The Optimal Selection through Allocating Values to Boolean Expressions in Tabulation Method

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Summary
For achieving the ideal size or the time spent for the data processing, the smallest circuits are motivations for the researchers. In the optimization of logical functions, the presented methods have not been benefited from the principal modeling. For this purpose, the optimization of the logical function has been accomplished through representing an appropriate model from the method of the linear modeling. Firstly, the optimization operations has been performed through studying the limitations and considering the coefficients in the linear model. Giving an applied example, the manner of using the represented view has been studied. The optimization method has been conducted with a particular method, which can in turn be regarded as a new work. The represented method can be an ideal option for the optimal selections in optimizing the logical circuits. At first optimizational operations, extracting a mathematical model and working on it, is necessary. There are some mathematical models for the optimizational purposes, however the represented approaches are complicated to some extent, and it is time-consuming to work with them in large scales. In the present research work, some special states have been suggested which is relatively simple and innovative method.

Key words
Optimization, Linear modeling, Logical functions, Selection weight, optimal selections.

1. Introduction

Due to the importance of saving in time and cost and the competition among the producers, finding the best state for the product with the above mentioned features seems necessary. In this paper, the object is selecting the least logical gates for a digital function. Some methods have been represented for the selections one of which is using the Karnaugh map method [1, 2]. In this method tables with 2n square-cells are provided where n is the number of input variables. Clearly, this method is not appropriate for the circuits with more than 5 or 6 inputs, and in practice it is impossible. On the other hand, the result obtained from simplifying the table depends on the initiation and may be different from the more suitable result. There is another suggested way to optimize the logical functions, called the tabulation method or Quine-Mc Cluskey method [2]. By using the combination of the output expressions in the function involved, this method simplifies them as much as possible. In this work, it is supposed that the reader is familiar with the tabulation method, and then a brief explanation is represented from the tabulation method for the familiarization [1, 2].

2. Tabulation method

In this method [4, 5, 6, 7] two Boolean expressions which are different from each other in the binary conversion with regard to the location of the bits are combined, for example two expressions 5 and 7 containing the binary forms 101 and 111, respectively are different only in the second location and their combination result is 1-1. The sign dashed (-) means that the resultant expression contains only two variables, and in this expression the second variable, is don't care. Now it is possible that one expression is not combined with the other expressions and so remains intact.
Furthermore, it is possible that two combinational expressions are combined again. In other words, the combination phases are repeated again. For example, the expression, 1-1 which is the combination result of 5 and 7 is combined with the expression 1-0 which is the combination result of 4 and 6 and their combination result is 1- -. The 1- - is the product of combining the collections, {4, 5, 6, 7}. If the input variables are assumed as x, y and z, for example the expression 6 the binary state of which is 110, the equation of $xy\bar{z}$ or 1-0 will be $x\bar{z}$, because the second variable has been combined and neutralized. Here, following example, explains the necessity of designing a mathematical model for optimization purpose. Assume that the output function of F(w, x, y, z) = E (1,4,6,7,8,9,10,11,15) is considered to be optimized. Based on the tabulation method the following expressions are the final result of the possible combinations (the variables are w, x, y and z. respectively).

-001 ($\overline{xy}z$) : {1,9}

01-0 ($\bar{w}x\bar{z}$) : {4,6}

011- ($\bar{w}xy$) : {6,7}

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-111	(xyz)	:	{7,15}	
1-11	(wyz)	:	{11,l5}	
10--	($w\bar{x}$)	:	{8,9,10,11}	(1)

Consequently it is concluded that:

$$F(w,x,y,z) = \bar{x}yz + \bar{w}x\bar{z} + \bar{w}xy + xyz + wyz + w\bar{x} \tag{2}$$

With little attention, it is observed that the existence or non-existence of some expressions does not change in the state of F(w,x,y,z).

In the case of, $F(w,x,y,z) = \bar{x}yz + \bar{w}x\bar{z} + xyz + w\bar{x}$ the needs of the problem are provided too. So using fewer gates will save the cost in designing the digital circuits.

Now how these selections are conducted so that contained the least gates? To solve this problem a table is suggested as follows where in the left side the simplified expressions and in its upside all of the min-terms have been included; they involve in the combinations of the final result expressions.

As it is observed in Table 1 line intersection related to one expression have been signed with the columns of the expressions of their combination product.

This table is studied so that the columns with only one crossed (*) sign are found. In this table the expressions of {1, 4, 8, 10} have this feature. Therefore these columns as well as the columns common in their sign crossed with the single-sign of the column linearly (i.e. they cover them) are selected in Table 1 with the sign * as Boolean expressions (see extreme left hand of Table 1).So the primary selections are the expressions in which the single-sign columns in their line have been signed with *.

After this stage it is observed that the expression xyz is the best selection among three expressions $\bar{w}$xy, xyz and wyz because it covers both of the expressions.

In the columns 7 and 15 of the Table 1 it was observed that selecting the expression of the fourth line is better than selecting the two third and fifth expressions, this clearly reduce the required gates. The question which is raised is that if the expressions which are not selected in the first main selections are more, and the interference of the expressions in the columns is complex, that estimation method can be useful, and if the result of the observations will be continuously clear? Certainly not.

As it was mentioned for the optimization operation a mathematic model is necessary, and extracted model is explained at the following sections.

3. Configurations of the linear modeling and the limitations

For example, assume that after performing the final operation on the binary data some columns remain according to Table 2.

For this reason that mostly the variables of each expression are different it will be important that how and with what priority the expressions and the columns are selected. It is supposed that X is the set of the variables from the expressions {A, B, ..., F} with the condition that all of the table columns are selected. Here, finding the least rate X is regarded. But how X is described in order that a certain operation can be accomplished on it for selecting the optimal selection? Some special definitions can help us in selecting X. If the variables as xj with the amounts 0 or 1 are attributed to the individual expressions (j = A, B, C, D, E, F) such that 1 means selection of the expression j and 0 means the lack of its selection. Then assuming that r, is the variables number of jth expression, the set rAxA+rBxB+rCxC+rDxD+rExE+rFxF can be attributed to X. It should be mentioned that the concept of the most rate for X by selecting xj = 1 or the least rate for X namely zero by selecting xj = 0 is the case with all of the variables. But it should be considered that the limitations prevent of it in which X equals zero which is necessitated by the columns in the optimization.

Considering that all of the expression columns of the binary expressions in table should be selected, it will be logical if a limitation is attributed to every column. For example the second column limitation in Table 2 can be written as the fallowing:

xB +xC +xE $\geq$ 1

The right hand side of this limitation states that at least one of the binary expressions in the column should be selected. Therefore for every column one limitation should be considered.

4. Linear modeling representation

In general it is possible to consider a model with some limitations as follows [3, 10]. The linear model suggested the minimizing the expression is

$$X = \sum_{j=1}^{n} \alpha_j x_j$$

(The linear model for optimization) where the limitations expression is as following:

$$\sum_{j=1}^{n} w_{1j} x_j \leq (\geq) \beta_1$$

$$\vdots \qquad\qquad \vdots$$

$$\sum_{j=1}^{n} w_{mj} x_j \leq (\geq) \beta_m \tag{3}$$

In the linear model and limitations, the variables and constant rates have been defined as the following:

(Constant Values) $\alpha_j, \beta_j, w_j \in R$

(Values) $x_j \in R$

i=1,2,...,m j=1,2,...,n

In this paper, the object is selecting the variables of x; in which firstly the rate of the linear model in lieu of the variables of the least or the most variables, secondly the condition limitations is established. In other words, considering the linear model and the above-mentioned limitations, some variables should be selected which minimalize the linear model preserving the limitations.

4.1. Analyzing the linear model in the binary system

Here an algorithm is represented which is able to optimize the linear models aforementioned. Before explaining algorithm it is remembered that the analysis of the problems, as following is conducted through "simplex" method in general [3]. This method is based on the matrixes, but since in large dimensions working with matrixes and relating their arrays and the programming is relatively complex. So a simple creative method is used. The linear model and limitations as well as the constant coefficients in the binary system utilizing the, Equation (3), is as follow,

$$X = \sum_{j=1}^{n} r_j x_j$$

(The linear model for optimization) The expression of the limitations is as follows:

$$\sum_{j=1}^{n} b_{ij} x_j \geq 1$$
$$\vdots$$
$$\sum_{j=1}^{n} b_{mj} x_j \geq 1 \tag{4}$$

In the linear model and the limitation of the binary system, the variables and constant values have been defined as following:

(Binary Values) xj =1 or 0

(Constant Values) $r_j \in R$

(Constant Values) bij =l or 0

i=1,2. ..., m, J=1,2,...,n

Studying the equation and the linear expressions (4) shows that by selecting xj = 0 in lieu of the individual is, X=0. But the limitations prevent of X=0; however some of the xj can become zero. This depends on considering the limitations and selecting xjs, and apart from establishing the limitations the linear model in lieu of these variables will be the least value.

lt is obvious that xjs with less coefficient (i.e. less rjs) and with the ability of establishing the possible limitations are in priority for selection. It should also be considered that being the value of other non-selected variables zero does not affect any to the limitations. So to avoid such a state, the minimalization is conducted step by step so that the conditions in the limitations are regarded.

4.2. The optimal selections through allocating value to the expressions

To consider the priority in the selections, the value of Selection Weight (Sw) for every xj is defined as follows. The selection value for every xj or Sw (xj) is the ratio of limitations in which xj in the linear model to coefficients allocated to xj.

Because the limitations in which xj participates is an additional factor or a direct factor to select xj and the allocated coefficient to xj in the linear model is a reducing or reverse factor for selecting xj. So if mj is the number of the limitations in which xj presents, and rj becomes the coefficient of xj in the linear model, the selection weight for xj is given by;

$$Sw(x_j) = \frac{m_j}{r_j} \tag{5}$$

Where mj is the appearance times of xj in the limitations and rj is amount of Boolean variable in expression, xj. The Sw (xj) is allocating the value to the Boolean expressions in the tabulation method in which it has been applied for the optimal selections.

If the Selection weight (Sw) is determined for all of the xj s in the linear model, it will be observed that one or some of the xj s contain the most selection weight.

It is clear that in the case of the equalization of the multi-variable Sws, the variable will have more selection weight with fewer coefficients in the linear model.

In the case of equal selection weights and the coefficients, the variable with equal Sws and coefficients in relative to the other variables will have less weight for the selection in the limitations with the variable having the most Sw (first phase). Because in the case of selection, the selection weight of the variable with more Sw will be reduced.

Now assume that a variable is selected by considering the above cases. This variable establishes several limitations of the problem limitations. So being zero or one about other variable in the mentioned limitations has no difference with them. For this reason, some changes will be created in the selection weight of the non-selected

variables. Therefore in the continuation, the limitations established by the selected variahles will not be considered and the rest of the work will be accomplished based on the remained variables and limitations.
It is obvious that after passing some finite phases, the selection weight of all of the variables will be zero, and after selection the optimal selections the operation will be finished. Studying the special state of the selection, it should be mentioned that the case of overlaying namely covering one or more expressions through one or more other expressions among the selected expressions requires a special optimal state. To highlight the overlaying for the purpose of more minimality of the variables (quantities), a solution will be represented with one example together with representing on algorithm thoroughly.

5. The optimal selections of algorithm design

In this selection, apart from giving an example, the optimal selections are executed of the remainder expressions step by step.
For example, the optimal selections in the columns remainder in Table 3 are ideal where the variables A, B, C, D, E equal 2, 3, 1, 2, and 1, respectively.
It should be mentioned that in Table 3 the top first row is labeled as ordinary numbers 1, 2, 3 and 4 respectively, sequential. The remained expressions in the Table 1 are the prime implicants A, B, C, D and E, where each expressions of prime implicants includes with defined Boolean variables [1, 2].
For the purpose of better selection the following linear model with the limitation related to the table 3 is considered.
Linear Model:

$$X = 2x_A + 3x_B + x_C + 2x_D + x_E$$

Limitation 1: $x_A + x_C \geq 1$

Limitation 2: $x_A + x_B + x_D \geq 1$

Limitation 3: $x_B + x_C \geq 1$

Limitation 4: $x_D + x_E \geq 1$ (6)

The algorithm phases of the optimal selections are:

I. Per, j=A, e, C, D, and E, $Sw(x_j) = \frac{m_j}{r_j}$ is calculated and the obtained rates have been tabulated in Table 4.
II. xj with the greatest Sw is selected.
In this example it is observed that Sw(xj) is maximum and so xj. is selected. Then the limitations with the selected variable are signed with cross (*).

Limitation 1: * $x_A + x_C \geq 1$

Limitation 2: $x_A + x_B + x_D \geq 1$

Limitation 3: * $x_B + x_C \geq 1$

Limitation 4: $x_D + x_E \geq 1$ (7)

If all of the limitations are signed the operation is terminated, if not, the operation is repeated. As it is observed in Table 5 the first phase is ended through selecting xC together with remained expressions. In the second period it is observed that the maximum value for expressions xD and xE, have equal Sws and equal 1, as shown in Table 5.
As it was explained in section 4.2, considering that xE has a less coefficient that xD(rE<rD), xE for the selection is in priority.
So for selecting xE and changing in Sws, we refer to the next phase. Here the equality in Sws as in previous phase has been repeated again, with difference that the coefficients of these two expressions are equal.
rA=rD (8)
Here at first we regard the limitations and observe that xA related to xC and xD to xE considering the Table 5 in the first phase, we observe that Sw(xC+=2 and Sw(xE)=1. So xD should be selected, because xD is out of the limitation with the most value between two expressions xC and xD, i.e. xC. In other words selecting xD reduces the value of xE which has less value than xC. In the later phases, expression with great Sw is selected.
Considering the limitations it is observed that how expression E is covered by expression D. Consequently it is concluded that the best selection is xD and xC in which the amount X is 3. Covering expression E through expression D is a certain phase which occurs rarely and depends on the data. This phase is called a particular state selection. In the next section the manner of particular state selecting is explained. The overall algorithm in the end has been represented in appendix.

5.1. Studying the state of certain (special) selection

The study of a certain selection is conducted through the final selection of the example mentioned in the algorithm; Table 6 is formed located in the left part of the selected variables and in the top of the table, the limitations lie regarding the numbers 1, 2, 3 and 4.
According to the tabulation, all of the columns should be selected in Table 6. Through studying Table 6, it is observed that the single-mark columns have been selected by the expression together with their details (other column signed in the mentioned line). In the end, it is observed that xC and xD cover all of the columns (this is the case without xE too). So through using the linear model to optimize the logical functions X have

been reduced to the linear expression $X_C + 2X_D$ which minimalizes the value X to 3.

6. Conclusion

In this paper it has been shown an appropriate algorithm in which the optimal selections are used to optimize the Boolean functions through the linear modeling.

The optimization method has been accomplished through a new method with allocating the values (weights) to the Boolean expressions. The optimization order has been explained in a flowchart, and in the optimization programmed algorithm of the Boolean functions, the secondary functions have been defined in the programming environment of C through which the optimization operation is accomplished. The method used in the optimization regarding to the Boolean expressions is completely different from the other common methods.

7. Appendix

Algorithm of the optimal-selections expressions representing the complete function of the program is necessary for holding a workshop. However, according to Figure 1 the total procedure has been represented. Considering the data value and the relationships between them, using the pointer has been preferred. The program has been represented in C++ language [9, 10] and can simplify the functions of circuits 2-32 bits. A 32-bit circuit may have 231 outputs and introducing these outputs in a programming language as arrays is not appropriate; since a data set with 3 bits creates no more than 8 outputs in the programme, other arrays will occupy the space memory without utilization.

In this programme 21 secondary functions have been used which apart from representing the flowchart, these functions are introduced. Flowchart of the programme algorithm has been indicated in Figure 1 with the necessary descriptions.

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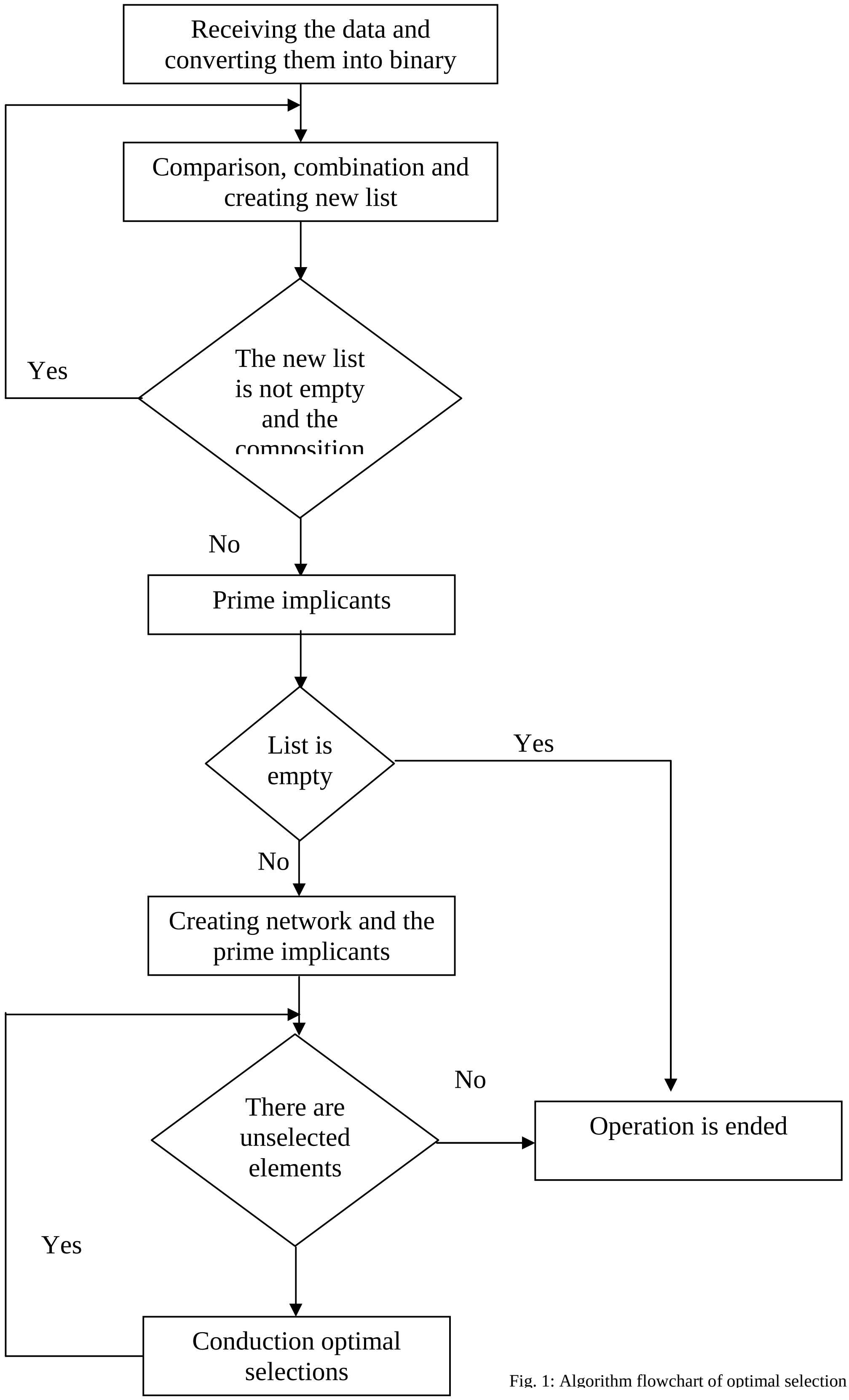

Fig. 1: Algorithm flowchart of optimal selection program

Table 1: Prime Implicant Selections

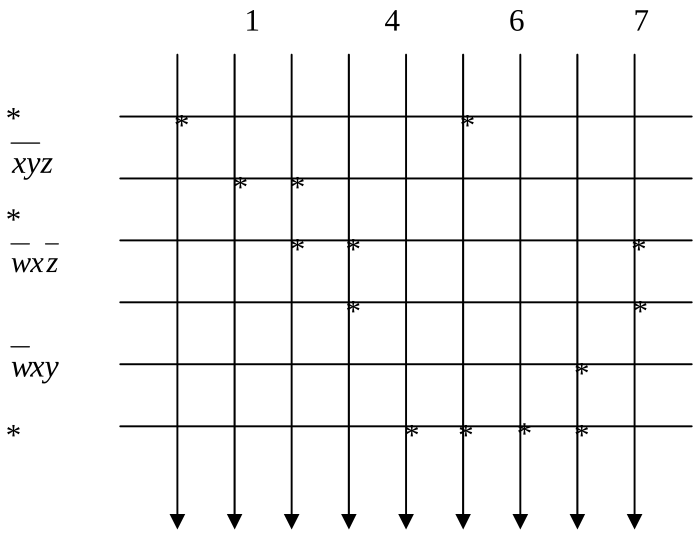

Table 2: The Observation of the Expressions Remained in Table 1

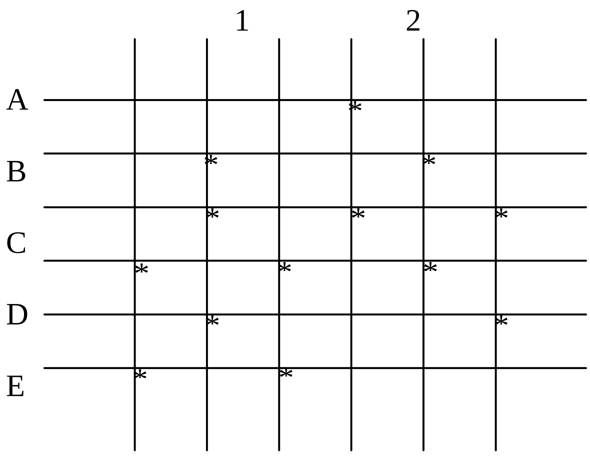

Table 3: Tabular example for the purpose of extracting the linear model together with limitations

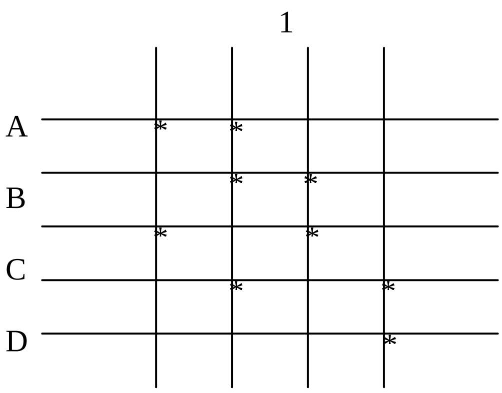

Table 4: Calculating Sw for the Expression.

x_j	r_j	$Sw(x_j)$
x_A	2	$2/2 = 1$
x_B	3	$2/3$
x_C	1	$2/1 = 2$
x_D	2	$2/2 = 1$
x_E	1	$1/1 = 1$

Table 5: Determining the Selection Weight to the Complete Phase

x_j	r_j	$Sw(x_j)$			
		First Phase	Second Phase	Third Phase	
x_A	2	$1 \rightarrow$	$1/2 \rightarrow$	$1/2 \rightarrow$	0
x_B	3	$2/3 \rightarrow$	$1/3 \rightarrow$	$1/3 \rightarrow$	0
x_C	1	$2 \rightarrow$	$0 \rightarrow$	$0 \rightarrow$	0
x_D	2	$1 \rightarrow$	$1 \rightarrow$	$1/2 \rightarrow$	0
x_E	1	$1 \rightarrow$	$1 \rightarrow$	$0 \rightarrow$	0

Table 6: Observing the Special Selection State

Variable	**Column**	**Limitation 1**	**Limitation 2**	**Limitation 3**	**Limitation 1**
$\mathbf{x_C}$		*		*	
$\mathbf{x_D}$			*		*
$\mathbf{x_E}$					*
Result Selection		♦	♦	♦	♦