

# Optimization of MIMO- FFT based OFDM system performance with minimum BER

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## Summary

Alomouti's space-time coding scheme for Multi-Input Multi-Output (MIMO) system has drawn much attention in 4G wireless technologies. Orthogonal frequency division multiplexing (OFDM) is a popular method for high data rate wireless transmission. OFDM may be combined with antenna arrays at the transmitter and receiver to increase the diversity gain and enhance the system capacity on time variant and frequency selective channels, resulting in Multi-Input Multi-Output (MIMO) configuration. This paper explores various physical layer research challenges in MIMO-OFDM system design including channel modeling, space time block code techniques, channel estimation and signal processing algorithms used for performing time and frequency synchronization in MIMO-OFDM system. The existing FFT based OFDM in MIMO the BER is more compare to wavelet based OFDM. But the wavelet based OFDM have more computational and hardware complexity in this paper we proposed mixed radix FFT algorithm with constant multiplier multipath delay feedback scheme. The performance MMO-OFDM is optimized with minimum BER.

## Key words:

*Multi-Input Multi Output (MIMO); orthogonal frequency division multiplexing (OFDM); Bit error rate (BER); signals to noise ratio (SNR); Single input single output (SISO); Space time block code (STBC)*

## 1. Introduction

Orthogonal frequency division multiplexing (OFDM) and space-time coding have been receiving increased attention due to their potential to provide increased capacity for next generation wireless system. OFDM supports high data rate traffic by dividing the incoming serial data streams into parallel low-rate streams, which are simultaneously transmitted on orthogonal sub-carriers [1]. For large enough and a sufficiently large guard interval, the channels as seen by each of the sub-carriers become approximately frequency flat and allow for high order modulation. Due to this desirable feature, OFDM has been

adopted in many commercial system such as the IEEE 802.11a, ETSI HIPERLAN type2 wireless LAN system and DAB, DVB-T broadcasting systems.

Space-time coding is a communication technique for wireless systems that realizes spatial diversity by introducing temporal and spatial correlation into the signals transmitted from different transmits antennas. Many space-time trellis and block codes have been proposed for flat fading channels. Most significantly, Alamouti discovered a very simple space-time block codes (STBC) for transmission with two antennas guaranteeing full spatial diversity and full rate. It lends itself to very simple decoding and has been adopted in third generation (3G) cellular systems such as W-CDMA. Recently many literatures proposed Space-time block coding schemes applicable to OFDM systems based on the Alamouti scheme [2]. When channel can be assumed to be approximately constant during to consecutive OFDM symbol durations, the Alamouti scheme is applied across two consecutive OFDM symbols and is referred to as the Alamouti STBC-OFDM or simply A-STBC-OFDM.

The combinations of the Multiple- Input Multiple – Output (MIMO) signal processing with orthogonal frequency – division multiplexing (OFDM) communication system is considered as a promising solution for enhancing the data rates of the next generation wireless communication systems operating in frequency-selective fading environment the high Throughput Task Group which establishes IEEE 802.11n standard is going to draw up the next generation wireless local area network (WLAN) proposal based on the 802.11a/g which is the current OFDM-based WLAN standards. The IEEE 802.11n standard based on the MIMO-OFDM system provides very high data throughput rate from the original data rate 54 Mb/s to the rate in excess of 600 Mb/s because the technique of the MIMO can increase the data rate by extending an OFDM-based system. However the IEEE 802.11n standard also increase the computational and hardware complexities greatly, compared with the current WLAN standard. It is a challenge to realize the physical

layer of the MIMO-OFDM system with minimal hardware complexity and power consumption.

The FFT/IFFT processor is one of the highest computational complexity modules in the physical layer or the IEEE 802.11n standard. If employing the traditional approach to solve the simultaneous multiple data sequences, several FFT processors are needed in the physical layer of a MIMO OFDM system. Thus the hardware complexity of the physical layer in MIMO OFDM system will be very high. This paper proposes as FFT processor with novel multipath pipelined architecture to deal with the issue of the multiple data sequences for MIMO OFDM applications. The 128/64 FFT with 1-4 simultaneous data sequences can be supported in our proposed processor with minimal hardware complexity. Furthermore, the power consumption can also be saved by using higher radix FFT algorithm.

## 2. Channel Models

### 2.1 Additive White Gaussian Noise Channel

With the transmitted signal vector X, the received signal vector is given by,  $Y=X+N$  where 'n' represents additive white Gaussian noise vector. It follows the normal distribution with Mean  $\mu$  and variance  $\sigma^2$ .

$$f(n) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(n - \mu)^2}{2\sigma^2}\right) \quad (1)$$

### 2.2 Flat Fading channel model

It is modeled as,  $Y= ax + n$  where a is the fading coefficients with PDF and n is the additive Gaussian Noise Vector.

$$f(a) = 2a \exp(-a^2) \quad \text{for } a > 0. \quad (2)$$

### 2.3 Frequency selective fading channel

In this model the channel is considered as a multi-path fading channel. It consists of multiple independent Raleigh faders, which is modeled as complex-valued random processes. By assuming uniform antenna pattern and uniform distributed incident power, the received signal at the receiver can be expressed as

$$y = \sum_j a_j^* x + n \quad (3)$$

Where 'n' is the additive white Gaussian noise and 'j' represents multi-path from transmitter.

## 3. MIMO System.

### 3.1 Space-Time Codes.

Space-time codes (STC) provide transmits diversity for the Multi-Input Multi-Output fading channel. There are two main types of STC's namely space-time block codes (STBC) and space-time trellis codes (STTC). Space-time block codes operate on a block input symbols, producing a matrix output whose columns represents time and rows represents antennas. Their main feature is the provision of full diversity with a very simple decoding scheme. On the other hand, Space-time trellis codes operate on one symbol at a time, producing a sequence of vector symbols whose length represents antennas. Like traditional TCM (TRELLIS CODED MODULATION) for a single-antenna channel, space-time trellis codes provide coding gain. Since they also provide full diversity gain, their key advantage over space-time block codes is the provision of coding gain [3]. Their disadvantage is that they are extremely hard to design and generally require high complexity encoders and decoders.

An STBC is defined by a  $p \times n$  transmission matrix G, whose entries are linear combination of  $x_1, \dots, x_k$  and their conjugates  $x_1^*, \dots, x_k^*$ , and whose columns are pair wise-orthogonal. when  $p=n$  and  $\{x_i\}$  are real, G is a linear processing orthogonal design which satisfies the condition that  $G^T G = D$ , where D is the diagonal matrix with the (i,i)th diagonal element of the form  $(11ix12+12ix22+\dots+1ni xn2)$ , with the coefficients  $11i, 12i, \dots, 1ni > 0$ . Without loss of generality, the first row of G contains entries with positive signs. If not, one can always negate certain columns of G to arrive at a positive row.

$$G_2 = \begin{pmatrix} x1 & x2 \\ -x2 & x1 \end{pmatrix} \quad G_4 = \begin{pmatrix} x1 & x2 & x3 & x4 \\ -x2 & x1 & -x4 & x3 \\ -x3 & x4 & x1 & x2 \\ -x4 & -x3 & x2 & x1 \end{pmatrix} \quad (4)$$

We assume the transmission at the base-band employs a signal constellation A with  $2b$  elements. At the first time slot,  $nb$  bits arrive at the encoder and select constellation signals  $c_1, \dots, c_n$ . Setting  $x_i = c_i$  for  $i=1 \dots, n$  in G yields a matrix C whose entries are linear combinations of the  $c_i$  and their conjugates. While G contains the in determinates  $x_1, \dots, x_n$ , C contains specific c constellation symbols (or linear combinations of them), which are transmitted from the n antennas as follows: At time t, the entries of row t of C are simultaneously transmitted from the n antennas, with the ith antenna sending the ith entry of the row. So each row of C gives the symbols sent at a certain antenna.

### 3.2 Receive Diversity

The base-band representation of the classical two-branch maximal ratio receive combining (MRRC) scheme. At a given time the signal is  $s_0$  is sent from the transmitter. The channel between the transmit antenna and receive antenna zero is denoted by  $h_0$  and between the transmit antenna and the receive antenna one is denoted by  $h_1$  where  $h_0 = \alpha_0 e^{j\theta_0}$   $h_1 = \alpha_1 e^{j\theta_1}$ .

Noise and interference are added at the two receivers. The resulting received base band signals are  $r_0 = h_0 s_0 + n_0$ ,  $r_1 = h_1 s_0 + n_1$ .

Where  $n_0$  and  $n_1$  represents complex noise and interference.

Assuming  $n_0$  and  $n_1$  are Gaussian distributed, the maximum likelihood decision rule at the receiver for this received signal is to choose signal  $s_i$  if and only if (iff).

$$d^2(r_0, h_0 s_i) + d^2(r_1, h_1 s_i) \leq d^2(r_0, h_0 s_k) + d^2(r_1, h_1 s_k) \quad (5)$$

Where  $d^2(x, y)$  is the squared Euclidean distance between signals  $x$  and  $y$  calculated by the following expression:

$$d^2(x, y) = (x - y)(x^* - y^*) \quad (6)$$

The receiver combining scheme for two-branch MRRC is as follows:

$$(\alpha_0^2 + \alpha_1^2 - 1) |s_i|^2 + d^2(s_i, s_k) \leq (\alpha_0^2 + \alpha_1^2 - 1) |s_k|^2 + d^2(s_i, s_k) \quad (7)$$

The maximal-ratio combiner may then construct the signal  $s_0$ , so that the maximum likelihood detector may produce  $s_0$ , which is a maximum likelihood estimate of  $s_0$ .

### 3.3 Alamouti's Transmit Diversity Scheme.

#### 3.3.1 Two Branch Transmit Diversity with One receiver

The base-band representation of the two branches transmit diversity scheme. The encoding and transmission sequence at a given symbol period, two signals are simultaneously transmitted from the two antennas. The signal transmitted from antenna zero is denoted by  $s_0$  and from antenna one by  $s_1$ . During the next symbol period signal  $(-s_1^*)$  is transmitted from antenna zero, and signal  $s_0^*$  is transmitted from antenna one where  $*$  is the complex conjugate operation. The encoding is done in space and time (Space-Time coding) [4]. The encoding may also be done in space and frequency. Instead of two adjacent symbol periods, two adjacent carriers may be used (Space-Frequency).

	Antenna 0	Antenna 1
Time t	$s_0$	$s_1$
Time t+T	$-s_1^*$	$s_0^*$

#### 3.3.2 Transmit diversity with receiver diversity

It is possible to provide a diversity order of  $2M$  with two transmit and  $M$  receive antennas. For illustration, we discuss the special case of two transmit and two receive antennas in detail. The generalization to  $M$  receive antennas is trivial.

The base band representations of the scheme with two transmit and two receive antennas. The encoding and transmission sequence of the information symbols for this configuration is identical to the case of a single receiver.

Similarly, for  $s_1$ , using the decision rule is to choose signal  $s_i$  iff

$$(\alpha_0^2 + \alpha_1^2 + \alpha_2^2 + \alpha_3^2 - 1) |s_i|^2 + d^2(s_i, s_k) \leq (\alpha_0^2 + \alpha_1^2 + \alpha_2^2 + \alpha_3^2 - 1) |s_k|^2 + d^2(s_i, s_k) \quad (8)$$

The combined signals are equivalent to that of four branches MRRC. Therefore the resulting diversity order from the new two branch transmit diversity scheme with two receivers is equal to that of the four branch MRRC scheme. It is interesting to note that the combined signals from the two receive antennas are the simple addition of the combined signals from each receive antenna. Hence conclude that using two transmit and  $M$  receive antenna and then simply add the combined signals from all the receive antennas to obtain the same diversity order as  $2M$ -Branch MRRC.

### 3.4 Channel Estimation

#### 3.4.1 Enhance Channel Estimation

Frequency domain and is written in matrix notation

$$Y = SH + N \quad (9)$$

Where  $Y$  is the Fourier transform of  $y$ ,  $S$  is the Fourier transforms of  $s$ ,  $N$  is Fourier transform of  $n$  and  $H$  is the Fourier transform of  $h$ .  $H$  can also be represented as

$$H = F.h \quad (10)$$

Where  $F$  is  $N \times N$  is the unitary FFT matrix. Therefore  $Y$  can be represented as,

$$Y = SF.h + N \quad (11)$$

$$Y = Qh + n \quad (12)$$

Where  $Q = XF$ . the estimated channel response in time domain can be obtained by the LS Estimator as,

$$h = (Q^H Q)^{-1} Q^H Y \quad (13)$$

Where  $Q^H$  denotes the Hermitian transpose. The successful implementation of the estimator depends on the existence of the inverse matrix  $(Q^H Q)$ . If the matrix  $(Q^H Q)$  is singular (or close to the singular), then the solution does not exist (or is not reliable)[5]. But it is a rare case.

### 3.5 Training Sequence used.

To increase the performance of the channel estimation for OFDM systems in the presence of ISI, Kim and Stuber propose this training sequence given by

$$X(n) = \begin{cases} A \exp(j2\pi(n/2)^2 / N) & n \in N \\ 0 & n \in M \end{cases} \quad (14)$$

Where  $N$  is the set of sub-carrier odd indices, where  $M$  is the set of sub-carrier even indices.

Transmitted data with pilot. It has alternative zeros. By doing so, the transformation of the training sequence in the time domain as the special property that its first half is identical to its second half, while the desirable peak to average power ratio of one is still retained. In our work, this training sequence is applied to the LS estimator for MIMO-OFDM systems.

### 3.6 Channel coefficients

The actual estimated coefficients through least square estimator and error between them. These coefficients are generated using Monte-Carlo simulation. The error is in the order of  $10^{-3}$ .

Estimated	Actual	Error
-0.7239 - 0.6893i	-0.7243 + 0.6895i	-0.0004
-0.0626 - 0.6063i	-0.0627 + 0.6063i	-0.0000
-0.1315 + 0.4757i	-0.1317 - 0.4766i	-0.0009
-0.3951 - 0.0034i	-0.3940 + 0.0030i	0.0011
0.0143 + 0.2363i	0.0138 - 0.2367i	-0.0004
-0.1753 + 0.0735i	-0.1752 - 0.0735i	0.0001
0.1065 + 0.0430i	0.1077 - 0.0429i	-0.0011
-0.0655 + 0.0239i	-0.0652 - 0.0252i	-0.0002
0.0411 + 0.0211i	0.0412 - 0.0209i	0.0000

## 4. MIMO-OFDM System

In the area of wireless communications, MIMO-OFDM is considered as a mature and well established technology. The main advantage is that it allows transmission over highly frequency selective channels at a reduced Bit Error

Rate (BER) with high quality signal. One of the most important properties of OFDM transmission is the robustness against multi-path delay spread [6]. This is achieved by having a long symbol period, which minimizes the inter symbol interference unfortunately this condition is difficult to fulfill in MIMO-OFDM systems, since the GI length is a system parameter, which is assigned by the transmitter but the maximum propagation delay is a parameter of the channel, which depends on the transmission environment. MIMO can be used either for improving the SNR of data rate. For improving the data rate, A-STBC-OFDM system is used.

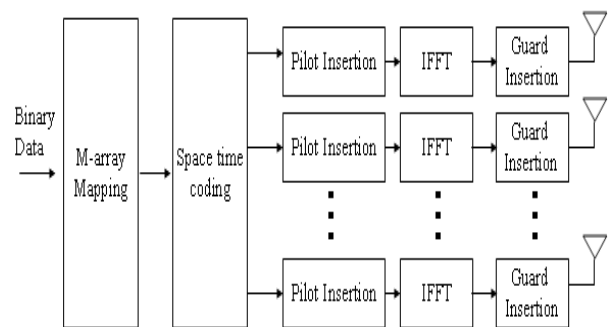


Fig. 1 Transmitter.

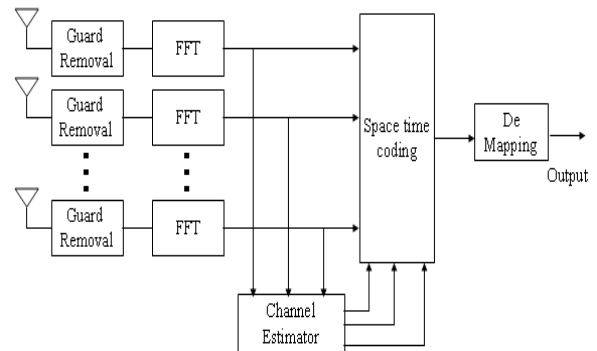


Fig. 2 Receiver.

### 4.1 Proposed FFT Algorithm.

Given a sequences  $x(n)$ , an  $N$ -points discrete Fourier transform (DFT) is defined as

$$X(k) = \sum_{n=0}^{N-1} x[n]W_N^{kn} \quad (15)$$

K=0,1,.....127.

Where  $x[n]$  and  $X(k)$  are complex number. The twiddle factor is

$$W_N^{nk} = e^{-j(2\pi nk/N)} = \cos\left(\frac{2\pi nk}{N}\right) - j \sin\left(\frac{2\pi nk}{N}\right) \quad (16)$$

Because 128-point FFT is not a power of 8, the mixed – radix FFT algorithm, including the radix-2 and radix-8 FFT algorithm, is needed. Since the algorithm has been derived in detail previously [7], it will be described briefly here

First let

N=128

$$n = 64n_1 + n_2, \{n_1 = 0,1 \text{ and } n_2 = 0,1 \dots 63\}.$$

$$k = k_1 + 2k_2, \{k_1 = 0,1 \text{ and } k_2 = 0,1 \dots 63\}.$$

Using (16), (14) can be rewritten as,

$$X(2k_2 + k_1) = \sum_{n_2=0}^{63} \sum_{n_1=0}^1 x[64n_1 + n_2] W_{128}^{(64n_1+n_2)(2k_2+k_1)} \quad (17)$$

$$= \sum_{n_2=0}^{63} \left\{ \sum_{n_1=0}^1 x(64n_1 + n_2) W_2^{n_1 k_1} W_{128}^{n_2 k_1} \right\} W_{64}^{n_2 k_2} \quad (18)$$

### 5. Simulation Results

The performance of the FFT and wavelet based OFDM under AWGN and flat fading channel is good. But in frequency selective fading channel the wavelet based OFDM gives better performance. The BER is less than  $10^{-2}$  may require upto 10dB SNR in wavelet based OFDM. But FFT algorithm BER less than  $10^{-2}$  may require 20dB SNR.

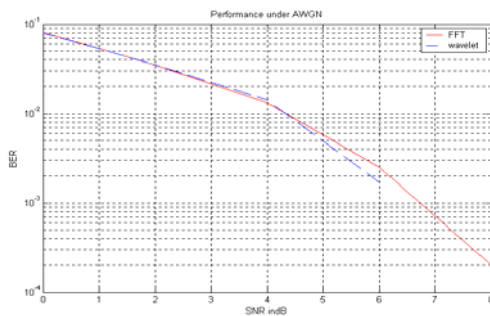


Fig. 3. Performance under AWGN

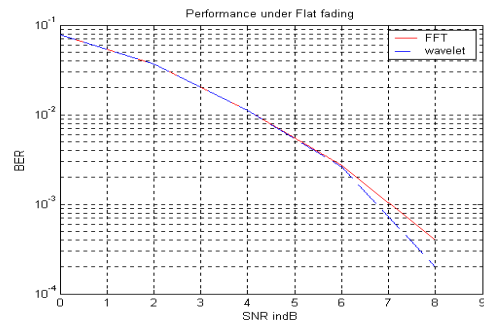


Fig.4. Performance under Flat Fading.

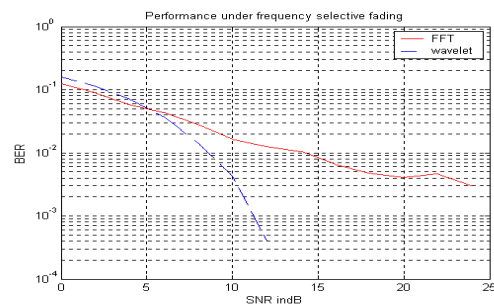


Fig. 5. Performance under Frequency selective Fading

The proposed FFT algorithm the BER is less than  $10^{-2}$  can achieve in 10dB SNR similar to wavelet based OFDM.

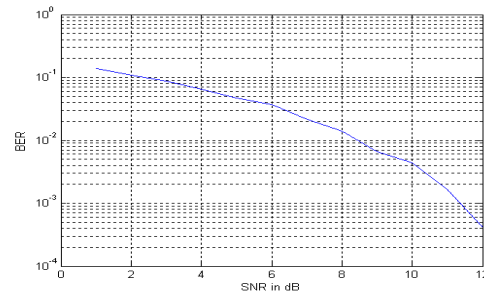


Fig. 6. Performance for Proposed System.

In fading channel, using typical modulation and coding schemes, reducing the effective bit error (BER) in MIMO system from  $10^{-2}$  to  $10^{-3}$  may require only 1-4 dB SNR. Achieve the same in SISO system required greater than 10 dB SNR.

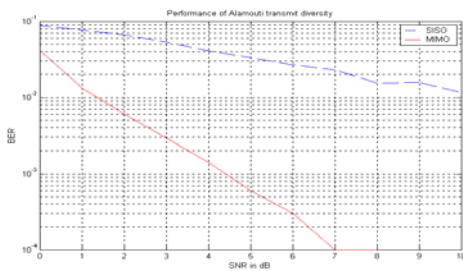


Fig.7. Performance of Alamouti's transmit diversity in proposed system.

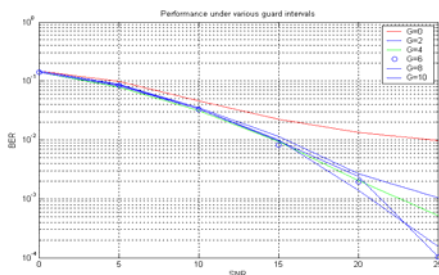


Fig. 8. Performance under various guard intervals in proposed system.

The performance of the MIMO-OFDM is increased when the guard interval is increased. When the guard interval is 10, the BER is decreased less than  $10^{-2}$  in SNR 15 dB.

## Conclusion

OFDM is an effective technique to combat multi-path delay spread for wideband wireless transmission. OFDM with multiple transmit and receive antennas form a MIMO system to increase system capacity. The system with STC (A-STBC-OFDM) and with high guard interval achieves the system requirements of high quality transmission and high data rate transmission. In frequency selective channel the BER is achieve minimum for the proposed FFT based OFDM compare to wavelet based OFDM in MIMO. The performance of the MIMO-OFDM system is Optimized with minimum bit error rate (BER).

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