An Improved Chan-Ho Location Algorithm for TDOA Subscriber Position Estimation

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Summary
In the 3rd generation (3G) telecommunication, the subscriber radio techniques based on time difference of arrival (TDOA) became one of the key technologies. Chan-Ho is a method that is widely used in solving the TDOA hyperbolic equations, but its performance is highly dependent on the pure distance term between the home base transceiver station (BTS) and the mobile handset. The proposed procedure uses another term that will utilize the relation between the mobile handset with two base transceiver stations. The performance of the proposed algorithm and the original Chan-Ho algorithm are evaluated and the simulation results show that the proposed algorithm has the advantage over the original.

Key words:
Cellular networks, position accuracy, Chan-Ho, hyperbolic, time difference of arrival (TDOA).

1. Introduction

With the rapid development and wide spread of the mobile communication technologies; all kinds of position location estimation applications, techniques and services has became the focus of the telecommunication operators, manufacturers and service providers. More urge was created after the demand of the United States Federal Communications Commission (FCC) compelled all cellular service providers to provide the users location information with a position error under 67% probability or up to 125 meters to E911 Public Security Service System [1]. Therefore offering the location service for a mobile station (MS) or mobile handset has became important research focus for service providers as well as engineers

Different position location estimation techniques based on signal strength (SS), angle of arrival (AOA), time of arrival (TOA), time difference of arrival (TDOA) and their combinations have been put to practice in recent years [2]. However, both TOA and TDOA are the extensive technologies in radio location systems based on cellular networks. TOA is obtained by calculating the time of signal arrival from the mobile station and the base transceiver station (BTS) directly. On the other hand, TDOA involves the calculation of time difference of signal arrival between two BTSs. Therefore, TDOA method has the advantage of reduced time synchronization over the TOA method in the non line-of-sight (NLOS) environments. As the time difference of arrival follows hyperbolic relationship with the distance, TDOA method results in a hyper-parabola for each pair of base transceiver stations [3, 4].

In this paper, we present a modified positioning method based on the Chan-Ho algorithm to solve the hyperbolic equations generated by the TDOA procedures. The TDOA hyperbolic equations model is presented in section two. Then the proposed improved Chan-Ho scheme is introduced in section three. The performance of the proposed algorithm and the original algorithm is evaluated and the simulation results as well as analysis are provided in section four.

2. TDOA Hyperbolic Model

Let *(x,y)* be the real coordinates of the mobile station (MS) (or mobile handset), and (x_i, y_i) is the coordinates of the ith BTS. Therefore the distance between the MS and BTSi is:

$$R_i = \sqrt{(x_i - x)^2 + (y_i - y)^2} \tag{1}$$

The difference of distances from the home BTS to the mobile station and that from the ith BTS to the mobile station can be identified as:

$$R_{i,1} = c\Delta\tau_{i,1} = R_i - R_1 \tag{2}$$

where c is the speed of light, and $\Delta\tau_{i,1}$ is the measured value of TDOA obtained by the cross correlation method.

Now assume that: $K_i^2 = X_i^2 + Y_i^2$ then equation (1) can be rewritten as:

$$R_i^2 = K_i^2 - 2X_i x - 2Y_i y + x^2 + y^2 \tag{3}$$

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We observe here that TDOA generated non-linear hyperbolic functions of (x) and (y) which is difficult to solve. The easiest way to solve a non linear equation is to break it down into several linear equations using the Chan-Ho method. By using the previous equation, we get:

$$\begin{aligned} R_i^2 &= (R_{i,1} + R_1)^2 \\ R_{i,1}^2 &+ 2R_{i,1}R_1 + R_1^2 \\ &= K_i^2 - 2X_i x - 2Y_i y + x^2 + y^2 \end{aligned} \quad (4)$$

Expanding equation (1) for *(i=1)*, generates:

$$R_1^2 = X_1^2 + Y_1^2 - 2X_1 x - 2Y_1 y + x^2 + y^2 \quad (5)$$

Now from combining the last two equations, we obtain:

$$\begin{aligned} R_{i,1}^2 &+ 2R_{i,1}R_1 \\ &= K_i^2 - K_1^2 - 2(X_i - X_1)x - 2(Y_i - Y_1)y \end{aligned} \quad (6)$$

By letting: $X_{i,1} = X_i - X_1$ and $Y_{i,1} = Y_i - Y_1$, we may simplify equation (6) to be as:

$$R_{i,1}^2 + 2R_{i,1}R_1 = K_i^2 - K_1^2 - 2X_{i,1}x - 2Y_{i,1}y \quad (7)$$

For a system involving three base transceiver stations, equation (7) will be rewritten as:

$$-2X_{2,1}x - 2Y_{2,1}y = 2R_{2,1}R_1 + R_{2,1}^2 - K_2^2 + K_1^2 \quad (8)$$

$$-2X_{3,1}x - 2Y_{3,1}y = 2R_{3,1}R_1 + R_{3,1}^2 - K_3^2 + K_1^2 \quad (9)$$

where *(i=2)* and *(i=3)* are for the two neighboring BTSs (other than the home BTS).

In the next section, the solution mechanism for the last two equations will be explained for the original Chan-ho and the proposed algorithms to determine the location coordinates of the mobile station.

3. The Proposed Improved Chan-Ho Model

In the Original Chan-Ho model [5], equation (8), and (9) can be represented in the form of the following matrices:

$$\begin{aligned} -2\begin{bmatrix} X_{2,1} & Y_{2,1} \\ X_{3,1} & Y_{3,1} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} \\ = 2\begin{bmatrix} R_{2,1} \\ R_{3,1} \end{bmatrix} R_1 + \begin{bmatrix} R_{2,1}^2 - K_2^2 + K_1^2 \\ R_{3,1}^2 - K_3^2 + K_1^2 \end{bmatrix} \end{aligned} \quad (10)$$

or,

$$\begin{aligned} \begin{bmatrix} x \\ y \end{bmatrix} = -\begin{bmatrix} X_{2,1} & Y_{2,1} \\ X_{3,1} & Y_{3,1} \end{bmatrix}^{-1} \\ \times \left\{ \begin{bmatrix} R_{2,1} \\ R_{3,1} \end{bmatrix} R_1 + \frac{1}{2}\begin{bmatrix} R_{2,1}^2 - K_2^2 + K_1^2 \\ R_{3,1}^2 - K_3^2 + K_1^2 \end{bmatrix} \right\} \end{aligned} \quad (11)$$

where *(x,y)* represents the location of the mobile station, and (R_1) is obtained from equation (5) in the previous section and ($R_{2,1}$) as well as ($R_{3,1}$) are obtained from equation (7), considering that:

$$\begin{aligned} K_1^2 &= X_1^2 + Y_1^2 \\ K_2^2 &= X_2^2 + Y_2^2 \\ K_3^2 &= X_3^2 + Y_3^2 \end{aligned} \quad (12)$$

After obtaining the first patch of *(x,y)* they can reiterate in the calculations again for more improvement. This can take from 1 to 4 cycles in many of the cases.

From equation (11), we can observe that the resulting values of *(x)* and *(y)*, are dependent on the value of (R_1) which is the distance between the home base transceiver station and the mobile station. We propose a non iterative Chan-Ho that adopts a different term that improves the accuracy.

We constructed a new value, which utilizes the different values resolved from two base stations transceivers, to estimate the right distance and therefore obtain the most accurate position. This value is derived from an expression that relates the error on each direction (vertical and horizontal) to the distance obtained, using the following expressions:

$\dfrac{(x_i - xo_i)}{R_i}$: for the x-axis ratio, and

$\frac{(y_i - yo_i)}{R_i}$: for the y-axis ratio.

where $(i\epsilon\{1,2\})$ is the BTS reference, (xo_i,yo_i) is the obtained coordinates of the mobile station (MS) by each BTS, (x_i,y_i) is the coordinates of the ith BTS, and (R_i) is the distance between the MS and BTSi. Therefore, for two sets of ratios, we get:

$$Q = \begin{bmatrix} \frac{(x_1 - xo_1)}{R_1} & \frac{(x_2 - xo_2)}{R_2} \\ \frac{(y_1 - yo_1)}{R_1} & \frac{(y_2 - yo_2)}{R_2} \end{bmatrix} \tag{13}$$

To generate a symmetrical matrix, we will multiply the matrix by its transpose. Then to understand the characteristics of this matrix we will obtain the eigenvalues, by calculating the trace; which will lead to the nearest accurate solution. The final value obtained will be corrected by taking the square root of the arithmetic mean of the trace, as following:

$$\omega = \sqrt{\frac{tr(Q'Q)}{2}} \tag{14}$$

Therefore the new calculation system will be:

$$\begin{bmatrix} x \\ y \end{bmatrix} = -\begin{bmatrix} X_{2,1} & Y_{2,1} \\ X_{3,1} & Y_{3,1} \end{bmatrix}^{-1} \times \left\{ \begin{bmatrix} R_{2,1} \\ R_{3,1} \end{bmatrix} \omega + \frac{1}{2} \begin{bmatrix} R_{2,1}^2 - K_2^2 + K_1^2 \\ R_{3,1}^2 - K_3^2 + K_1^2 \end{bmatrix} \right\} \tag{15}$$

4. Simulation and Analysis

The time difference of arrival mechanism for position estimation utilizes the timing information from the mobile user to the base transceiver station only (reverse link). So for this simulation, TDOA will be measured on the reverse link of a randomly located mobile station as well as BTSs as shown in figure (1). Each BTS has its own radio coverage range. The MS was located at the point (2,2) and associated with BTS1 as its home base transceiver station.

Generally, TDOA position location system can be implemented in two different ways. Either by subtracting the time of arrival at two BTSs, which requires the implementation of the absolute TOA mechanism or by cross-correlating the received signals from the two BTSs. The second is the mechanism deployed in our simulation for a WCDMA cellular network.

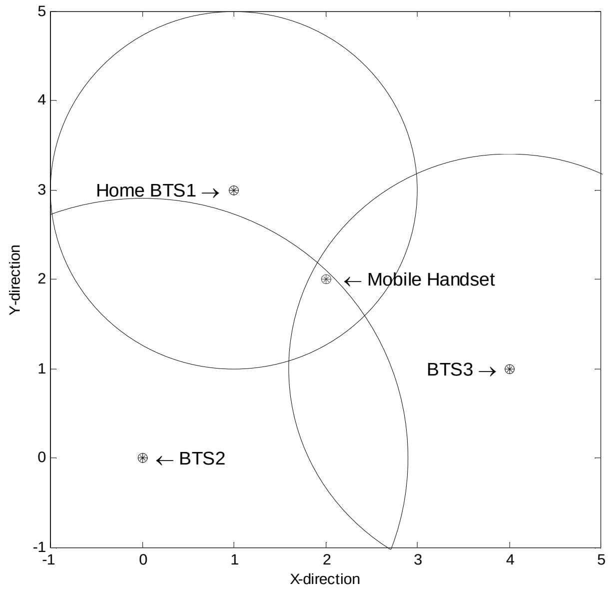

Fig. (1) Initial Cellular Network Setup

Now, let $s(t)$ be the signal generated by the mobile station that will propagate through the transmission channel. So we can calculate $x_1(t)$ and $x_2(t)$, the received signals at the home BTS and a neighbor BTS, as following:

$$x_1(t) = a_1 s(t - \Delta t_1) + n_1(t) \tag{16}$$

$$x_2(t) = a_2 s(t - \Delta t_2) + n_2(t) \tag{17}$$

where $n_1(t)$ and $n_2(t)$ are the random additive white Gaussian noise (AGWN) with zero mean [6] such that they are uncorrelated with $s(t)$, Δt_1 and Δt_2 are the traveling time between the MS and the BTSs, and a_1 and a_2 are the amplitudes of the signals. If $(\tau = \Delta t_2 - \Delta t_1)$, then the TDOA can be estimated from the maximum value of crosscorrelation $R_{x1x2}(\tau)$, that is expressed as [7]:

$$R_{X_1X_2}(\tau) = \int_{-\infty}^{\infty} x_1(\tau) x_2(\tau - t) dt \tag{18}$$

The original Chan-Ho and the improved algorithms are simulated for $(n=1000)$ measurements (with random iteration for the original between 1 and 4 cycles), and the results of the obtained positions points are shown in gray for the original and in black for the improved (fig. 2). Since the original Chan-Ho uses the term (R_1) that obtained from BTS1; and the modified uses the term (ω) obtained from BTS1 and BTS2, most of the estimated points are located within the range of the home BTS, and the intersection area of BTS1 and BTS2, respectively. However, the second set of estimated points is condensed and closer to the original mobile handset location rather than being spaced in case of the original algorithm, which indicates the higher performance of the improved algorithm.

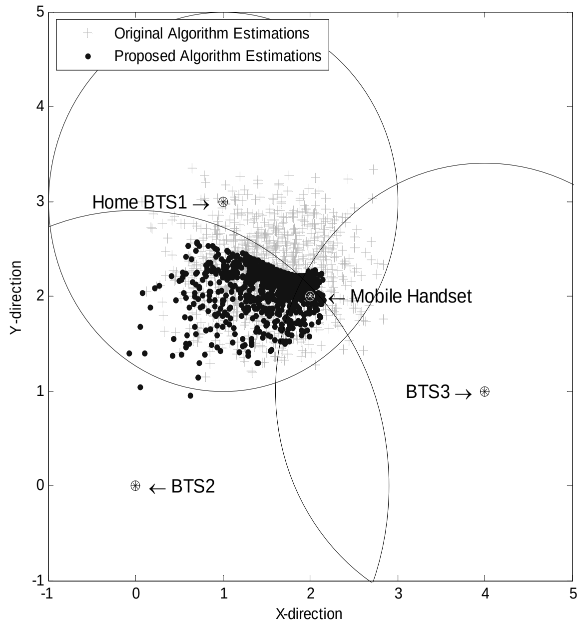

Fig. (2) Comparison of location estimate of the original algorithm results the proposed algorithm results and the true node location

Moreover, the cumulative probability distribution functions (CDF) of the absolute position error (equation 19) between the original (MS) position and the obtained values, were obtained from the simulation investigations.

$$\Delta d = \sqrt{(x - x_o)^2 + (y - y_o)^2} \tag{19}$$

The comparisons of the errors for the two methods are presented in next figure and the graph shows that the improved method offers a better accuracy where it provides a closer position from the first run of the measurement with no extra iterations.

5. Conclusions

In this paper an improved Chan-Ho algorithm is applied in mobile position location estimation on cellular networks. Theoretical analysis and simulation results show that the proposed algorithm has a better stable positioning performance in random noise environment than the original Chan-Ho. Since this algorithm is not iterative it can be considered also low in computational complexity over the other.

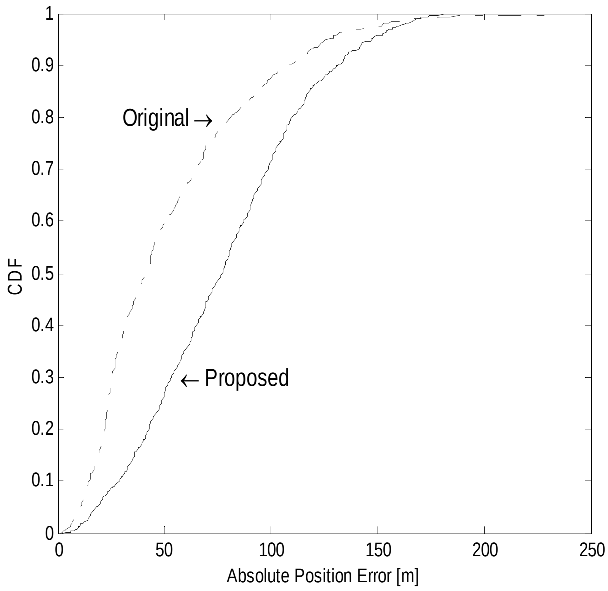

Fig.(3) The CDFs of absolute position error for the methods

References

[1] F. C. C. C. G. Affairs, "FCC Consumer Facts," Wireless 911 Services, http://www.fcc.gov/cgb/consumerfacts/wireless911srvc.html, [10 08, 2008].

[2] P. Myllymäki, H. Tirri, P. Kontkanen et al., Location estimation in wireless telecommunication networks, US 7228136, 2007.

[3] C. Mensing, and S. Plass, "Positioning Algorithms for Cellular Networks Using TDOA." pp. IV-IV.

[4] W. Chin-Der, and L. Ming-Hui, "Data fusion methods for accuracy improvement in wireless location systems." pp. 471-476 Vol.1.

[5] Y. T. Chan, and K. C. Ho, "A simple and efficient estimator for hyperbolic location," Signal Processing, IEEE Transactions on, vol. 42, no. 8, pp. 1905-1915, 1994.

[6] Y. T. Chan, and K. C. Ho, "Joint time-scale and TDOA estimation: analysis and fast approximation," Signal Processing, IEEE Transactions on, vol. 53, no. 8, pp. 2625-2634, 2005.

[7] W. C. Lee, Mobile Cellular Telecommunications: Analog and Digital Systems, 2nd ed.: McGraw-Hill Professional, February , 1995.

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