray scale in cover image pixel $P_{(i,j)}$ in the interval between $2^{n-2} \le P_{(i,j)} \le 15\times2^{n-2}$ as shown in Fig.(13&14). Otherwise the embedding error Ω are grow one by one according to the values of gray scale in cover image pixel $P_{(i,j)}$ in the range from $17 \le \Omega \le 32$ as shown in the first half of case.1 and in the last half in case.8, when the values of gray scale in the cover image pixel $P_{(i,j)}$ in the intervals from $0 \le P_{(i,j)} < 2^{n-2}$ as shown in Fig.13 with case.1 and from $15\times2^{n-2} < P_{(i,j)} < 256$ as shown in Fig.14 with case.8. Notices that in the proposed method the embedding errors Ω are minimized to the half (½), where are the embedding errors decreased one by one as shown in Fig.16 in each case, when the values of gray scale image pixel $P_{(i,j)}$ in the interval between $2^{n-2} \le P_{(i,j)} \le 15\times2^{n-2}$ with compared by the direct replacement of embedding watermark bits $Ƥ_{(i,j)}$ as shown the embedding errors Ω' are constant Ω' =32. On the other hand from the

Fig.16, theoretically can be calculated the summation of embedding errors in the all intervals, but will be neglects the gray scale values $P_{(i,j)}$ from intervals $0 \leq P_{(i,j)} < 2^{n-2}$ and from $15\times2^{n-2} < P_{(i,j)} < 256$ as shown in Fig.(13&14). Where are almost of gray scale images out of these intervals. Our proposed algorithm minimized the embedding error from (2^{n-1}) in the maximum pyramid of embedding error to the medial pyramid of embedding error (2^{n-2}) as shown in Fig.16, then the summation number of embedding errors ***'i'*** are start from ***one*** to (2^{n-2}) are growth one by one, the formula will be useful to calculate the summation of embedding errors ***'i' as***:

$$\sum_{i=1}^{2^{n-2}} i = 1+2+...+2^{n-2} = \frac{2^{n-2}\left(2^{n-2}+1\right)}{2} = 2^{n-3}\left(2^{n-2}+1\right) \rightarrow (6.3)$$

Hence from Eq.(6.3) the summation of embedding errors ***'i'*** can be calculated the average of embedding errors between the cover image and watermarked image derived by:

$$\text{The average of embedding error} = \frac{2^{n-3}\left(2^{n-2}+1\right)}{2^{n-2}} = \frac{2^{n-2}+1}{2} \rightarrow (6.4)$$

When the $P_{(i,j)} \neq {}''p_{(i,j)}$. Suppose that all the pixels in the cover image are used for the embedding of watermark bit by proposed method, theoretically, the average of worst mean square error between the cover image and watermarked image can be derived by Eq(4.3):

$$\text{Averg.WMSE}^* = \frac{1}{M\times N}\sum_{i=1}^{M}\sum_{j=1}^{N}\left[\frac{2^{n-2}+1}{2}\right]^2 = \left[\frac{2^{n-2}+1}{2}\right]^2 \rightarrow (6.5)$$

Notice that the $\text{WMSE}=(2^{n-1})^2$ by the direct replacement by the simple LSB_n and MSB_n substitution method are constant of embedded $\text{error}=2^{n-1}$. But with proposed method ($\text{APAP-MPOEE-FOBCB}_{MSB6}$) the max-worst mean square error $\text{Max.WMSE}^*=(2^{n-2})^2$, $\text{Min.WMSE}^*=1$, and the average worst mean square error are obtained in Eq.(6.5). Moreover the embedding errors are minimized to the half (½) from the max-embedding errors Ω in the watermarked image as shown in Fig.16. Let's WMSE and Max.WMSE^* be the worst mean square error between watermarked image and cover image are obtained the $\text{WMSE}=(2^{n-1})^2$ by the direct replacement of simple LSB_n or MSB_n substitution and the Max.WMSE^* by proposed method ($\text{APAP-MPOEE-FOBCB}_{MSB6}$). Theoretically, by combining WMSE and Max.WMSE^*, we have

$$\frac{Max.WMSE^*}{WMSE} = \frac{(2^{n-2})^2}{(2^{n-1})^2} \quad (6.6)$$

$$\text{Max.}WMSE^* = \frac{Max.WMSE^*}{WMSE}WMSE \quad (6.7)$$

$$Max.WMSE^* = \frac{(2^{n-2})^2}{(2^{n-1})^2}WMSE \quad (6.8)$$

From Eq.(6.8) and when $n=6$ reveals that the $\text{Max.WMSE}^* = \frac{1}{4}WMSE$, $\text{averg.WMSE}^* = 0.0705WMSE$ and the $\text{Min.WMSE}^* = \frac{1}{1024}WMSE$, this result of our analyzed shows that the average of embedding errors in Eq.(6.4) = 8.5 and WMSE^* obtained by proposed scheme are proved efficient and better than obtained by the previous methods and modified algorithms.

7. Performance Results

The experimental results have been computed and applying to measure the performance result by comparative study between the previous methods, modified algorithms and proposed method (APAP-MPOEEMSB6) in two parts (i)- Theoretically analysis, and (ii)- Applied on the different benchmark of six-test-images and two quantum of watermark bit embedded, to study the performance of enhancement grey scale image quality means (fidelity), imperceptibility, capacity and robustness under of the mechanism different image attacks.

7.1 The experimental result computed Theoretically:

Experimental results have been computed theoretically. Suppose that all the pixels in the cover image are used for the embedding of watermark bit by the list of substitutions of previous methods, modified algorithms and proposed method (APAP-MPOEE_{MSB6}), theoretically, have been measure in the Max, Min and *the average number of embedding errors Ω, worst mean square error WMSE, $WMSE^*$ and worst peak signal to noise ratio ($PSNR_{worst}$)* between the cover image $F_{(i,j)}$ and watermarked image $f_{(i,j)}$. Have been seen that the Table.(1 to 4) tabulates the comparisons results. Furthermore, in the worst number of embedding errors Ω, there are only 256 possible pixel values for eight-bit gray scale images. Theoretically, in the Table.1 tabulates the number of embedding errors Ω for some of n =1 to eight-bit of gray scale images in (LSB_n and MSB_n). It could be seen that the image quality of the watermarked image is degraded drastically when ***n*** growth up one by one. In this letter, the number of embedding errors Ω in proposed method set of $MSB_6 = 16_{Max}$, 8.5_{avrg} and 1_{Min} are lowest than with compared of the list substitutions of previous methods and modified algorithms.

Table.1 The number of embedding errors are compared between the proposed method, previous methods, and modified algorithms.

Comparison between Methods of embedded watermark bits:	Level	The number of embedding errors, $1 < n \le 8$, $P_{(i,j)} \neq p'_{(i,j)}$	There are only 256 possible pixel values for eight-bit gray scale images. *Theoretically*, the number of embedding errors							
			LSB_1	LSB_2	LSB_3	LSB_4	MSB_5	MSB_6	MSB_7	MSB_8
Simple LSB or MSB substitution method	Constant	2^{n-1}	1	2	4	8	16	32	64	128
Method of Wang-Lin-Lin scheme & PAP-algorithm-1 set of MSB_6	Max	2^{n-1}	1	2	4	8	16	32	64	128
	Average	$\frac{2(2^{n-1} - 2^{n-3} + 1) - 1}{2}$	1	2	3.5	6.5	12.5	24.5	48.5	96.5
	Min	$2^{n-2} + 1$	1	2	3	5	9	17	33	65
Simple k-LSB substitution method using 'k' capacity of embedded watermark bits	Constant	$2^k - 1$	1	3	7	15	31	63	127	255
Method of Chi-Kwong-L.M.Cheng scheme, using 'k' capacity of embedded watermark bits in k-LSB	Max	2^{k-1}	1	2	4	8	16	32	64	128
	Average	$\frac{2^{k-1} + 1}{2}$	1	1.5	2.5	4.5	8.5	16.5	32.5	64.5
	Min	1	1	1	1	1	1	1	1	1
PAP-algorithm-2 set of MSB_6	Constant	2^{n-1}	1	2	4	8	16	32	64	128
Method of Aiad and Abdul scheme & PAP-algorithm-3 set of MSB_6	Max	2^{n-1}	1	2	4	8	16	32	64	128
	Average	$\frac{2^{n-1} + 1}{2}$	1	1.5	2.5	4.5	8.5	16.5	32.5	64.5
	Min	1	1	1	1	1	1	1	1	1
Proposed method	Max	2^{n-2}	1	1	2	4	8	16	32	64
	Average	$\frac{2^{n-2} + 1}{2}$	1	1	1.5	2.5	4.5	8.5	16.5	32.5
	Min	1	1	1	1	1	1	1	1	1

Moreover in the proposed method, the embedding errors are minimized to the half ½ from the maximum embedding errors $\Omega=2^{n-1}=32$ in the watermarked image as shown in Table.1 theoretically.

Table.2: Theoretically the WMSE and WMSE* with comparisons.

Comparison between Methods of embedded watermark bits:	Level	The worst mean square error of embedding watermark bits ($WMSE^*$), when $1 < n \le 8$ & $P_{(i,j)} \neq p'_{(i,j)}$	Suppose that all the pixels in the cover image are used for the embedded watermark bits by the list of substitution methods. *Theoretically*, the worst mean square error are:							
			LSB_1	LSB_2	LSB_3	LSB_4	MSB_5	MSB_6	MSB_7	MSB_8
Simple LSB or MSB substitution method	Constant	$(2^{n-1})^2$	1	4	16	64	256	1024	4096	16384
Method of Wang-Lin-Lin scheme & PAP-algorithm-1 set of MSB_6	Max	$(2^{n-1})^2$	1	4	16	64	256	1024	4096	16384
	Average	$\left[\frac{2(2^{n-1} - 2^{n-3} + 1) - 1}{2}\right]^2$	1	4	12.25	42.25	156.25	600.25	2352.25	9312.25
	Min	$(2^{n-2} + 1)^2$	1	4	9	25	81	289	1089	4225
Simple k-LSB substitution method using 'k' capacity of embedded watermark bits	Constant	$(2^k - 1)^2$	1	9	49	225	961	3969	16129	65025
Method of Chi-Kwong-L.M.Cheng scheme, using 'k' capacity of embedded watermark bits in k-LSB	Max	$(2^{k-1})^2$	1	4	16	64	256	1024	4096	16384
	Average	$\left[\frac{2^{k-1} + 1}{2}\right]^2$	1	2.25	6.25	20.25	72.25	272.25	1056.25	4160.25
	Min	1	1	1	1	1	1	1	1	1
PAP-algorithm-2 set of MSB_6	Constant	$(2^{n-1})^2$	1	4	16	64	256	1024	4096	16384
Method of Aiad and Abdul scheme & PAP-algorithm-3 set of MSB_6	Max	$(2^{n-1})^2$	1	4	16	64	256	1024	4096	16384
	Average	$\left[\frac{2^{n-1} + 1}{2}\right]^2$	1	2.25	6.25	20.25	72.25	272.25	1056.25	4160.25
	Min	1	1	1	1	1	1	1	1	1
Proposed method	Max	$(2^{n-2})^2$	1	1	4	16	64	256	1024	4096
	Average	$\left[\frac{2^{n-2} + 1}{2}\right]^2$	1	1	2.25	6.25	20.25	72.25	272.25	1056.25
	Min	1	1	1	1	1	1	1	1	1

However the worst mean square error WMSE and $WMSE^*$, theoretically calculated in the Table.2 tabulates the WMSE for all eight-bit of gray scale images in (LSB_n and MSB_n). It could be seen that the image quality of thewatermarked image is degraded drastically when growth depth in the MSB_n one by one. In this letter the $WMSE^*$ in proposed method set of $MSB_6=256_{Max}$, 72.25_{avrg} and 1_{Min} are lowest with compared of the list substitutions of previous methods. On the other hand, let the worst mean square error WMSE obtained by simple LSB_n or MSB_n substitution method equal = $(2^{n-1})^2$ and the simple k-LSB substitution method using 'k' capacity of embedded watermark bits=$(2^k - 1)^2$ as shown in Table.2. Furthermore the $WMSE^*$ obtained by the list substitutions of previous methods and proposed method, it is be the worst mean square error between the watermarked image and cover image as shown in Table.2. Theoretically are computed in the Table.3, tabulates the combining between the WMSE and $WMSE^*$ for all eight-bit of gray scale images in (LSB_n and MSB_n), reveals that $WMSE^* = \frac{WMSE^*}{WMSE} WMSE$. It could be seen that in the MSB6 the combining are reveals that the $Max.WMSE^* = \frac{1}{4} WMSE$ $averg.WMSE^* = 0.0705\, WMSE$ and the $MinWMSE^* = \frac{1}{1024} WMSE$, obtained by the proposed method is better than that obtained by the list substitutions of previous methods and modified algorithms as shown in Table.3. ***Finally***, the Table.4 tabulates the worst case of $PSNR_{worst(dB)}$ for each LSB_n and MSB_n of gray scale image.

Table.3: The combining WMSE within WMSE* and comparisons.

Comparison between Methods of embedded watermark bits:	Level	The worst mean square error of embedding watermark bits ($WMSE^*$), when $1<n\leq 8$ & $P_{(i,j)} \neq p'_{(i,j)}$	WMSE= $(2^{n-1})^2$ or $(2^k-1)^2$, $WMSE^* = \frac{WMSE^*}{WMSE} WMSE$ Theoretically the $WMSE^*$:-								
			LSB_1	LSB_2	LSB_3	LSB_4	MSB_5	MSB_6	MSB_7	MSB_8	
Method of Wang-Lin-Lin scheme & PAP-algorithm-1 set of MSB_6	Max	$(2^{n-1})^2$	WMSE	WMSE	WMSE	WMSE	WMSE	WMSE	WMSE	WMSE	
	Average	$\left[\frac{2(2^{n-1}-2^{n-3}+1)-1}{2}\right]^2$	1	1	0.7656	0.6601	0.6103	0.5861	0.5742	0.5683	× WMSE
	Min	$(2^{n-2}+1)^2$	1	1	0.5625	0.3906	0.3164	0.2822	0.2658	0.2578	
Method of Chi-Kwong-L.M.Cheng scheme, using 'k' capacity of embedded watermark bits in k-LSB	Max	$(2^{k-1})^2$	1	0.4444	0.3265	0.2844	0.2663	0.2579	0.2539	0.2519	
	Average	$\left[\frac{2^{k-1}+1}{2}\right]^2$	1	0.25	0.1275	0.09	0.0751	0.0685	0.0654	0.0639	
	Min	1	1	0.1111	0.0204	0.0044	0.0010	0.0002	6.20E-05	1.53E-05	
PAP-algorithm-2 set of MSB_6	Constant	$(2^{n-1})^2$	WMSE	WMSE	WMSE	WMSE	WMSE	WMSE	WMSE	WMSE	
Method of Aiad and Abdul scheme & PAP-algorithm-3 set of MSB_6	Max	$(2^{n-1})^2$	WMSE	WMSE	WMSE	WMSE	WMSE	WMSE	WMSE	WMSE	
	Average	$\left[\frac{2^{n-1}+1}{2}\right]^2$	1	0.5625	0.3906	0.3164	0.2822	0.2658	0.2578	0.2539	× WMSE
	Min	1	1	0.25	0.0625	0.0156	0.0039	0.0009	0.0002	6.10E-05	
Proposed method	Max	$(2^{n-2})^2$	1	0.25	0.25	0.25	0.25	0.25	0.25	0.25	
	Average	$\left[\frac{2^{n-2}+1}{2}\right]^2$	1	0.25	0.1406	0.0976	0.0791	0.0705	0.0664	0.0644	
	Min	1	1	0.25	0.0625	0.0156	0.0039	0.0009	0.0002	6.10E-05	

It could be seen that the image quality of the watermarked image is degraded drastically when n growth one by one initial to the eight-bit.

Hence that theoretically the worst case of $PSNR_{worst}$ can be computed by Eq.(4.7) as:

$$PSNR_{worst} = 10 \times \log_{10} \frac{255^2}{WMSE} \quad dB$$

and

$$PSNR_{worst} = 10 \times \log_{10} \frac{255^2}{WMSE^*} \quad dB$$

Table.4 The worst cases of PSNRworst and comparison.

Comparison between Methods of embedded watermark bits:	Level	The worst mean square error of embedding watermark bits ($WMSE^*$), when $1<n\leq 8$ & $P_{(i,j)} \neq p'_{(i,j)}$	Suppose that all the pixels in the cover image are used for the embedded watermark bits by the list of substitution methods. *Theoretically*, in the worst PSNR (dB) are:							
			LSB_1	LSB_2	LSB_3	LSB_4	MSB_5	MSB_6	MSB_7	MSB_8
Simple LSB or MSB substitution method	Constant	$(2^{n-1})^2$	48.130	42.110	36.089	30.069	24.048	18.027	12.007	5.986
Method of Wang-Lin-Lin scheme & PAP-algorithm-1 set of MSB_6	Max	$(2^{n-1})^2$	48.130	42.110	36.089	30.069	24.048	18.027	12.007	5.9866
	Average	$\left[\frac{2(2^{n-1}-2^{n-3}+1)-1}{2}\right]^2$	48.130	42.110	37.249	31.872	26.192	20.347	14.415	8.4402
	Min	$(2^{n-2}+1)^2$	48.130	42.110	38.588	34.151	29.045	23.521	17.760	11.872
Simple k-LSB substitution method using 'k' capacity of embedded watermark bits	Constant	$(2^k-1)^2$	48.130	38.588	31.228	24.608	18.303	12.143	6.054	0
Method of Chi-Kwong-L.M.Cheng scheme, using 'k' capacity of embedded watermark bits in k-LSB	Max	$(2^{k-1})^2$	48.130	42.110	36.089	30.069	24.048	18.027	12.0072	5.9866
	Average	$\left[\frac{2^{k-1}+1}{2}\right]^2$	48.130	44.608	40.172	35.066	29.542	23.781	17.893	11.939
	Min	1	48.130	48.130	48.130	48.130	48.130	48.130	48.130	48.130
PAP-algorithm-2 set of MSB_6	Constant	$(2^{n-1})^2$	48.130	42.110	36.089	30.069	24.048	18.027	12.0072	5.9866
Method of Aiad and Abdul scheme & PAP-algorithm-3 set of MSB_6	Max	$(2^{n-1})^2$	48.130	42.110	36.089	30.069	24.048	18.027	12.0072	5.9866
	Average	$\left[\frac{2^{n-1}+1}{2}\right]^2$	48.130	44.608	40.172	35.066	29.542	23.781	17.893	11.939
	Min	1	48.130	48.130	48.130	48.130	48.130	48.130	48.130	48.130
Proposed method	Max	$(2^{n-2})^2$	48.130	48.130	42.110	36.089	30.069	24.048	18.027	12.007
	Average	$\left[\frac{2^{n-2}+1}{2}\right]^2$	48.130	48.130	44.608	40.172	35.066	29.542	23.7811	17.8931
	Min	1	48.130	48.130	48.130	48.130	48.130	48.130	48.130	48.130

In the Table.4 shown the proposed method set-of-MSB_6=$(24.048_{(dB)})_{Max}$, $(29.542_{(dB)})_{avg}$, and $(48.130_{(dB)})_{Min}$ are higher than with compared of the list substitutions of previous methods and modified algorithm. Where the $PSNR_{worst}$ are obtained by the proposed method is better than that obtained by the list substitutions of previous methods and modified algorithm as shown in Table.4. Furthermore, the proposed method of an adaptively pixel adjustment process based on medial pyramid of embedding error set of the Most-Significant-Bit-n ($APAP\text{-}MPOEE_{MSB6}$), the algorithm is requires a checking between the MSB_6 in the pixel of cover image and EMB of the embedded watermark bit before embedding the watermark bits depending on the nearest of adaptively pixel in the medial pyramid of embedding error to inform the watermarked image "$P_{(i,j)}$. Thus theoretically are proved efficient and better than obtained by the previous methods and modified algorithms.

7.2 The experimental result applied on different benchmark:

The experimental results have been applied on different benchmark six-test-images (Lena, Boat, Baboon, jet, Birds and Pills) to study the performance of enhancement grey scale image quality (fidelity), imperceptibility, capacity and robustness under of the mechanism different image attacks. In order to compare the performance results of the proposed a novel method APAP-MPOEE set-of-the MSB_6 by applying falling-off-boundary in corners board of cover image with random pixel manipulation in spatial domain ($APAP\text{-}MPOEE\text{-}FOBCB_{MSB6}$), with the state-of-the-art-algorithms are required, Kevin Curran-Xuelong Li and Roisin Clarke[1] proposed an investigation into the use LSB substitution in digital watermarking by simple MSB_6 substitution, Ran Zan Wang, Chi-Fang Lin and Ja-Chen Lin[20] proposed hiding data in images by optimal moderately significant bit replacement used LSB_4, Chi-Kwong Chan and L.M.Cheng[2] proposed hiding data in images by simple LSB substitution using k-right most LSBs substitution by applying OPAP, Aiad Ibraheem-Abdul Sada[3] proposed hiding data using LSB_3 by applying LPAP and our method of robust digital watermarking based falling-off-boundary in corners board gray scale images[9], by the way will be compare the modified algorithms of previous methods (PAP-algorithm-1, PAP-algorithm-2 and PAP-algorithm-3) by applying the falling-off-boundary in corners board of cover image set-of-the MSB_6 with random pixel manipulation in spatial domain with proposed method. A set of standard six-test grey scale images (Lena, Boat, Baboon, jet, Birds and Pills) 512×512 gray scale level images has been used as a cover images as shown in Table.5 and two quantum of watermark bit embedded, by using a different size of binary watermark image 45×45 and 16×16 as shown in Fig.17, are used to tested the performance of capacity, by insertion with redundantly distributed the watermark bits over many pixels of the cover image, by using a small binary watermark image 16×16, are added simultaneously in the falling-off-boundary in corners board pixel in the cover image as a capacity data bits to increase watermark intensity as a power of the embedded watermark, this procedure maybe increasing the robustness, as well as the increasing the watermark intensity will be cause the cost of the degradation of watermarked image. However the max-bits can be embedded 2048bits in the cover image.

(a)-Watermark (16×16) (b)-Watermark (45×45)

Fig.17: The binary watermark image.

7.2.1Imperceptibility:

To measure the performance and compare between the state-of-the-art algorithms are required[1][2][3][9][20] and modified algorithms of previous methods under computed the average of performance results. We evaluated imperceptibility to sense the degree of distortion resulting from pixel value changes in watermarked image f(M,N) by the most popular difference distortion measures of pixel based metrics. These measures metrics are all based on the difference between the cover F(M,N), and watermarked image:

7.2.1.1 Average absolute difference: AD is used to measure the variation of the embedding errors occurred by the embedded watermark bits between the cover images and watermarked image can be computed by Eq.(4.1) as shown the results in Table.5. Whereas the AD of the proposed method is equal $(0.032777)_{average}$, where the proposed method are lowest than with compared between previous methods and modified algorithms. Since that in the modified algorithm (PAP-algorithm-3), the AD are equal $(0.068829)_{average}$, where are lower with compared of the previous methods. Thus the AD in the other methods of previous methods and modified algorithms are higher.

Table 5: The performance results of AD with comparisons.

Comparisons between state-of-the-art algorithms:	AD have been applied on different benchmark test images					
	Lena	Boat	Baboon	jet	Birds	Pills
Method of Wang-Lin-Lin scheme	1.168892	1.137920	1.142609	1.151752	1.136826	1.136818
Method of OPAP K-LSB$_1$, k=1	0.205574	0.205532	0.205383	0.205204	0.204319	0.204983
Method of OPAP k-LSB$_{1,2}$, k=2	0.290031	0.290611	0.289585	0.289799	0.290260	0.289192
Method of OPAP k-LSB$_{1,2,3}$, k=3	0.577080	0.581444	0.575672	0.579075	0.581032	0.572990
Method of OPAP k-LSB$_{1,2,3,4}$, k=4	1.169395	1.125023	1.144703	1.157658	1.149666	1.148590
Method of Aiad and Abdul scheme	1.250572	1.247662	1.256798	1.263229	1.240578	1.255844
Method of an investigation into simple MSB$_6$ substitution	16.025024	16.151733	15.819946	16.538330	16.013550	16.237183
Method, Filling-of-boundary in corners board set-of-MSB$_6$	0.119873	0.121826	0.127075	0.122559	0.121948	0.125122
Applying method of optimal moderately by FOBICB-set-of-MSB$_6$ (PAP-algorithm-1)	0.109798	0.111042	0.106998	0.094780	0.090996	0.098965
Applying method of hiding data using FOBICB set of MSB$_6$ with modified LSB$_{1,2,3,4}$ and MSB$_6$. (PAP-algorithm-3)	0.061203	0.072075	0.068748	0.063946	0.076675	0.070324
Applying method of OPAP algorithm by FOBICB-set-of-MSB$_6$. (PAP-algorithm-2)	0.119873	0.121826	0.127075	0.122559	0.121948	0.125122
Proposed method of APAP-MPOEE applying Filling-of-boundary in corners board set-of-MSB$_6$	0.029156	0.036804	0.033241	0.030457	0.034088	0.032917

7.2.1.2 Normalized average absolute difference: NAD is used to measure the variation of changes the range of pixel intensity values between the cover image and watermarking image can be computed by Eq.(4.2) as shown the results in Table.6. Whereas in the proposed method the NAD=$(0.000253)_{average}$, are the lowest than with compared of the list substitutions of previous methods and modified algorithms. Moreover the NAD of modified PAP-algorithm-3 is $(0.000532)_{average}$, where the PAP-algorithm-3 is better than with compared of the other modified algorithms and previous methods.

Table 6: The performance results of NAD with comparisons

Comparisons between state-of-the-art algorithms:	NAD have been applied on different benchmark test images					
	Lena	Boat	Baboon	jet	Birds	Pills
Method of Wang-Lin-Lin scheme	0.009358	0.008795	0.008896	0.006500	0.010368	0.008925
Method of OPAP K-LSB$_1$, k=1	0.001646	0.001589	0.001599	0.001158	0.001863	0.001609
Method of OPAP k-LSB$_{1,2}$, k=2	0.002322	0.002246	0.002255	0.001636	0.002647	0.002270
Method of OPAP k-LSB$_{1,2,3}$, k=3	0.004620	0.004494	0.004482	0.003268	0.005299	0.004499
Method of OPAP k-LSB$_{1,2,3,4}$, k=4	0.009362	0.008696	0.008912	0.006534	0.010486	0.009018
Method of Aiad and Abdul scheme	0.010012	0.009643	0.009785	0.007130	0.011315	0.009860
Method of an investigation into simple MSB$_6$ substitution	0.128299	0.124840	0.123165	0.093341	0.146052	0.127481
Method, Filling-of-boundary in corners board set-of-MSB$_6$	0.000960	0.000942	0.000989	0.000692	0.001112	0.000982
Applying method of optimal moderately by FOBICB-set-of-MSB$_6$ (PAP-algorithm-1)	0.000879	0.000858	0.000833	0.000535	0.000830	0.000777
Applying method of hiding data using FOBICB set of MSB$_6$ with modified LSB$_{1,2,3,4}$ and MSB$_6$. (PAP-algorithm-3)	0.000490	0.000557	0.000535	0.000361	0.000699	0.000552
Applying method of OPAP algorithm by FOBICB-set-of-MSB$_6$. (PAP-algorithm-2)	0.000960	0.000942	0.000989	0.000693	0.001112	0.000982
Proposed method of APAP-MPOEE applying Filling-of-boundary in corners board set-of-MSB$_6$	0.000233	0.000284	0.000259	0.000172	0.000311	0.000258

7.2.1.3 Mean square error: The MSE is the cumulative squared error between the watermarked image $f_{(M,N)}$ and the cover image $F_{(M,N)}$ can be computed by Eq.(4.3). The Table.7 tabulates the MSE, whereas a lower value for MSE is in proposed method equal $(0.358758)_{average}$, where are lesser error with compared between previous methods and modified algorithms. Moreover the MSE of modified PAP-algorithm-3 is equal $(1.551337)_{average}$, where the PAP-algorithm-3 is better than with compared of the other modified algorithms. Moreover the lower value of MSE, means that the lower the error.

Table 7: The performance results of MSE with comparisons.

Comparisons between state-of-the-art algorithms:	MSE have been applied on different benchmark test images					
	Lena	Boat	Baboon	jet	Birds	Pills
Method of Wang-Lin-Lin scheme	8.327126	8.021168	8.089958	8.163006	8.077568	8.037651
Method of OPAP K-LSB$_1$, k=1	0.458771	0.458591	0.457703	0.457867	0.454464	0.456493
Method of OPAP k-LSB$_{1,2}$, k=2	1.385742	1.390625	1.381290	1.387402	1.385101	1.377754
Method of OPAP k-LSB$_{1,2,3}$, k=3	5.750397	5.797272	5.735722	5.796764	5.802826	5.676796
Method of OPAP k-LSB$_{1,2,3,4}$, k=4	23.897247	22.528656	23.321171	23.569286	23.340851	23.173866
Method of Aiad and Abdul scheme	3.758286	3.740513	3.780121	3.822701	3.726509	3.771149
Method of an investigation into simple MSB$_6$ substitution	512.800781	516.855469	506.238281	529.226563	512.433594	519.589844
Method, Filling-of-boundary in corners board set-of-MSB$_6$	3.835938	3.898438	4.066406	3.921875	3.902344	4.003906
Applying method of optimal moderately by FOBICB-set-of-MSB$_6$ (PAP-algorithm-1)	3.248180	3.270924	2.987247	2.434158	2.262512	2.617588
Applying method of hiding data using FOBICB set of MSB$_6$ with modified LSB$_{1,2,3,4}$ and MSB$_6$. (PAP-algorithm-3)	1.361740	1.584038	1.538620	1.429089	1.781166	1.613369
Applying method of OPAP algorithm by FOBICB-set-of-MSB$_6$. (PAP-algorithm-2)	3.835938	3.898438	4.066406	3.921875	3.902344	4.003906
Proposed method of APAP-MPOEE applying Filling-of-boundary in corners board set-of-MSB$_6$	0.304180	0.420105	0.366882	0.323944	0.375168	0.362270

7.2.1.4 Normalized mean square error: We know that the distance measure is normalized to a range between 0 and 1. It is independent of the range of gray scale values in the images can be computed by Eq.(4.4). The Table.8 tabulates the NMSE, whereas the proposed method are get a great-ideal of NMSE equal $(0.00001)_{average}$, with are compared of the list substitutions of the previous methods and modified algorithms. On the other hand the same case in the modified algorithms (PAP-algorith-3) equal $(0.000083)_{average}$, where the proposed method is better than compared with each other methods. Here the NMSE of proposed method suggests the images are very similar in spatial layout and gray scale values.

Table 8: The performance results of NMSE with comparisons.

Comparisons between state-of-the-art algorithms:	NMSE have been applied on different benchmark test images					
	Lena	Boat	Baboon	jet	Birds	Pills
Method of Wang-Lin-Lin scheme	0.000475	0.000428	0.000458	0.000245	0.000574	0.000413
Method of OPAP K-LSB$_1$, k=1	0.000026	0.000024	0.000026	0.000014	0.000032	0.000023
Method of OPAP k-LSB$_{1,2}$, k=2	0.000079	0.000074	0.000078	0.000042	0.000098	0.000071
Method of OPAP k-LSB$_{1,2,3}$, k=3	0.000328	0.000309	0.000325	0.000174	0.000413	0.000292
Method of OPAP k-LSB$_{1,2,3,4}$, k=4	0.001364	0.001202	0.001320	0.000707	0.001660	0.001191
Method of Aiad and Abdul scheme	0.000214	0.000200	0.000214	0.000115	0.000265	0.000194
Method of an investigation into simple MSB$_6$ substitution	0.029260	0.027574	0.028653	0.015884	0.036436	0.026711
Method, Filling-of-boundary in corners board set-of-MSB$_6$	0.000219	0.000208	0.000230	0.000118	0.000277	0.000206
Applying method of optimal moderately by FOBICB-set-of-MSB$_6$ (PAP-algorithm-1)	0.000185	0.000175	0.000169	0.000073	0.000161	0.000135
Applying method of hiding data using FOBICB set of MSB$_6$ with modified LSB$_{1,2,3,4}$ and MSB$_5$. (PAP-algorithm-3)	0.000078	0.000085	0.000087	0.000043	0.000127	0.000083
Applying method of OPAP algorithm by FOBICB-set-of-MSB$_6$. (PAP-algorithm-2)	0.000219	0.000208	0.000230	0.000118	0.000277	0.000206
Proposed method of APAP-MPOEE applying Filling-of-boundary in corners board set-of-MSB$_6$	0.000017	0.000022	0.000021	0.000010	0.000027	0.000019

7.2.1.5 Signal to noise ratio: We know that the higher ratio means that the less obtrusive of the embedding errors in the watermarked image. The SNR can be computed by Eq.(4.6). Thus the Table.9 tabulates the SNR; the proposed method is getting a higher of SNR equal $(47.34791367\ _{dB})_{average}$, with compared of the list substitutions of the previous methods and modified algorithms. Moreover the SNR is a technical term used to characterize the quality of the watermarked image. On the other hand have been see that the modified algorithm (PAP-algorith-3) are get SNR approximately equal $(40.98167283_{dB})_{average}$, where the proposed method is better than compared with each other methods. Here the SNR of proposed method suggests the *image fidelity (IF)* is very similar in spatial layout and gray scale values with the lowest distortion in gray scale image. Thus this is the prove imperceptibility and *image fidelity* of our proposed method, where are great higher SNR.

Table.9: The performance results of SNR with comparisons

Comparisons between state-of-the-art algorithms:	SNR have been applied on different benchmark test images					
	Lena	Boat	Baboon	jet	Birds	Pills
Method of Wang-Lin-Lin scheme	33.231720	33.686246	33.392360	36.108440	32.408291	33.838476
Method of OPAP K-LSB$_1$, k=1	45.820713	46.114363	45.865988	48.619551	44.906103	46.295423
Method of OPAP k-LSB$_{1,2}$, k=2	41.019846	41.296522	41.068973	43.804919	40.066282	41.498050
Method of OPAP k-LSB$_{1,2,3}$, k=3	34.839693	35.096386	34.885942	37.595085	33.844702	35.348735
Method of OPAP k-LSB$_{1,2,3,4}$, k=4	28.653192	29.201270	28.794319	31.503477	27.799930	29.239783
Method of Aiad and Abdul scheme	36.686773	36.999311	36.696766	39.403238	35.768075	37.125031
Method of an investigation into simple MSB$_6$ substitution	15.337184	15.594931	15.428273	17.990525	14.384721	15.733161
Method, Filling-of-boundary in corners board set-of-MSB$_6$	36.597955	36.819717	36.379715	39.292004	35.567842	36.864929
Applying method of optimal moderately by FOBICB-set-of-MSB$_6$ (PAP-algorithm-1)	37.320269	37.581918	37.719111	41.363453	37.935188	38.710755
Applying method of hiding data using FOBICB set of MSB$_6$ with modified LSB$_{1,2,3,4}$ and MSB$_5$. (PAP-algorithm-3)	41.095728	40.730967	40.600509	43.676349	38.974053	40.812431
Applying method of OPAP algorithm by FOBICB-set-of-MSB$_6$. (PAP-algorithm-2)	36.597955	36.819717	36.379715	39.292004	35.567842	36.864929
Proposed method of APAP-MPOEE applying Filling-of-boundary in corners board set-of-MSB$_6$	47.605362	46.495044	46.826555	50.122240	45.738841	47.299440

7.2.1.6 Peak signal to noise ratio: We know that the typical PSNR values range between 20 and 40 *dB*, where higher is better for quality image and The PSNR can be computed by Eq.(4.8) as shown the results in the Table.10 tabulates the PSNR, here in the modified algorithm (PAP-algorithm-3), the PSNR are equal $(46.2399895_{dB})_{average}$, where are higher than with compared of the previous methods and modified algorithm. But in the proposed method is getting a higher of PSNR equal $(52.6062305_{dB})_{average}$, with compared of the list substitutions of the previous methods and modified algorithms. Thus where the quality degradations could hardly be perceived by human eye, then the PSNR of proposed method suggests the high quality image and *image fidelity (IF)*, where are very similar closeness in spatial layout of gray scale values between cove image and watermarked image. So that this is the prove imperceptibility and *image fidelity* for our proposed method, where are greatens higher PSNR.

Table.10: The performance results of PSNR with comparisons.

Comparisons between state-of-the-art algorithms:	PSNR (dB) have been applied on different benchmark test images					
	Lena	Boat	Baboon	jet	Birds	Pills
Method of Wang-Lin-Lin scheme	38.925852	39.088428	39.051341	39.012303	39.057997	39.079512
Method of OPAP K-LSB$_1$, k=1	51.402418	51.516544	51.524969	51.523413	51.555809	51.536459
Method of OPAP k-LSB$_{1,2}$, k=2	46.703865	46.698703	46.727954	46.708782	46.715988	46.739086
Method of OPAP k-LSB$_{1,2,3}$, k=3	40.690663	40.498567	40.544923	40.498947	40.494408	40.589771
Method of OPAP k-LSB$_{1,2,3,4}$, k=4	34.527257	34.603451	34.453300	34.407339	34.449637	34.480819
Method of Aiad and Abdul scheme	42.290906	42.401492	42.355747	42.307101	42.417782	42.366067
Method of an investigation into simple MSB$_6$ substitution	21.031317	20.997112	21.087254	20.894387	21.034428	20.974197
Method, Filling-of-boundary in corners board set-of-MSB$_6$	42.292088	42.221898	42.038696	42.195866	42.217548	42.105965
Applying method of optimal moderately by FOBICB-set-of-MSB$_6$ (PAP-algorithm-1)	43.014402	42.984100	43.378092	44.267315	44.584894	43.951791
Applying method of hiding data using FOBICB set of MSB$_6$ with modified LSB$_{1,2,3,4}$ and MSB$_5$. (PAP-algorithm-3)	46.789861	46.133148	46.259490	46.580212	45.623759	46.053467
Applying method of OPAP algorithm by FOBICB-set-of-MSB$_6$. (PAP-algorithm-2)	42.292088	42.221898	42.038696	42.195866	42.217548	42.105965
Proposed method of APAP-MPOEE applying Filling-of-boundary in corners board set-of-MSB$_6$	53.299495	51.897225	52.485536	53.026103	52.388548	52.540476

7.2.2 Image fidelity:

Have been see that the Table.11 tabulates the results of Image fidelity (IF) are computed by Eq.(4.9). Here in the modified algorithm (PAP-algorithm-3), the Image fidelity are equal $(0.999916167)_{average}$, where are lower than with compared of the previous methods and modified algorithm. But on the other hand the image fidelity of the proposed method are getting ideal value equal $(0.999980667)_{average}$, with compared of the list substitutions of the previous methods and modified algorithms. Thus where the quality degradations could hardly be perceived by human eye, then the image fidelity of proposed method suggests a high quality of watermarked image and higher imperceptibility. Where are very similar in spatial layout of gray scale values between cove image and watermarked image. Furthermore the image fidelity measure is also commonly referred to as an image metric, the traditional image fidelity standard are the mean squared-error (MSE), signal-to-noise ratio (SNR), and the peak signal-to-noise ratio (PSNR) for cover image with watermarked image. **From the above Table.(6 to 10) in proposed technique** measuring, **First**: The measuring value of NAD equal $(0.000253)_{average}$, are signifies lesser error in the watermarked image. **Second:** The MSE has been obtained a lowest average of the squared difference between the intensity of the cover image and watermarked image at each pixel location equal $(0.358758)_{average}$, with ideal value in the NMSE equal $(0.00001)_{average}$,. **Third:** the SNR getting a higher ratio equal $(47.34791367_{dB})_{average}$, means that the lesser obtrusive of the embedding errors (called noise) in the watermarked image.

Table.11: The performance results of image fidelity with comparisons.

Comparisons between state-of-the-art algorithms:	Image fidelity have been applied on different benchmark test images					
	Lena	Boat	Baboon	jet	Birds	Pills
Method of Wang-Lin-Lin scheme	0.999525	0.999572	0.999542	0.999755	0.999426	0.999587
Method of OPAP K-LSB$_1$, k=1	0.999974	0.999976	0.999974	0.999986	0.999968	0.999977
Method of OPAP k-LSB$_{1,2}$, k=2	0.999921	0.999926	0.999922	0.999958	0.999902	0.999929
Method of OPAP k-LSB$_{1,2,3}$, k=3	0.999672	0.999691	0.999675	0.999826	0.999587	0.999708
Method of OPAP k-LSB$_{1,2,3,4}$, k=4	0.998636	0.998798	0.998680	0.999293	0.998340	0.998809
Method of Aiad and Abdul scheme	0.999786	0.999800	0.999786	0.999885	0.999735	0.999806
Method of an investigation into simple MSB$_6$ substitution	0.970740	0.972426	0.971347	0.984116	0.963564	0.973289
Method, Filling-of-boundary in corners board set-of-MSB$_6$	0.999781	0.999792	0.999770	0.999882	0.999723	0.999794
Applying method of optimal moderately by FOBICB-set-of-MSB$_6$ (PAP-algorithm-1)	0.999815	0.999825	0.999831	0.999927	0.999839	0.999865
Applying method of hiding data using FOBICB set of MSB$_6$ with modified LSB$_{1,2,3,4}$ and MSB$_6$. (PAP-algorithm-3)	0.999922	0.999915	0.999913	0.999957	0.999873	0.999917
Applying method of OPAP algorithm by FOBICB-set-of-MSB$_6$. (PAP-algorithm-2)	0.999781	0.999792	0.999770	0.999882	0.999723	0.999794
Proposed method of APAP-MPOEE applying Filling-of-boundary in corners board set-of-MSB$_6$	0.999983	0.999978	0.999979	0.999990	0.999973	0.999981

Whereas the SNR is a technical term used to characterize the quality of the watermarked image detection. **Four:** the PSNR is getting a higher of PSNR equal $(52.6062305_{dB})_{average}$,; here a logically a higher value of PSNR is high quality watermarked image. *Thus form our observation the values of (Image fidelity, NAD, MSE, NMSE, SNR, and PSNR)* in proposed method are demonstrates better performance than of the list substitutions of the previous methods and modified algorithms, these are proves of the imperceptibility, higher quality watermarked image means *image fidelity* of our proposed novel method as shown the watermarked image in the Fig.18.(a, b, c, d, e and f) and Fig.18.(g), shown the extracted watermark image using as logo/hidden.

7.2.3 Robustness:

We evaluated robustness of the proposed method (APAP-MPOEE-FOBCB$_{MSB6}$), under major digital signal processing operations (attacks): *watermark degrading attacks*, *watermark removal attacks* and *geometric transformations attacks*, by using different benchmark six-test-images (Lena, Boat, Baboon, jet, Birds and Pills), and under using different size of binary watermark image 45×45 and 16×16 as shown in Fig.17, where are used as a tested the performance of capacity, by insertion with redundantly distributed the watermark bits over many pixels of the cover image, by using a small binary watermark image 16×16, where are added simultaneously in the falling-off-boundary in corners board pixel to improve the capacity and to ensure robustness. They are good representatives of the more general attacks. Thus will be measure the similarity between the original watermark and extracted watermark, after applying attacks by NCC in Eq.(4.10) and SM in in Eq.(4.11), whereas the similarity values NCC and SM of about 0.75 or above is considered acceptable and with computing psnr between the original watermark and extracted watermark.

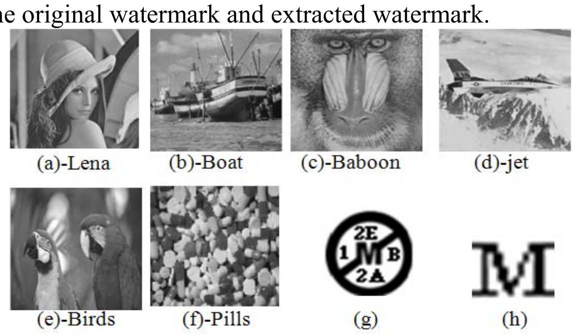

Fig.18: (a, b, c, d, e and f): The watermarked image and (g & h)-The extracted watermark image.

7.2.3.1 The watermark degrading attacks:

For adding the Gaussian noise, Salt & Pepper noise and Speckle noise to the watermarked image, the added noise is a watermark degrading attack, where are add noise can be used as an attacks to remove the watermark. From the performance results are obtained under degrading attacks as shown in the Table.12, comparisons between proposed method (APAP-MPOEE-FOBCB$_{MSB6}$) and FOBCB-$_{MSB6}$ method to study the performance robust results between the constant of the embedding error and proposed method of an adaptively pixel adjustment process based on medial pyramid of embedding error set of the MSB_6.

a)-Effect of Gaussian Noise: In this experiment we add Gaussian noise to the watermarked image the performance results as shown in Table.12. The extracted watermark image comparator by using NCC, SM and *psnr* with measuring the PSNR of watermarked image after attacked. The observations of the proposed method are robust under effect of Gaussian noise attacks applying in the different benchmark test images with different size of watermark image inserted. Thus the results are shown in Table.10 shown the PSNR of watermarked image obtained after added Gaussian noise equal $(47.50548083_{dB})_{average}$, with inserted watermark image 45×45, and equal $(47.439372_{dB})_{average}$, with inserted watermark image 16×16, as seen that by applying proposed method the NCC=$(0.895808)_{average}$, *psnr*=$(8.95452783_{dB})_{average}$, and SM=$(0.90746467)_{average}$, with the extracted watermark image 16×16, and NCC=$(0.8407545)_{average}$, *psnr*=$(8.25548883_{dB})_{average}$, and SM=$(0.8572943)_{average}$, with watermark image 45×45. Results show the proposed method (APAP-MPOEE-$FOBCB_{MSB6}$) deals with Gaussian noise excellently and good robustness. The extracted watermark can maintain a good similarity with the original one even after the watermarked image is adding Gaussian noise and with compared the $FOBCBMSB_6$ method within constant of embedding error.

b)- Effect of Salt & Pepper Noise: In this experiment we add Salt & Pepper noise to the watermarked image the performance results as shown in Table.12, the PSNR of watermarked image obtained after added Salt & Pepper noise equal $(18.4405495_{dB})_{average}$, with inserted watermark image 45×45, and equal $(18.423221167_{dB})_{average}$, with inserted watermark image 16×16, as seen that by applying proposed method of (APAP-MPOEE-$FOBCB_{MSB6}$) the NCC= $(0.988029167)_{average}$, *psnr*=$(19.48169567_{dB})_{average}$, and SM=$(0.9916935)_{average}$, with the extracted watermark image 16×16, and NCC=$(0.97139767)_{average}$, *psnr*= $(15.7933993_{dB})_{average}$, and SM=$(0.974951)_{average}$, with watermark image 45×45. Results show the proposed method (APAP-MPOEE-$FOBCB_{MSB6}$) deals with Salt & Pepper noise excellently and good robustness. The extracted watermark can maintain a higher similarity with the original one even after the watermarked image is adding Salt & Pepper noise with heavily degraded of watermarked image and with compared the $FOBCBMSB_6$ method within constant of embedding error.

Table.12 Performance results under degrading attacks and comparison

Different capacity of W(i,j) inserted	Methods	Different benchmark test images	watermarked image	The watermark degrading attacks:											
				Gussion Noise				Salt & Pepper Noise				Speckle Noise			
			PSNR(dB)	PSNR	NCC	*psnr*	SM	PSNR	NCC	*psnr*	SM	PSNR	NCC	*psnr*	SM
Inserted watermark image size of 45×45	The method FOBCB-MSB6	Lena	42.292088	41.48	0.98	16.7	0.98	18.48	0.973	15.82	0.98	41.66	0.98	17.27	0.982
		Boat	42.221898	41.42	0.98	17.6	0.984	18.57	0.968	15.28	0.97	41.59	0.98	18.01	0.985
		Baboon	42.038696	41.25	0.99	17.5	0.983	18.72	0.975	16.63	0.98	41.43	0.98	17.04	0.981
		Jet	42.195866	41.39	0.98	16.8	0.981	18.03	0.974	15.43	0.97	41.09	0.98	15.99	0.976
		Birds	42.217548	41.41	0.99	18.9	0.988	18.41	0.968	15.51	0.97	41.66	0.99	20.05	0.991
		Pills	42.105965	41.32	0.98	16.9	0.981	18.26	0.984	16.73	0.98	41.44	0.98	18.01	0.985
	Proposed Method APAP-MPOEE by FOBCB-MSB6	Lena	53.299495	47.69	0.86	8.88	0.876	18.54	0.975	16.63	0.98	48.58	0.85	8.915	0.876
		Boat	51.897225	47.28	0.79	8.52	0.86	18.59	0.968	15.28	0.97	48.06	0.75	7.737	0.83
		Baboon	52.485536	47.47	0.84	7.92	0.846	18.73	0.975	16.63	0.98	48.2	0.88	8.123	0.859
		Jet	53.026103	47.64	0.84	7.96	0.849	18.05	0.974	15.43	0.97	46.49	0.81	7.131	0.816
		Birds	52.388548	47.44	0.84	8.37	0.86	18.42	0.968	15.51	0.97	48.55	0.87	9.884	0.901
		Pills	52.540476	47.51	0.88	7.89	0.853	18.31	0.968	15.28	0.97	47.99	0.91	8.881	0.882
Inserted watermark image size of 16×16	The method FOBCB-MSB6	Lena	42.294378	41.19	1	24.1	0.997	18.52	0.994	24.08	1	41.37	0.99	24.08	0.997
		Boat	41.107306	40.47	0.99	24.1	0.997	18.48	0.983	18.06	0.99	40.62	0.99	24.08	0.997
		Baboon	42.305376	41.48	0.99	24.1	0.997	18.72	0.983	18.06	0.99	41.66	0.99	21.07	0.994
		Jet	41.803226	41.08	1	21.1	0.995	18.05	0.994	19.31	0.99	40.78	0.99	19.31	0.992
		Birds	42.195866	41.41	0.99	24.1	0.997	18.48	0.994	19.31	0.99	41.63	0.99	21.07	0.994
		Pills	42.613255	41.74	0.99	21.1	0.994	18.21	0.983	18.06	0.99	41.86	0.99	21.07	0.994
	Proposed Method APAP-MPOEE by FOBCB-MSB6	Lena	53.299495	47.57	0.89	9.17	0.91	18.59	0.989	21.07	0.99	48.46	0.94	10.47	0.936
		Boat	51.897225	46.8	0.82	8.06	0.881	18.5	0.983	18.06	0.99	47.5	0.86	9.031	0.908
		Baboon	52.485536	47.54	0.92	9.31	0.918	18.7	0.989	21.07	0.99	48.31	0.92	7.955	0.89
		Jet	53.026103	47.64	0.9	8.9	0.908	18.07	0.994	19.31	0.99	46.46	0.87	8.285	0.893
		Birds	52.388548	47.35	0.91	8.52	0.901	18.42	0.983	18.06	0.99	48.45	0.91	11.29	0.946
		Pills	52.540476	47.73	0.93	9.77	0.926	18.26	0.989	19.31	0.99	48.24	0.93	10.1	0.931

c)- Effect of Speckle Noise: In this experiment we add Speckle noise to the watermarked image the performance results as shown in Table.12, the PSNR of watermarked image obtained after added Speckle noise equal $(47.97788183_{dB})_{average}$, with inserted watermark image 45×45, and equal $(47.902916167_{dB})_{average}$, with inserted watermark image 16×16, as seen that by applying proposed method of (APAP-MPOEE-$FOBCB_{MSB6}$) the NCC= $(0.9049485)_{average}$, *psnr*= $(9.5221683_{dB})_{average}$, and SM $(0.91736783)_{average}$, with the extracted watermark image 16×16, and NCC= $(0.84341883)_{average}$, *psnr*= $(8.4450285_{dB})_{average}$, and SM= $(0.860574167)_{average}$, with watermark image 45×45. Results show the proposed method (APAP-MPOEE-$FOBCB_{MSB6}$) deals with Speckle noise excellently and good robustness. The extracted watermark can maintain a high similarity with the original one even after the watermarked image is adding Speckle noise and with compared the $FOBCBMSB_6$ method.

7.2.3.2 Geometric transformations attack:

Most attacks will first apply the geometric transformation for e.g. horizontal flip, rotation, cropping, scaling, and JPEG compression then save the images with geometric transformation. It makes sense to test robustness of watermarking system to geometric transformation. From the performance results are obtained under the geometric transformations attacks as shown in the Table.13 comparisons between proposed method (APAP-MPOEE-$FOBCB_{MSB6}$) and FOBCB-$_{MSB6}$ method to study the performance robust results between the constant of the embedding error with the proposed method of an adaptively pixel adjustment process based on medial pyramid of embedding error set of the MSB_6.

Table.13 Performance results under geometric attacks & comparison

Different capacity of W(i,j) inserted	Methods	Different benchmark test images	Geometric transformations attacks									
			Scaling (60%)			Rotation 30°CW			Geometric distortion			
			NCC	*psnr*	SM	NCC	*psnr*	SM	PSNR	NCC	*psnr*	SM
Inserted watermark image size of 45×45	The method FOBCB-MSB6	Lena	0.846	7.59	0.84	0.809	5.44	0.75	37.34	0.8	6.64	0.8
		Boat	0.846	7.59	0.84	0.781	5.57	0.75	38.94	0.78	7.477	0.82
		Baboon	0.846	7.59	0.84	0.816	5.5	0.76	40.18	0.81	6.791	0.8
		Jet	0.846	7.59	0.84	0.799	5.42	0.75	40.72	0.86	6.542	0.81
		Birds	0.846	7.59	0.84	0.834	5.64	0.77	40.56	0.83	7.098	0.82
		Pills	0.846	7.59	0.84	0.845	5.51	0.77	40.28	0.8	6.842	0.81
	Proposed Method APAP-MPOEE by FOBCB-MSB6	Lena	0.846	7.59	0.84	0.801	5.33	0.75	38.7	0.55	3.868	0.59
		Boat	0.846	7.59	0.84	0.781	5.57	0.75	40.99	0.4	3.905	0.53
		Baboon	0.846	7.59	0.84	0.816	5.5	0.76	43.57	0.59	3.624	0.59
		Jet	0.846	7.59	0.84	0.799	5.42	0.75	44.7	0.57	3.61	0.58
		Birds	0.846	7.59	0.84	0.834	5.64	0.77	44.24	0.57	3.561	0.58
		Pills	0.846	7.59	0.84	0.845	5.51	0.77	43.79	0.67	3.595	0.62
Inserted watermark image size of 16×16	The method FOBCB-MSB6	Lena	0.884	8.17	0.89	0.906	7.65	0.88	37.19	0.89	7.748	0.88
		Boat	0.884	8.17	0.89	0.873	7.27	0.87	38.3	0.9	9.311	0.91
		Baboon	0.884	8.17	0.89	0.895	7.55	0.88	40.26	0.87	8.172	0.89
		Jet	0.884	8.17	0.89	0.906	7.27	0.87	40.33	0.91	8.519	0.9
		Birds	0.884	8.17	0.89	0.934	7.65	0.89	40.51	0.89	9.031	0.91
		Pills	0.884	8.17	0.89	0.912	7.85	0.89	40.51	0.85	8.519	0.9
	Proposed Method APAP-MPOEE by FOBCB-MSB6	Lena	0.884	8.17	0.89	0.906	7.65	0.88	38.68	0.75	4.54	0.75
		Boat	0.884	8.17	0.89	0.873	7.27	0.87	40.86	0.75	4.26	0.75
		Baboon	0.884	8.17	0.89	0.895	7.55	0.88	43.59	0.75	4.492	0.75
		Jet	0.884	8.17	0.89	0.876	7.55	0.87	44.71	0.75	4.215	0.75
		Birds	0.884	8.17	0.89	0.934	7.65	0.89	44.16	0.76	4.638	0.76
		Pills	0.884	8.17	0.89	0.912	7.85	0.89	43.86	0.82	4.589	0.77

a)- Re-scaling: The robustness against re-scaling is tested by first resizing the watermarked image to the scaled factor 60% of its size and then enlarging the image to its original size. Whereas the performance results as shown in Table.13, as seen that by applying proposed method of (APAP-MPOEE-FOBCB$_{MSB6}$) the NCC= $(0.883978)_{average}$, *psnr*=$(8.171754\ _{dB})_{average}$, and SM=$(0.891396)_{average}$, with the extracted watermark image 16×16, and NCC=$(0.846011)_{average}$, *psnr*= $(7.586503_{dB})_{average}$, and SM=$(0.837889)_{average}$, with watermark image 45×45. The results show the proposed method is resilient in the scaling image of factor 60%, deals to excellently and good robustness. The extracted watermark can maintain a good similarity in both inserted watermark image with the original one even after the watermarked image is re-scaling and with the similarity comparison of the FOBCBMSB$_6$ method.

b)- Re-rotation: In this experiment is tested by first rotate the watermarked image small angle rotation 30°CW and then re-rotate the watermarked image to the same angle rotation 30°CCW to its original size, the performance results as shown in Table.13, as seen that by applying proposed method of (APAP-MPOEE-FOBCB$_{MSB6}$) the NCC=$(0.89917383\ )_{average}$, *psnr*=$(7.58603167_{dB})_{average}$, and SM= $(0.8791805\ )_{average}$, with the extracted watermark image 16×16, and NCC=$(0.8127705)_{average}$, psnr=$(5.492718167_{dB})_{average}$,and SM=$(0.75595883)_{average}$, with watermark image 45×45. The results show the proposed method is resilient in the rotation image of 30°CW, deals to excellently and good robustness. The extracted watermark can maintain a good similarity in both inserted watermark image with the original one even after the watermarked image is re-rotate and with the comparison of the FOBCBMSB$_6$ method.

c)- Geometric distortion with JPEG compression: JPEG called unintentionally attack is currently one of the most widely used compression algorithms for images in order to reduce the file size and save limited bandwidth. In this experiment, the watermarked images are compressed by JPEG standard and the performance results as shown in Table.13, as seen that the PSNR of watermarked image obtained after compressed equal $(42.66553183_{dB})_{average}$, with inserted watermark image 45×45, and equal $(42.64152167_{dB})_{average}$, and with inserted watermark image 16×16, by proposed method. Furthermore the NCC= $(0.75001633)_{average}$, *psnr*=$(4.4554005_{dB})_{average}$, and SM= $(0.75015367)_{average}$, of the extracted watermark image 16×16, and NCC=$(0.556741)_{average}$, *psnr*=$(3.693767\ _{dB})_{average}$, and SM=$(0.582182167)_{average}$ of the extracted watermark image 45×45. Thus the results shown that the proposed technique deals with geometric distortion of JPEG compression excellently and good robustness only with extracted watermark 16×16 are inserted with redundantly distributed of the watermark bits over many pixels in the cover image, it is added simultaneously in the falling-off-boundary in corners board pixel to improve the capacity and to ensure robustness. While the FOBCB$_{MSB6}$ method is great a higher robustness with both extracted watermark image. Otherwise the proposed method is robust under extracted watermark 16×16 and low level with extracted watermark 45×45.

d)- Horizontal flip: In this experiment we flipped (Horizontal or vertical) the watermarked image without losing any value as well as the proposed scheme is resilience to flipping. Thus the watermark can be well detected and extracted with higher robustness and without any effected distortion.

e)- Cropping: The image cropping is a disposition geometrical attack; in this experiment we crop in the medial of the watermarked image and then the watermark image can be well detected and extracted with high robustness in both methods. Thus the proposed scheme is resilience to cropping.

7.2.3.3The watermark removal attacks:

a)-Changing in lower order bit manipulation of gray values: In this experiment, if the attacker knows that the image has watermark embedded, then the attacker would only have to replace all LSB bits with a '1' fully defeating the effects or complement the LSB bits and the watermark cannot be recovered from lower order bits LSB$_{1,2,3,4}$. So that the proposed method (APAP-MPOEE-FOBCB$_{MSB6}$) and the FOBCBMSB$_6$ method prevents the attacker to detect or replacing bits, where the PSNR obtained after changing in lower order bit of the LSB$_{1,2,3,4}$= 31.48 dB and then can be well detected and extracted watermark with high robustness in both methods.

b)- Altered image: The altered image called a removal attack well here the extraction/detection process for still image is presented. In this experiment we altered image of the watermarked image with other face or other scene. Since that the embedding of watermark bits are in the falling-off-boundary in corners board of cover image. Whereas the watermark image can be well detected and extracted with high robustness in both methods.

c)- Drawing image: The image drawing is a removal attack; in this experiment we drawing on the watermarked image. Thus the watermark image can be well detected and extracted with higher robustness in both methods. Whereas that the embedding of watermark bits are in the falling-off-boundary in corners board of cover image.

8. Conclusion and Future Work

The digital watermarking technology is a way to apply digital information hiding techniques to prevent malicious and non-malicious attacks to detect hidden information. Have been proposed a novel method, fidelity and robust watermark embedding method satisfies the requirements and problems at the same time in an acceptable manner, called adaptively pixel adjustment process based on medial pyramid of embedding error applying in the falling-off-boundary in corners board of the cover image set-of-the Most-Significant-Bit-6 with the random pixel manipulation blind in spatial domain (APAP-MPOEE-$FOBCB_{MSB6}$). Furthermore the proposed APAP-MPOEE-$FOBCB_{MSB6}$ method using as a embedding watermark bits in the boundary in corners board of the cover image and before embedding requires a checking between the MSB_6 in the boundary in corners board pixel of the cover image and EMB of the embedded watermark bit, depending on the nearest of the adaptively pixel in the medial pyramid of embedding error to inform the watermarked image. Experimental results of the proposed method was computed **(i)** Theoretically are effectiveness in the average of worst case and minimized the embedding error from $(2^{n-1})_{Max}$ in the maximum pyramid of embedding error to the medial pyramid of embedding error $(2^{n-2})_{medial}$, where are the embedding error are restricted between the minimum of embedding error '1' to the medial pyramid of embedding error $(2^{n-2})_{medial}$, as well as the number of embedding errors Ω are obtained lowest in the 16_{Max}, 8.5_{avrg} and 1_{Min} , but within combining between the WMSE and WMSE* reveals that the $Max.WMSE^* = \frac{1}{4}WMSE$ are better, and in worst case of $PSNR_{worst}$ equal $(24.048_{(dB)})_{Max}$, $(29.542_{(dB)})_{avg}$, and $(48.130_{(dB)})_{Min}$ are higher in almost cases of a theoretically computed are proved better than that obtained by the list substitutions of previous methods and modified algorithm. **(ii)** by the way with applied on the different benchmark of six gray scale images and two quantum of watermark bit embedded are compared with an previous works and modified algorithms was found better. Thus the proposed method leads to imperceptible embed watermark bits with preserving a higher fidelity and good robustness in all different benchmark of test-images the watermarks were extracted from watermark degrading, removal and geometric transformations attacks to an acceptable degree of SM and NCC with extracted watermark image 16×16 are inserted with redundantly distributed of the watermark bits over many pixels in the cover image, it is added simultaneously in the falling-off-boundary in corners board pixel are proved the capacity and ensures robustness, when the watermarks thus embedded were found perceptually non-obstructive on six different gray scale images. ***Finally***, *the proposed method*, there is no perceptual distortion (fidelity) in the original and watermarked image with PSNR=$(52.6062305_{dB})_{average}$, which means that the proposed novel technique has satisfied the criteria that an efficient watermark should be unobtrusive imperceptibility, robust, discreet and easily extracted in spatial domain. By the way from our study of previous methods and modified the algorithms, the our performance results are obtained by computed theoretically and applied on the different benchmark of six gray scale images, we observe that the modified algorithm (PAP-algorithm-3) great a good performance PSNR=$(46.2399895_{dB})_{average}$ with compared of the previous works and modified algorithms.

Further research works may be carried out in spatial domain watermarking to generate: (1)-Higher robustness digital watermarking image by applying proposed method by using a factor controller of embedding error. (2)-Applying proposed method on the color image to achieve more increasing the capacity and higher robustness.

References

[1] Kevin Curran, Xuelong Li and Roisin Clarke, "An Investigation into the Use of the Least Significant Bit Substitution Technique in Digital Watermarking", American Journal of Applied Science Vol.2, No.3, p.648-654, 2005.

[2] Chi-Kwong Chan, L.M. Cheng, "Hiding data in images by simple LSB substitution", the Journal Pattern Recognition Society 37, p.469–474, 2004.

[3] Aiad Ibraheem and Abdul-Sada, "Hiding Data Using LSB-3", the Journal Basrah Researches Sciences, Vol. 33. No.4 p.81-88, 19/DEC/ 2007.

[4] Fabien A. P. Petitcolas, Microsoft Research, "Watermarking schemes evaluation", IEEE Signal Processing Magazine, Vol.17, No.5, p.58–64, September 2000.

[5] Ali Al-Haj, "Combined DWT-DCT Digital Image watermarking", the Journal of Computer Science Vol.3, No.9, p.740-746, 2007.

[6] Franco A. Del Colle and Juan Carlos Gomez, "DWT based Digital Watermarking Fidelity and Robustness Evaluation",

the Journal of Computer Science and Technology, Vol. 8, No.1, April 2008.

[7] Cheng-Hsing Yang, "Inverted pattern approach to improve image quality of information hiding by LSB substitution", the Journal Pattern Recognition Society, Vol.41, No.8, p.2674-2683, August, 2008.

[8] Yusnita YUSOF and Othman O. KHALIFA, "Analysis on Perceptibility and Robustness of Digital Image Watermarking Using Discrete Wavelet Transform", IJCSES International Journal of Computer Sciences and Engineering Systems, Vol.2, No.4, October 2008.

[9] Mehemed Bashir Aliwa, Tarek El-Ahmady El-Tobely, Mahmood M. Fahmy, Mohamed EL Said Nasr, Mohamed Hashem Abd El-Aziz, "Robust Digital Watermarking Based Falling-off-Boundary in Corners Board-MSB-6 Gray Scale Images", IJCSNS International Journal of Computer Science and Network Security, Vol.9 No.8 p. 227-240, 30 August 2009.http://paper.ijcsns.org/07_book/html/200908/200908033.html

[10] Sanghyun Joo, Youngho Suh, Jaeho Shin, and Hisakazu Kikuchi, "A New Robust Watermark Embedding into Wavelet DC Components", ETRI Journal, Vol.24, No.5, p:401-404, October 2002.

[11] Martin Kutter, Fabien A. P. Petitcolas, lan,"Fair evaluation methods for image watermarking systems", Journal of Electronic Imaging, Vol.9, No.4, p.445-455, October 2000.

[12] Adil Haouzia and Rita Noumeir,"Methods for image authentication: a survey", Multimed Tools Appl 2008, 39:1–46, Springer Science, Business Media, LLC,2008.

[13] Er.Ashish Bansall and Dr. Sarita Singh Bhadouria "Network Security and Confidentiality with Digital Watermarking", 2007 Inaugural IEEE International Conference on Digital Ecosystems and Technologies, p.325-328, 2007.

[14] Yusnita Yusof and Othman O. Khalifa, Member, IEEE," Digital Watermarking For Digital Images Using Wavelet Transform", Proceedings of the 2007 IEEE International Conference on Telecommunications and Malaysia International Conference on Communications, Penang, Malaysia, p.665 – 669, 14-17 May 2007.

[15] Eugene P. Genov, "Digital Watermarking of Bitmap Images", International Conference on Computer Systems and Technologies, CompSysTech, Publications 2007.

[16] Kamran Hameed, Adeel Mumtaz, and S.A.M. Gilani, "Digital Image Watermarking in the Wavelet Transform Domain", Proceedings of world academy of science, engineering and technology, VOL.13. p.86-89, MAY 2006.

[17] Bijay Shrestha, Charles G. O and Nicolas H. Younan, ‘’ JPEG2000: IMAGE QUALITY METRICS’’, ASPRS 2005 Annual Conference Baltimore,Maryland,March,7-11, 2005.

[18] Santi Prasad Maity, Malay Kumar Kundu, "Robust and Blind Spatial watermarking in Digital image", Proceedings of the Third Indian Conference on Computer Vision, Graphics & Image Processing (ICVGIP 2002), Ahmadabad, India, December 16-18, 2002.

[19] Muhammad Shafique Shaikh and Yasuhiko Dot, "A watermarking scheme for digital images using multilevel wavelet de compodition", Malaysian Journal of Computer Science, Vol.16, No.1, p.24-36, June 2003.

[20] Ran-Zan Wang, Chi-Fang Lin, Ja-Chen Lin, "Hiding data in images by optimal moderately significant-bit replacement", IEE Electronics Letters. Vol.36, No.25, p.2069–2070, 7 December 2000.

[21] Matt L. Miller, Ingemar J. Cox and Jean-Paul M.G. Linnartz Ton Kalker, "A review of watermarking principles and practices", Published in “Digital Signal Processing in Multimedia Systems, Ed. K. K. Parhi and T. Nishitani, Marcell Dekker Inc., p.461-485, 1999.

[22] M. Kutter and F. A. P. Petitcolas, "A fair benchmark for image watermarking systems", Electronic Imaging '99. Security and Watermarking of Multimedia Contents, vol. 3657, Sans Jose, CA, USA, p.25-27 January 1999.

[23] Fernando P´erez-Gonz´alez and Juan R. Hern´andez, "A Tutorial on Digital Watermarking", In Proc. of the 33rd IEEE Annual Carnahan Conference on Security Technology, Madrid, Spain, October 1999.

[24] S.P.Mohanty, et al., ”A DualWatermarking Technique for Images”,Proc.7th ACM-MM’99 International Multimedia Conference, ,Part.2,pp.49-51,Orlando,USA,Oct. 1999.

[25] Lesley R. Matheson, Stephen G. Mitchell, Talal G. Shamoon, Robert E. Tarjan and Francis Zane, "Robustness and Security of Digital Watermarks", Proceedings of the Second International Conference on Financial Cryptography,Lecture Notes In Computer Science; Vol. 1465,P:227–240, Pub:Springer-Verlag London, UK, 1998.

[26] Neil F. Johnson and Sushil Jajodia, "Exploring Steganography: Seeing the Unseen", IEEE computer, vol.31, No.2, p.26-35, February 1998.

[27] R.G.van Schyndel, A.Z.Tirke and C.F.Osborne, "A Digital Watermark, First IEEE Image Processing Conference, Houston TX, vol.2, p.86-90, November 15-17, 1994.

[28] Tirkel, A., Rankin, G., Schyndel, R., Ho, W., Mee, N., & Osborne, C. " Electronic watermark", Proc of Digital Image Computing, Technology and Applications, p.666-673, 1993.

[29] Yubing Wang, "Survey of Objective Video Quality Measurements", EMC Corporation Hopkinton, MA 01748, USA, March 2006.

[30] J. Cummins, P. Diskin, S. Lau and R. Parlett, “Steganography and Digital Watermarking”, Student Seminar Report, School of Computer Science, University of Birmingham,2004.

[31] Bhaskar Koukuntla, Reshma Kumar, Hoanhla and Rew Lah, "STEGANOGRAPHY", Technical Report, GROUP: GRAND, 11/21/2005.

[32] Lin Liu, "A Survey of Digital Watermarking Technologies", Technical Report, Computer Vision Laboratory, Department of Electrical and Computer Engineering, State University of New York at Stony Brook, NY 11794-2350, USA, 2005.

[33] Mauro Barni and Franco Bartolini, "Watermarking Systems Engineering Enabling Digital Assets Security and Other Applications", Copyright ©2004 by Marcel Dekker, Inc. New York, ISBN: 0-8247-4806-9.

[34] Stefan Katzenbeisser and Fabien A. P. Petitcolas, “Information hiding Techniques for Steganography and Digital watermarking", Copyright ©2000 Artech house, inc.

[35] J. Cox, Matthew L. Miller, and Jeffrey A. Bloom "Digital Watermarking Morgan Kaufmann Series in Multimedia Information and Systems", pub:Elsevier, San Francisco, Copyright.2002 by Academic Press,UnitedStatesof America.

[36] Eric Cole, "Hiding in Plain Sight: Steganography and the Art of Covert Communication", Published by Wiley Publishing, Inc., Indianapolis, Indiana, Copyright © 2003.
[37] David Salomon, "Data Compression The Complete Reference", Fourth Edition, British Library Cataloguing in Pub:Springer-Verlag London Limited Copyright © 2007.
[38] Chun-Shien Lu, Institute of Information Science Academia Sinica, Taiwan, ROC, "Multimedia Security: Steganography and Digital Watermarking Techniques for Protection of Intellectual Property", Idea Group Inc (Idea Group Pub: Hershey,London,Melbourne, Singapore), Copyright © 2005.
[39] Michael Arnold, Martin Schmucker and Stephen D. Wolthusen, "Techniques and Applications of Digital Watermarking&Content Protection",Copyright2003 Artech House Computer Security Series Boston, London,inc.
[40] Website Wikipedia Encyclopedia http://en.wikipedia.org/ wiki /PSNR.
[41] Website Multimedia Systems and Applications, http://bmrc. berkeley.edu/courseware/cs294/fall97 /assignment/psnr.html
[42] MSE, PSNR: Image Compression, http://www. debugmode. com/imagecmp/index.htm
[43] Normalized mean square error, Graphics Magick, ‘Compare images’, http://www.Graphics magick.org/api/compare.html
[44] The math works, Documentation video and image processing, http://www.mathworks.com/access/helpdesk/ help/toolbox/vipblks/ref/psnr.html
[45] Wikipedia,SNR, http://wapedia.mobi/en/Signal_to _noise_ ratio_(image_processing)
[46] Mm Wns SNR, PCO.imaging, http://www.pco.de/filea dm in/user_upload/db/download/pco_cooKe_kb_ snr_0504.pdf.

Mehemed Bashir Aliwa received the B.Sc. Degree in Computer Engineering on September 1, 1992 from the Engineering Academy, Tajoura-Libya and the M.Sc. Degree in Computer Engineering on March.2008 from the Arab Academy for Science, Technology and Maritime Transport College of Engineering and Technology, Alexandria-Egypt. He is currently pursuing the Ph.D. degree at the Electrical Engineering (Computer and Control Engineering) of the Faculty of Engineering TANTA University, Egypt. From 1992 to 1996 his was working in the research center of military industrialization, from 1996 to 1997 as a Lecturer at the School of Electronic Support and from 1997 to 2005 as a director, office of the global information network and the office of training and maintenance in computer system in Authority operations and training Libyan armed forces. His research interests include digital watermarking, hiding information, and routing protocol in mobile ad-hoc networks.