Discrete Modified Smith Predictor for an Unstable Plant with Dead Time Using a Plant Predictor

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Summary
In this paper, a discrete control method for an unstable plant with dead time is proposed. The plant is controlled by means of a modified Smith predictor and a predicted-state feedback technique composed of a plant predictor and an observer. Because we use a plant predictor that calculates future outputs and states of the plant, we can design a controller as if the system had no dead time. The state feedback controller with the plant predictor can place the poles of the plant at designed locations. Thus, the method can stabilize the system, even if the plant has unstable poles. In addition, a modified minor feedback eliminates the extra dead time component and a steady-state error caused by input-side disturbance. In simulation studies, we show that our proposed method is effective for unstable plants with dead time.
Key words:
Smith Predictor, Plant Predictor, Predicted State Feedback, Dead Time, Unstable Plant

1. Introduction

A Smith predictor [1] is an efficient control method for a plant with dead time. However, if the plant has unstable poles, it cannot stabilize the system, and if the plant has integrators, input-side disturbances cause a steady-state error. To overcome these problems, many methods have been proposed. For instance, Paor et al. [2], [3] proposed a modified Smith predictor that has a constraint on the ratio of dead time to a time constant. Majhi et al. [4] proposed a new Smith predictor with three controllers that are tuned by simple tuning formulas. Liu et al. [5] proposed an analytical two-degree-of- freedom control scheme for a first- and second-order unstable plant. Rao et al. [6], [7] presented an enhanced Smith predictor that consists of one tuning parameter and offers better performance. In addition to modifications to the structure of the Smith predictor, new control methods using predictors that calculate the future signals of a system have been proposed. The method of Watanabe et al. [8] is based on an output prediction for a plant. Furukawa et al. [9] proposed a control strategy based on a predicted-state feedback technique with an observer. The method of Tan et al. [10] is based on the generalized predictive control approach. Del-Muro-Cuellar et al.[11] used an observer-based predictor with dead time partitions to stabilize an unstable plant.

In this paper, we propose a discrete control method for an unstable plant with a long dead time using a plant predictor. The plant is controlled by means of a predicted state feedback technique and a modified Smith predictor composed of the plant predictor and an observer. The feedback technique can stabilize the system, even if the plant has unstable poles. The modified feedback can eliminate a steady-state error caused by an input-side disturbance.

2. Plant Predictor

2.1 Basic Equations

In this method, we consider the following plant, which is controllable and observable, with dead time d.

$$\boldsymbol{x}_p(k+1) = \boldsymbol{A}_p \boldsymbol{x}_p(k) + \boldsymbol{b}_p u_p(k-d) \quad (1)$$

$$y_p(k) = \boldsymbol{c}_p \boldsymbol{x}_p(k) \quad (2)$$

A plant predictor is derived recursively as follows:

$$\begin{aligned}
\boldsymbol{x}_p(k+1) &= \boldsymbol{A}_p \boldsymbol{x}_p(k) + \boldsymbol{b}_p u_p(k-d) \\
\tilde{\boldsymbol{x}}_p(k+2) &= \boldsymbol{A}_p x_p(k+1) + \boldsymbol{b}_p u_p(k-d+1) \\
&= \boldsymbol{A}_p^2 \boldsymbol{x}_p(k) + \boldsymbol{A}_p \boldsymbol{b}_p u_p(k-d) + \boldsymbol{b}_p u_p(k-d+1) \\
&\ \vdots \\
\tilde{\boldsymbol{x}}_p(k+d) &= \boldsymbol{A}_p^d \boldsymbol{x}_p(k) + \boldsymbol{A}_p^{d-1} \boldsymbol{b}_p u_p(k-d) + \cdots + \boldsymbol{b}_p u_p(k-1) \\
&= \boldsymbol{A}_p^d \boldsymbol{x}_p(k) + \sum_{i=1}^{d} \boldsymbol{A}_p^{d-i} \boldsymbol{b}_p u_p(k-d+i-1)
\end{aligned} \quad (3)$$

If the states of the plant are not observed directly, the following observer is used.

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$$\begin{aligned}\hat{\boldsymbol{x}}_p(k+1) &= \boldsymbol{A}_p \hat{\boldsymbol{x}}_p(k) \\ &+\boldsymbol{b}_p u_p(k-d) + \boldsymbol{L}_o\left(y_p(k) - \hat{y}_p(k)\right)\end{aligned} \tag{4}$$

$\boldsymbol{L_o}$ is the observer gain. Note that the observer includes dead time d in the manipulated variable.

2.2 Predicted-State Feedback

A state feedback control cannot stabilize a system with a long dead time. However, the plant predictor solves this problem. Fig.1 is a block diagram of a predicted state-feedback system with a predictor. The characteristic equation of the closed-loop system is given by

$$\begin{aligned}&\det\begin{bmatrix} z\boldsymbol{I}-\boldsymbol{A}_p & 0 & -\boldsymbol{b}_p z^{-d} \\ -\boldsymbol{L_o c}_p & z\boldsymbol{I}-\boldsymbol{A}_p+\boldsymbol{L}_o\boldsymbol{c}_p & -\boldsymbol{b}_p z^{-d} \\ 0 & -\boldsymbol{F}_1\boldsymbol{A}_p^d & 1-\boldsymbol{F}_1\left(I-\boldsymbol{A}_p^d z^{-d}\right)\left(z\boldsymbol{I}-\boldsymbol{A}_p\right)^{-1}\boldsymbol{b}_p \end{bmatrix} \\ &= \det\left(z\boldsymbol{I}-\boldsymbol{A}_p+\boldsymbol{L}_o\boldsymbol{c}_p\right)\det\left(z\boldsymbol{I}-\boldsymbol{A}_p\right)\det\left(1-\boldsymbol{F}_1\left(z\boldsymbol{I}-\boldsymbol{A}_p\right)^{-1}\boldsymbol{b}_p\right) \\ &= \det\left(z\boldsymbol{I}-\boldsymbol{A}_p+\boldsymbol{L}_o\boldsymbol{c}_p\right)\det\left(z\boldsymbol{I}-\boldsymbol{A}_p-\boldsymbol{b}_p\boldsymbol{F}_1\right)\end{aligned} \tag{5}$$

The characteristic equation clearly has no dead time components, so the feedback can stabilize the system, even for plants with unstable poles. Note that the z-transform of the plant predictor (3) can be written as follows.

$$\begin{aligned}&\boldsymbol{A}_p^d \boldsymbol{x}_p(k) + \sum_{i=1}^{d}\boldsymbol{A}_p^{d-i}\boldsymbol{b}_p u_p(k-d+i-1) \\ &\longrightarrow \boldsymbol{A}_p^d \boldsymbol{X}_p(z) + \left(\boldsymbol{I}-\boldsymbol{A}_p^d z^{-d}\right)\left(z\boldsymbol{I}-\boldsymbol{A}_p\right)^{-1}\boldsymbol{b}_p U_p(z)\end{aligned} \tag{6}$$

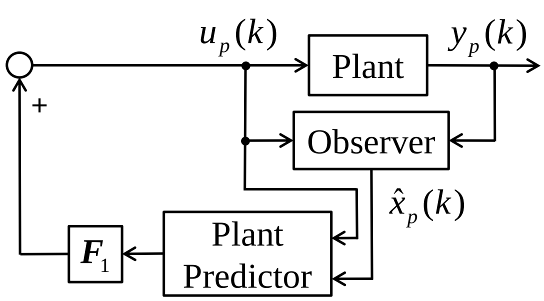

Fig. 1 Predicted State Feedback

3. Modified Smith Predictor

3.1 Modified Minor Feedback

In this section, we propose a modified Smith predictor with a plant predictor. The minor feedback of a conventional Smith predictor shown in Fig. 2 consists of a plant model and a dead-time-free component of the model. That is, it is composed of a plant output and a future plant output. To cut down on the number of delay devices in the system, we realized this idea using the observer output and the plant predictor output.

$$v(k) = \tilde{y}_p(k+d) - \hat{y}_p(k) \tag{7}$$

However, the feedback causes a steady-state error, if an input-side step disturbance is added. To overcome this problem, we insert a disturbance rejection controller Gdc(z) into the feedback as shown in Fig. 3.

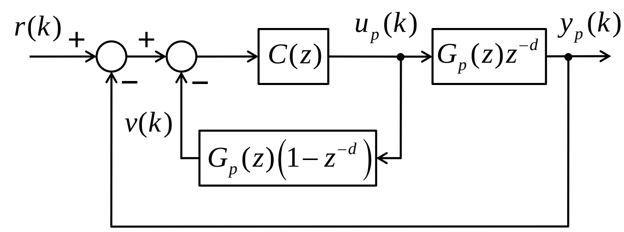

Fig. 2 Smith Predictor

3.2 Design of Disturbance Rejection Controller $G_{dc}(z)$

The controller Gdc(z) eliminates a steady-state error caused by an input-side disturbance. We consider it a proportional controller, that is, Gdc(z) = Kdc. The proportional gain Kdc is obtained by applying the final value theorem to the disturbance response GD(z), which is given by

$$\begin{aligned}G_D(z) &= \begin{bmatrix}\boldsymbol{E}^{\#} & \boldsymbol{0}\end{bmatrix}\begin{bmatrix}\boldsymbol{A}^{\#} & \boldsymbol{B}^{\#} \\ \boldsymbol{C}^{\#} & \boldsymbol{D}^{\#}\end{bmatrix}^{-1}\begin{bmatrix}\boldsymbol{F}^{\#} \\ \boldsymbol{0}\end{bmatrix} \\ &= \frac{Q(z)\boldsymbol{c}_p\left(z\boldsymbol{I}-\boldsymbol{A}_p-\boldsymbol{b}_p\boldsymbol{F}_1\right)^{-1}\boldsymbol{b}_p}{1+F_2(z-1)^{-1}\boldsymbol{c}_p\left(z\boldsymbol{I}-\boldsymbol{A}_p-\boldsymbol{b}_p\boldsymbol{F}_1\right)^{-1}\boldsymbol{b}_p} z^{-d}\end{aligned} \tag{8}$$

where,

$$\boldsymbol{A}^{\#} = \begin{bmatrix} z\boldsymbol{I}-\boldsymbol{A}_p & \boldsymbol{0} \\ -\boldsymbol{L}_o\boldsymbol{c}_p & z\boldsymbol{I}-\boldsymbol{A}_p+\boldsymbol{L}_o\boldsymbol{c}_p\end{bmatrix} \tag{9}$$

$$\boldsymbol{B}^{\#} = \begin{bmatrix} \boldsymbol{0} & -\boldsymbol{b}_p z^{-d} \\ \boldsymbol{0} & -\boldsymbol{b}_p z^{-d}\end{bmatrix} \tag{10}$$

$$\boldsymbol{C}^{\#} = \begin{bmatrix} G_c(z)\boldsymbol{c}_p\left(K_{dc}+1\right) & G_c(z)\boldsymbol{c}_p\left\{\boldsymbol{A}_p^d-\left(K_{dc}+1\right)\boldsymbol{I}\right\} \\ \boldsymbol{0} & -\boldsymbol{F}_1\boldsymbol{A}_p^d\end{bmatrix} \tag{11}$$

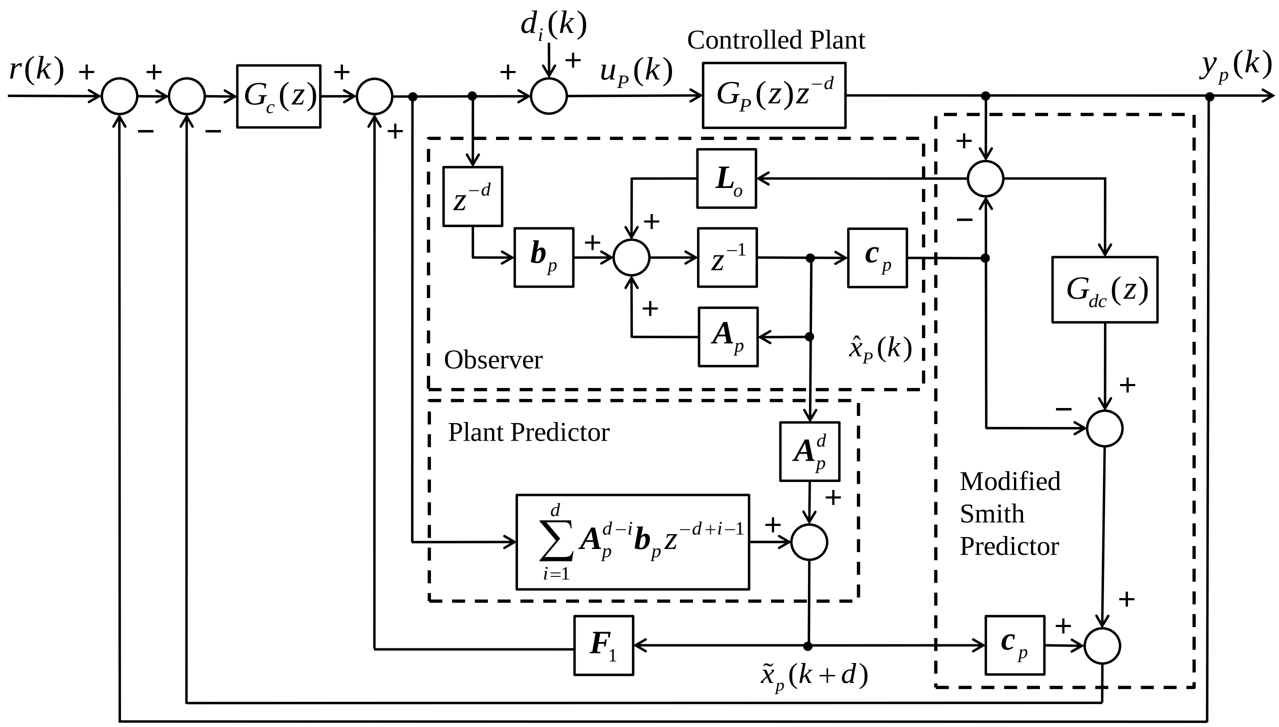

Fig. 3 Block Diagram of the Proposed Method

$$
\boldsymbol{D}^{\#} = \begin{bmatrix} 1 & G_c(z)\boldsymbol{c}_p\left(\boldsymbol{I}-\boldsymbol{A}_p^d z^{-d}\right)\left(z\boldsymbol{I}-\boldsymbol{A}_p\right)^{-1}\boldsymbol{b}_p \\ -1 & 1-\boldsymbol{F}_1\left(\boldsymbol{I}-\boldsymbol{A}_p^d z^{-d}\right)\left(z\boldsymbol{I}-\boldsymbol{A}_p\right)^{-1}\boldsymbol{b}_p \end{bmatrix} \quad (12)
$$

$$
\boldsymbol{E}^{\#} = \begin{bmatrix} \boldsymbol{c}_p & \boldsymbol{0} \end{bmatrix} \quad (13)
$$

$$
\boldsymbol{F} = \begin{bmatrix} \boldsymbol{b}_p z^{-d} \\ \boldsymbol{0} \end{bmatrix} \quad (14)
$$

$$
\begin{aligned} Q(z) &= 1-\left(\boldsymbol{F}_1 - G_c(z)\boldsymbol{c}_p\right)\sum_{i=1}^{d}\boldsymbol{A}_p^{d-i}\boldsymbol{b}_p z^{-d+i-1} \\ &-\left[G_c(z)\boldsymbol{c}_p\left(K_{dc}+1\right)+\left\{\boldsymbol{F}_1-G_c(z)\boldsymbol{c}_p\right\}\boldsymbol{A}_p^d\right]\boldsymbol{H}^{-1}\boldsymbol{b}_p z^{-d} \end{aligned} \quad (15)
$$

$$
\boldsymbol{H} = z\boldsymbol{I}-\boldsymbol{A}_p+\boldsymbol{L}_o\boldsymbol{c}_p \quad (16)
$$

To eliminate the steady-state error, the following equation must be satisfied:

$$
\lim_{z\to 1}\frac{Q(z)\boldsymbol{c}_p\left(z\boldsymbol{I}-\boldsymbol{A}_p-\boldsymbol{b}_p\boldsymbol{F}_1\right)^{-1}\boldsymbol{b}_p}{1+G_c(z)\boldsymbol{c}_p\left(z\boldsymbol{I}-\boldsymbol{A}_p-\boldsymbol{b}_p\boldsymbol{F}_1\right)^{-1}\boldsymbol{b}_p} = 0 \quad (17)
$$

When a controller $G_c(z)$ is chosen by

$$
G_c(z) = \frac{F_{22}}{z-1} \quad (18)
$$

the gain K_{dc} is obtained as follows.

$$
K_{dc} = \frac{\boldsymbol{c}_p\sum_{i=1}^{d}\boldsymbol{A}_p^{d-i}\boldsymbol{b}_p+\boldsymbol{c}_p\left(\boldsymbol{A}_p^d-\boldsymbol{I}\right)\left(\boldsymbol{I}-\boldsymbol{A}_p+\boldsymbol{L}_o\boldsymbol{c}_p\right)^{-1}\boldsymbol{b}_p}{\boldsymbol{c}_p\left(\boldsymbol{I}-\boldsymbol{A}_p+\boldsymbol{L}_o\boldsymbol{c}_p\right)^{-1}\boldsymbol{b}_p} \quad (19)
$$

3.3 Reference Response

The reference response is given by

$$
\begin{aligned} G_R(z) &= \begin{bmatrix} \boldsymbol{E}^{\#} & \boldsymbol{0} \end{bmatrix}\begin{bmatrix} \boldsymbol{A}^{\#} & \boldsymbol{B}^{\#} \\ \boldsymbol{C}^{\#} & \boldsymbol{D}^{\#} \end{bmatrix}^{-1}\begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{N}^{\#} \end{bmatrix} \\ &= \frac{G_c(z)G_a(z)}{1+G_c(z)G_a(z)} z^{-d} \end{aligned} \quad (20)
$$

where,

$$
\boldsymbol{N}^{\#} = \begin{bmatrix} G_c(z) \\ 0 \end{bmatrix} \quad (21)
$$

$$
G_a(z) = \boldsymbol{c}_p\left(z\boldsymbol{I}-\boldsymbol{A}_p-\boldsymbol{b}_p\boldsymbol{F}_1\right)^{-1}\boldsymbol{b}_p \quad (22)
$$

The reference response is equal to that of the conventional Smith predictor. In addition, the controller $G_{dc}(z)$ has no effect if plant model is accurate. Thus, we can set the desired reference response with parameters $\boldsymbol{F_1}$ and $\boldsymbol{F}_2$.

4. Simulation

4.1 Example 1

Consider the unstable first-order plant with a long dead time studied by Rao et al. [7].

$$G_p(z) = \frac{4}{4s-1} e^{-4s} \tag{23}$$

To set the system parameters, the plant is discretized by a zero-order hold at sampling time Ts = 0.01[sec]. The discrete state formation and the output equation of the plant are as follows.

$$x_p(k+1) = 1.0025 x_p(k) + 0.01 u_p(k-400) \tag{24}$$

$$y_p(k) = x_p(k) \tag{25}$$

The feedback gains are F1 = −5.8705 and F2 = 0.015, and the observer gain is Lo = 0.0059. From (19), the gain Kdc is set to 4.0169. The result is shown in Fig. 4. A unit set-point input is introduced at time t = 0 [sec], and an input-side disturbance of magnitude −0.05 is added at time t = 125 [sec]. The figure shows that the disturbance response is faster and smoother than that of Rao et al. We assume a +5% estimated error in the dead time and a −5% estimated error in the time constant. Fig. 5 shows that the proposed method achieves more robust stability and better disturbance rejection.

4.2 Example 2

Consider the unstable second-order plant with dead time studied by Liu et al. [5].

$$G_p(z) = \frac{1}{(s-1)(0.5+1)} e^{-1.2s} \tag{26}$$

To set the system parameters, the plant is discretized by a zero-order hold at sampling time T_s = 0.01 [sec]. The discrete state formation and the output equation of the plant are as follows.

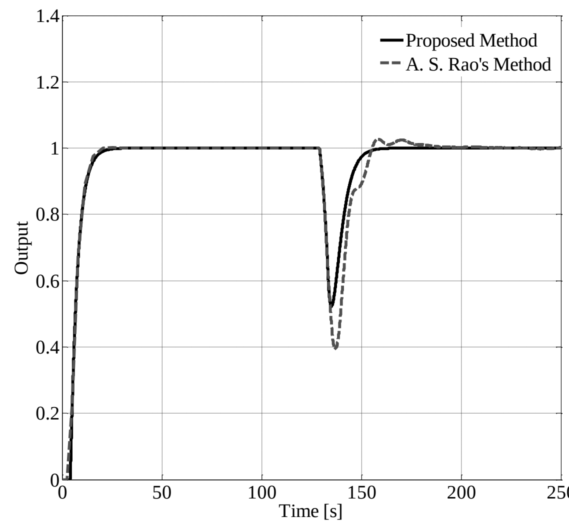

Fig.4 Response of Example 1

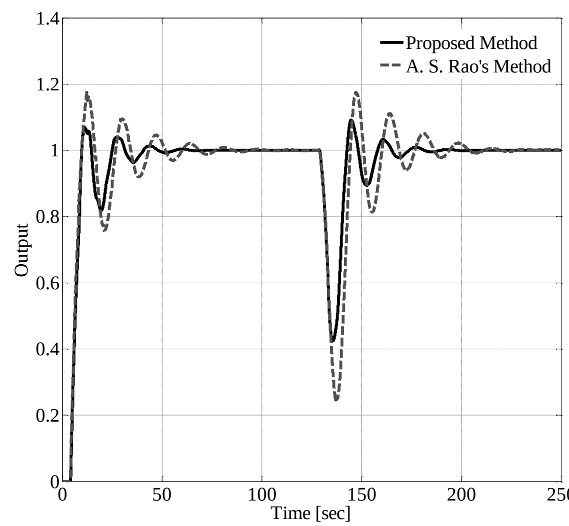

Fig.5 Response of Example 1 with Estimated Error

$$\boldsymbol{x}_p(k+1) = \begin{bmatrix} 0.9702 & -0.0199 \\ 0.0199 & 1.02 \end{bmatrix} \boldsymbol{x}_p(k) + \begin{bmatrix} 0.0099 \\ 0.0001 \end{bmatrix} u_p(k-120) \tag{27}$$

$$y_p(k) = \begin{bmatrix} 0 & 1 \end{bmatrix} \boldsymbol{x}_p(k) \tag{28}$$

The feedback gains are F1 = [−3.5172, −8.4016] and F2 = 0.01, and the observer gain is Lo = [−0.0099, 0.0226]T. From (19), the gain Kdc is set to 5.4065. Fig. 6 shows the result for a perfect plant model, and Fig. 7 is the response when we assume a +10% estimated error in the dead time and a +20% estimated error in the unstable time constant.

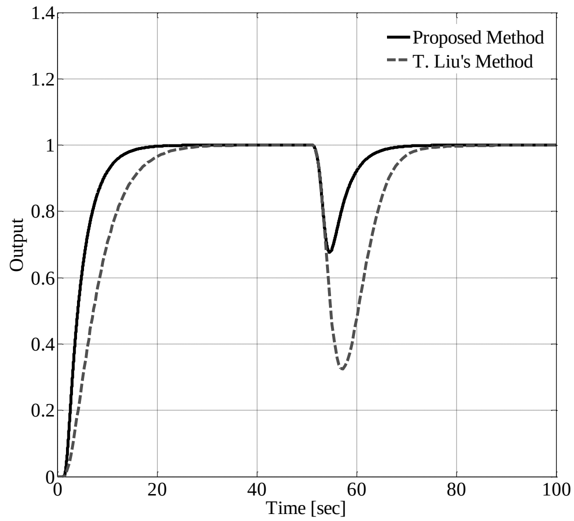

Fig.6 Response of Example 2

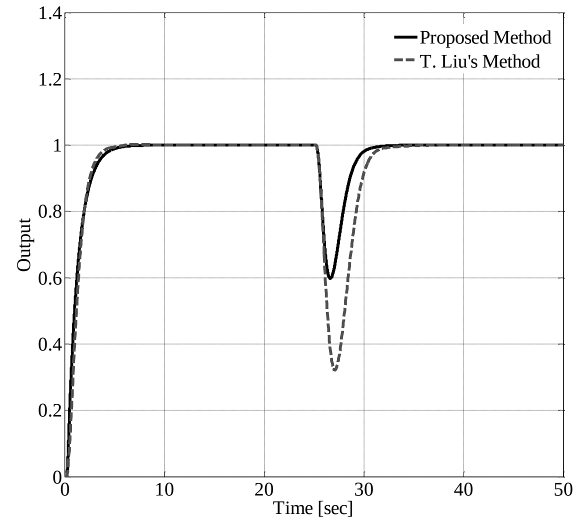

Fig.8 Response of Example 3

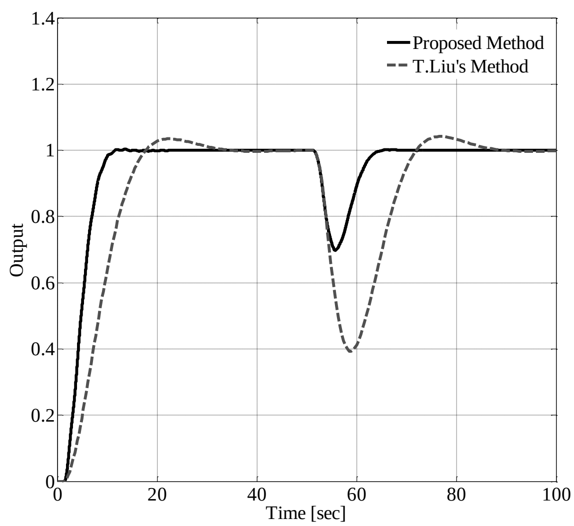

Fig.7 Response of Example 2 with Estimated Error

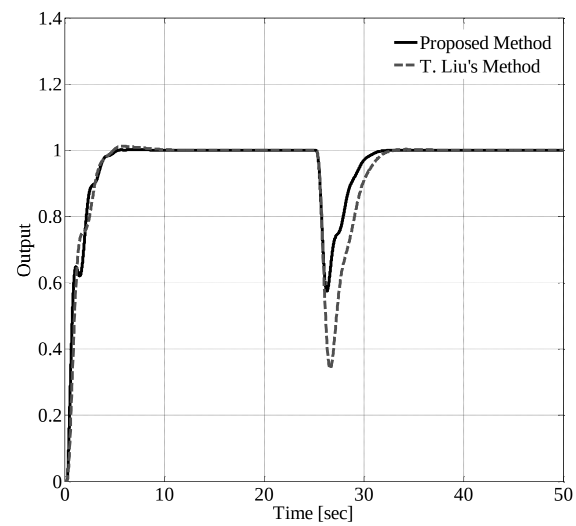

Fig.9 Response of Example 3 with Estimated Error

A unit set-point input is introduced at time t = 0 [sec], and an input-side disturbance of magnitude −0.05 is added at time t = 50 [sec]. The proposed method clearly achieves a better reference response and disturbance response.

4.3 Example 3

We consider the unstable integral plant

$$G_p(z) = \frac{1}{s(s-1)} e^{-0.2s} \qquad (29)$$

which was studied by Liu et al. [5]. To set the system parameters, the plant is discretized by a zero-order hold at sampling time Ts = 0.001 [sec]. The discrete state formation and output equation of the plant are as follows.

$$\boldsymbol{x}_p(k+1) = \begin{bmatrix} 1.001 & 0 \\ 0.001 & 1 \end{bmatrix} \boldsymbol{x}_p(k) + \begin{bmatrix} 0 \\ 0.001 \end{bmatrix} u_p(k-200) \qquad (30)$$

$$y_p(k) = \begin{bmatrix} 0 & 1 \end{bmatrix} \boldsymbol{x}_p(k) \qquad (31)$$

The feedback gains are F1 = [−26.99, −339.75] and F2 = 0.3, and the observer gain is Lo = [0.0069, 0.0037]T. From (19), the gain Kdc is set to 0.8874. Fig. 8 shows the result for a perfect plant model, and Fig. 9 is the response when we assume a +20% estimated error in dead time and a −20% estimated error in the unstable time constant. A unit set-point input is introduced at time t = 0 [sec] and a negative unit input-side disturbance is added at time t = 50 [sec]. These results also show that the proposed method performs better than that of Liu et al.

5. Conclusion

We have proposed a modified Smith predictor with a plant predictor for an unstable plant with dead time. A predicted state feedback technique can stabilize the system, even if the plant has unstable poles. A modified minor feedback with a proportional controller can eliminate a steady-state error caused by input-side step disturbance. In simulation studies, we have demonstrated the effectiveness of the proposed method.

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