Synthesis of Antenna Array by Complex-valued Genetic Algorithm

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Summary
The synthesis of the radiation pattern of linear antenna arrays is an interesting problem in radiating systems. A complex-valued genetic algorithm (GA) for optimization of beam forming in linear array antennas is presented in this paper. Unlike conventional GA using binary coding, this method directly represents the array excitation weighting vectors as complex number chromosomes and improves genetic operator methods based on the complex-valued encoding. The algorithm enhances searching efficiency greatly, and avoids effectively premature convergence. Numerical results are presented to illustrate the advantages of the proposed technique over conventional pattern synthesis methods.

Key words:
Antenna array; genetic algorithm; complex-number; synthesis; optimization;

1. Introduction
Array antenna constitutes one of the most versatile classes of radiators due to their capacity for beam shaping, beam steering and high gain [1]. In array-pattern synthesis, the main concern is to find an appropriate weighting vector to yield the desired radiation pattern. Various analytical and numerical techniques have been developed to meet this challenge. Examples of analytical techniques include the well-known Taylor method and Chebyshev method [2]. In recent years, there are several methods which are developed to from nulls in the antenna pattern in the directions of interference signals in the literature [3-10]. The most widely used optimization techniques in antenna array pattern synthesis are steepest decent algorithms [5], genetic algorithms [9], ant colony optimization [10], and so on.

In this paper, an effective method based on genetic algorithm [11] (GA) is proposed for synthesizing a linear antenna array. As an excellent search and optimization algorithm, GA has gained more and more attention and has very wide applications [12, 13]. In recent years, genetic algorithms have also been applied to array beamforming. Haupt [14] applied GA to determine which element should be turned on, in thinned linear and planar arrays to obtain low sidelobe. Yan and Lu [15] used a GA for array pattern synthesis, where the phase and magnitude are restricted to certain discredited values for easy implementation. Yeo and Lu [16] used an improved GA for the correction of the failure of array. Mahanti etc. [17] proposed floating-point Genetic algorithm for the design of a reconfigurable antenna arrays by phase-only control. Li [18] used hybrid genetic algorithm to synthesize the shaped-beam array antennas. In this paper, an improved GA based on the reference [19] is applied to synthesis of the linear antenna array. Since the conventional GA easily gets stuck in local minima and result in prematurity because of the single crossover method, in this paper we expend the coding space of simple GA, and propose an improved complex-valued genetic algorithm. Numerical examples based on single null and multi-nulls are presented to show the effectiveness of this approach.

We arranged the rest of the paper as follows: Section 2 described the problem formulation of the linear array synthesis. In section 3, the improved complex-valued genetic algorithm based on the problem of synthesis of array was given. Numerical simulation experiments and results were presented and comparisons with real-coded GA were made in section 4. Finally, we presented conclusions in section 5.

2. Overview of linear array synthesis
We consider a linear array of 2N isotropic antennas, symmetrically and equally spaced a distance d apart along the x-axis with its center at the origin. It is shown in Fig. 1.

Figure 1: The structure of a 2N-element radiation antenna array.
The free space far-field pattern \( F(\Phi) \) in azimuth plane (x-y plane) with symmetric amplitude distributions is given by Eqn. (1):

\[
F(\Phi) = \sum_{i=1}^{N} f_i e^{jkd(i-1)\cos\Phi}
\]

Here the elements are numbered from the array center and array center is at the origin. Where \( n \) is element number, \( d \) is element spacing and \( k = \frac{2\pi}{\lambda} \) represents the wave number, \( \lambda \) is wavelength, \( \Phi \) denotes azimuth angle of the far-field point measured from x-axis, \( f_i \) is excitation amplitude of the \( n \)th element. All the elements have the same excitation phase. Normalized power pattern, \( P(\Phi) \) in dB can be expressed as follows:

\[
P(\Phi) = 10 \log_{10} \left( \frac{|F(\Phi)|}{|F(\Phi)|_{\text{max}}} \right) = 20 \log_{10} \left( \frac{|F(\Phi)|}{|F(\Phi)|_{\text{max}}} \right)
\]

Then maximum sidelobe level (MSLL) can be computed by:

\[
\text{MSLL} = \max_{\Phi \in \Phi} \{ F(\Phi) \}
\]

where, \( S \) is the sidelobe area of pattern, if the width of zero-power of main beam is \( 2\theta_0 \), then

\[
S = \{ \theta | 0 \leq \theta \leq 90^\circ - \theta \text{ or } 90^\circ + \theta \leq \theta \leq 180^\circ \}
\]

When it is calculated practically, \( S \) should be dispersed by a certain interval (e.g. 0.4°).

### 3. Complex-valued genetic algorithms for antenna arrays

Natural evolution is a search for the fittest in the species space. The success of life on earth demonstrates the effectiveness of this search process. Based on natural evolution [20], genetic algorithms capitalize on tools that work well in nature. It is considered a sophisticated search algorithm for complex, poorly understood mathematical search spaces. Living beings are encoded by chromosomes, with GA’s one encodes the possible solutions in the form of data structures. Thus GAs are capable of arriving at an optimal salutation without the benefit of explicit knowledge concerning the solution space. As a complex-number coded GA (CGA), it is not needed to transform the variable to binary string. Differing from real-coded GA, complex number coded GA uses complex number to represent the variables which are needed to be optimized.

#### A. Coding of chromosomes

Most GAs use binary coding and binary genetic operations [12]. The proposed approach, however, applies complex number genetic operations on array weighting vectors. Hence, each chromosome is a vector of complex number and dimension of the vector is equivalent to the number of array elements. Chromosomes can be showed as:

\[
W = [w_1, w_2, \ldots, w_N]
\]

where, complex number chromosomes \( w_n \) (\( W_n = a_n + ib_n \)) are gene of \( n \)th elements, \( a_n \) and \( b_n \) are the real part and imaginary part of the \( n \)th complex-number chromosomes, respectively. They respond to the value of current amplitude and phase of antenna array respectively.

#### B. Initialization of population

There are a lot of methods of choosing the initialization of population in the literature, such as minimum mean square error (MMSE), windows method (e.g. Chebyshev window), etc. In this paper, considering the universality of algorithm, we choose the random numbers as initial population.

\[
A = \text{Rand}_{\text{popsize}*N}, \quad B = \text{Rand}_{\text{popsize}*N}
\]

where popsize is the size of population, \( N \) is the number of elements. Rand\(_{\text{popsize}*N}\) is a real number random matrix, \( A = [A_1, A_2, \ldots, A_{\text{popsize}}]^T \) and \( B = [B_1, B_2, \ldots, B_{\text{popsize}}]^T \) is popsize\(*N\) directions initial real part and imaginary part matrix.

#### C. Selection operator

We used elitist selection and roulette wheel selection, that is to say, retaining the best individuals in every generation unchanged to the next generation, and other individuals of population were chose by fitness proportionate selection. In this paper, that means for the random number \( r_i \) and the cumulative probability \( q_i \), if it is \( q_{i-1} \leq r_i \leq q_i \), the \( i \)th real part chromosome \( A_i \) and imaginary part chromosome \( B_i \) are chose meanwhile.

#### D. Reproduction operator

In the reproduction operator, we used the crossover operator and mutation operator which had been described in the literature [19]. That is to say, the arithmetic crossover was used in real-part and imaginary part respectively, and the adaptive mutation was used in real-part and Muhlenbein mutation was used in imaginary-part for mutation operator.

#### E. Fitness function

In general, it is desired that the generation of the null with the depth of NLVL in given \( N_n \) directions \( \Phi_i (i=1,2,\ldots,N_n) \),
and achievement in MSLL approach with a certain number
SLVL is the objective of the problem. So the fitness
function to be minimized for optimal synthesis of array
can be defined in Eqn. (6)

\[\text{Fitness} = |\text{MSLL}-\text{SLVL}| + |\text{MNL}-\text{NLVL}| \quad (6)\]

\[\text{MNL} = \max_{i=1}^{N} \{F(\Phi)\} \quad (7)\]

Where, \(\alpha\) and \(\beta\) are weighting coefficients to control the
relative importance given to each term of Eqn. (4),\(\alpha=0.8, \beta=0.2.\)

F. The parameter of genetic operator

Table 1 gives the main parameters’ meaning and values of
the proposed complex-valued genetic algorithm adopted in
the following simulations results. And the parameters of
real-valued genetic algorithm for comparison are taken
from the literature [21].

Table 1: The meaning of parameters and their values of CGA adopted in
simulation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Meaning</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>Population size</td>
<td>50</td>
</tr>
<tr>
<td>(P_c)</td>
<td>Probability of crossover</td>
<td>0.5</td>
</tr>
<tr>
<td>(\tau)</td>
<td>Mutation precision</td>
<td>15</td>
</tr>
<tr>
<td>(\lambda)</td>
<td>Adaptive mutation</td>
<td>2</td>
</tr>
<tr>
<td>(f_{\text{max}})</td>
<td>The largest value of current fitness</td>
<td>0.001</td>
</tr>
<tr>
<td>(n_{\text{elite}})</td>
<td>The number of best individuals which are retained in elite selection</td>
<td>2</td>
</tr>
</tbody>
</table>

4. Simulation results

To illustrate the effectiveness of the proposed algorithm,
simulations are presented here. Considering a uniform
array antenna composed with 10 elements, the element
space is \(\lambda/2.\) Using complex number chromosome represent elements’ excitation, GA can be used to optimize
the pattern by adjust the element’s excitation.

4.1 Experiment1: one nulls

Considering a uniform array antenna was composed with
10 elements, the element space was \(\lambda/2,\) and the current
amplitude of element \(I_n \in (0.0,10.0),\) i.e. the feasible
solution \(\Omega = \{I_1, I_2, ..., I_{10}\}.\) To generate a null in 70°, the width of zero-power was in
pattern 20=20° and in the objective function had been
expressed by Eqn.(6), SLVL=-35.0dB, NLVL=-80.0dB.

Figure 2 depicted the fitness progress curves of RGA and
CGA, obtained by the same original values when null
direction was 70°. Notice that the convergence was observed
for CGA when the iteration reached about 130 generations
and converges at the generation 392. AS shown in Fig. 3,
the pattern of antenna array in case of number of null
direction was 70°. From this figure, the direction of both
RGA and CGA achieved the null in 70°, but the side
level which CGA gained was less than the RGA gained.
In order to make the comparison more obviously, a log file
of the GA progress was recorded. From the results which
were shown in table 2, we can see that for our CGA model,
the depth of null was -80.042821dB. It was better than the
desire which null depth was -80dB basically and better
than the RGA’s -79.80699. Moreover, the maximum
sidelobe level (MSLL) was -34.998549dB which was
gained in the direction of 56.2°, the value was near the
desire value -35dB. While in RGA the value of MSLL was
-31.739991dB which was gained in 74.1°. The other one
which is worth to be noticed is success rate. Success rate
indicates an algorithm’s robustness. In simulation
experiments, both RGA and CGA were excluded 100
times independently, and the times of success (fval<0.001)
were recorded. The success rate of CGA was 82% while
the RGA was 15%. So it can be said that, the proposed
algorithm had a better adapting performance for antenna
array synthesis problem.

4.2 Experiment 2: multi-nulls

To illustrate the effectiveness of the proposed algorithm,
some computer simulations were presented in the
following.

1. Number of nulls \(N_n=3,\) Nulls direction \(\varphi=64, 70\)
and 76.
2. Number of nulls \(N_n=4,\) Null direction \(\varphi=44 110 138 157.\)
3. Number of nulls \(N_n=6,\) Null direction \(\varphi=23.5 43.4 50.4 55 64.8 69.8.\)

In the simulation experiments 1 and 2, SLVL=-35 and
NLVL=-80. The current amplitude of elements \(I_n \in (0.0,10.0),\) i.e. the feasible solution \(\Omega = \{I_1, I_2, \ldots, I_{10}\}.\) To generate a null in 70°, the width of zero-power was in
pattern 20=20° and in the objective function had been
expressed by Eqn.(6), SLVL=-35.0dB, NLVL=-80.0dB.

Table 3 gave the quantitative value of simulation
experiment 2. Table 4 gave the normalized excitation
current amplitudes of elements. In figure 4, (a), (b) and (c)
showed three patterns of antenna arrays which were

synthesized by the proposed algorithm. Here, the dot line in (a) is the results which came from the literature [21]. In literature [21], the maximum null was -75.265dB, and the maximum sidelobe level was -30.3657dB. From the tables and figures, we can see that nerveless the number of null were, the proposed complex-valued can satisfy the requirements of the synthesis of antenna array, not only reached the depth of nulls in prescribed direction but also made the sidelobe level under the certain level.

5. Conclusion

We improved synthesis of antenna array technique by traditional genetic algorithm, and used complex-valued genetic algorithm to synthesize the linear antenna array for confirming current amplitude of elements. It included the generation of nulls and the achievement of the sidelobe level together with the depth of null reaching the desired value. In this paper, the proposed complex-valued genetic algorithm was utilized as an array pattern synthesis method. Four simulation experiments were implemented along with the number of nulls was 1, 3, 4 and 6 respectively. Simulation results indicated that the proposed algorithm could enlarge the optimal space, thus improving the performance of global search in the genetic algorithm. Consequently, promising results could be gained by the proposed complex-valued genetic algorithm.

References

Figure 2: Convergence curve for RGA and CGA (φ=70º).

Figure 3: Pattern of antenna array (φ=70º).

Table 2: The comparison value in synthesis of antenna array (φ=70º).

<table>
<thead>
<tr>
<th>generation</th>
<th>RGA</th>
<th>CGA</th>
</tr>
</thead>
<tbody>
<tr>
<td>fval</td>
<td>2.6463</td>
<td>0.0097</td>
</tr>
<tr>
<td>MNL (dB)</td>
<td>-79.808699</td>
<td>-80.042821</td>
</tr>
<tr>
<td>MSLL (dB)</td>
<td>-31.739991</td>
<td>-34.998549</td>
</tr>
<tr>
<td>Success rate (%)</td>
<td>15</td>
<td>82</td>
</tr>
</tbody>
</table>

Table 3: The quantitative value of simulation experiment 2

<table>
<thead>
<tr>
<th>n=3</th>
<th>n=4</th>
<th>n=6</th>
</tr>
</thead>
<tbody>
<tr>
<td>fval</td>
<td>0.0093</td>
<td>0.2719</td>
</tr>
<tr>
<td>MNL</td>
<td>-79.9684</td>
<td>-80.0001</td>
</tr>
<tr>
<td>MSLL</td>
<td>-34.9963</td>
<td>-34.6601</td>
</tr>
</tbody>
</table>

Table 4: normalized excitation current amplitudes of elements

<table>
<thead>
<tr>
<th>Nn=3</th>
<th>Nn=4</th>
<th>Nn=6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,20</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>2,19</td>
<td>1.9621</td>
<td>1.8675</td>
</tr>
<tr>
<td>3,18</td>
<td>2.4086</td>
<td>2.5010</td>
</tr>
<tr>
<td>4,17</td>
<td>3.0098</td>
<td>3.1545</td>
</tr>
<tr>
<td>5,16</td>
<td>4.7149</td>
<td>3.5521</td>
</tr>
<tr>
<td>6,15</td>
<td>4.8847</td>
<td>3.7293</td>
</tr>
<tr>
<td>7,14</td>
<td>5.3780</td>
<td>4.4026</td>
</tr>
<tr>
<td>8,13</td>
<td>5.4123</td>
<td>4.9406</td>
</tr>
<tr>
<td>9,12</td>
<td>5.5018</td>
<td>5.5171</td>
</tr>
<tr>
<td>10,11</td>
<td>5.5363</td>
<td>5.5565</td>
</tr>
</tbody>
</table>

Here, the dot line in (a) is the results which came from the literature [21].
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