Improved Image De-Noising in Haar – Wavelet Domain Using Neighboring Coefficients

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Summary

In this paper we present the performances of the Discrete Haar Transform in image de-noising using the improved NeighShrink method. Beside the fact that the Discrete Haar Transform is an orthogonal transform, it can be very rapidly calculated with a specific algorithm. The results yielded by this method through simulations were compared to the ones obtained using the VisuShrink and original NeighShrink methods and proved to be better.

Key words:

Image de-noising, Haar Transform, Thresholding Haar Coefficientss.

1. Introduction

Generally, noise supression is a major part of the image processing systems. An image is inherently more or less corrupted by noise in the acquiring stage as well as during the processing. The aim of the de-noising stage is to improve the quality of an image affected by a given type of noise. Image quality is most frequently measured by the peak value of the signal-to-noise ratio (PSNR) or simply by the signal-to-noise ratio (SNR) [1]. Some of the traditional de-noising methods include a linear processing stage like Wiener filtering [2], and others resort to a non-linear technique in the case of a Gaussian-type noise [3].

Recently, the noise supression methods most frequently used are achieved in the wavelet transform domain. The most widely used method was proposed by Donoho [4]; in this approach the noise supression is done by thresholding the wavelet coefficients using an universal threshold. The methods based on wavelet coefficient thresholding are mainly focused on a statistical modeling of the wavelet coefficients and an optimal choice of the threshold values [5].

The de-noising of images affected by Gaussian noise using the technique of wavelet coefficient thresholding is particularly efficient due to its feature of retaining most of the image energy in a few transform coefficients. The most common noise type is the Gaussian additive noise. Thus for an image J(i,j) corrupted by a Gaussian additive noise one can write:

$$J(i,j) = I(i,j) + Z(i,j) \qquad (1)$$

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where I(i,j) is the image un-affected by noise and Z(i,j) is the Gaussian-type noise described by a Gaussian distruibution N(0,σ). An issue to be solved is to estimate the original image as exactly as possible according to various criteria. In the wavelet domain, if an orthogonal transform is used, this problem can be formulated as follows:

$$Y(i,j) = W(i,j) + N(i,j) \qquad (2)$$

where Y(i,j) are the wavelet coefficients affected by noise, W(i,j) are the wavelet coefficients un-affected by noise and N(i,j) is the Gaussian noise.

The technique of image noise filtering in the wavelet domain can be summarized as follows:

i) Calculate the Wavelet Transform of the image affected by noise;
ii) Apply a technique of thresholding the wavelet coefficients affected by noise;
iii) Calculate the inverse wavelet transform obtaining the noise-free filtered image;

In this work we propose the use of a modified version of the NeighShrink method, namely the method proposed by Cho and Chen [6], using the Discrete Haar Transform as a wavelet transform.

2. Haar-Wavelet Transform

The Haar Transform is based on the Haar functions. A Haar function is a function whose dilated and translated versions form a Riesz basis for the square-summable function space $L^2(R)$. Most of the Haar functions are derived from a given scaling function ϕ which satisfies a scaling equation:

$$\phi(x) = \sum_{i \in Z} c_i \phi(2x - i) \qquad (3)$$

Given such a function, the space V^0 is defined as the closure of the linear space generated by the integer translations of the function ϕ, $\phi_i^0(x) = \phi(x-i), i \in Z$ and for each $j \in Z$, the space V^j is considered as a closure of the linear space generated by the integer translations of the dilated ϕ function, $\phi_i^j(x) = \phi(2^j x - i), i \in Z$.

These 2^j functions ϕ_i^j form the V^j space basis. Thus an infinite chain of vector spaces is obtained: $V^0 \subset V^1 \subset V^2 \subset ... \subset V^j \subset V^{j+1} \subset ...$.

Based on these functions the so-called mother Haar functions are defined as:

$$\psi(x) = \sum_{i \in Z} (-1)^i c_{1-i} \phi(2x - i) \tag{4}$$

Corresponding to the functions ϕ_i^j the functions $\psi_i^j(x) = \psi(2^j x - i)$, $i \in Z$, are derived, which form a basis in W^j. For each j index it can be written:

$V^j = V^{j-1} \oplus W^{j-1} = V^{j-2} \oplus W^{j-2} \oplus W^{j-1} = V^0 \oplus W^0 \oplus W^1 \oplus ... \oplus W^{j-2} \oplus W^{j-1}$

The Haar transform of a signal is in fact a projection of the signal onto the Haar basis. For the two-dimensional case the following separable bases are considered:

$$\psi^1(x_1, x_2) = \psi(x_1)\phi(x_2), \qquad \psi^2(x_1, x_2) = \phi(x_1)\psi(x_2),$$
$$\psi^3(x_1, x_2) = \psi(x_1)\psi(x_2) \tag{5}$$

The 2D Haar Transform can be derived from the 1D Haar Transform. Thus the 1D transform can be applied row-wise and then column-wise to an image. This procedure yields four Haar coefficient channels, namely: low-low (LL), low-high (LH), high-low (HL) si high-high (HH).

LL	HL
LH	HH

Fig. 1 The four Haar coefficient channels

Following the above procedure, a single-level decomposition of an image is realized. The images that must have the size 2^{2N} can be decomposed into a maximum number of N levels which is similarly done only on level LL.

3. NeighShrink Modified Method

The Discrete Haar Transform can be implemented through recursively applying a filter bank made up of a low-pass filter and a high-pass filter on the same low frequency Haar coefficients. This implies that the Haar coefficients are correlated within a small neighborhood. Thus a large coefficient will be surrounded also by large coefficients. Based on this property of the Haar coefficients, a thresholding method is proposed in [7], which takes into account also neighboring coefficients of the thresholded coefficient. The image decomposition through a wavelet (in particular Haar) transform will give four regions of wavelet coefficients (LL, LH, HL, HH) for each decomposition level. Due to the fact that in the low-frequency coefficient region (LL) it is desired to maintain the coefficient values, the thresholding procedure will be applied only to the coefficients in the high-frequency regions (LH, HL, HH). Thus, denoting by $d(i,j)$ the Haar coefficients of the regions of interest, for each coefficient a centered squared neighborhood $B(i,j)$ is defined. It must be mentioned that the procedure of Haar coefficient thresholding is done independently for each coefficient region.

Let us define $S^2(i,j) = \sum d^2(i,j)$ over the neighborhood $B(i,j)$. Each coefficient to be thresholded will be first shrinked as follows:

$$d(i,j) = d(i,j) \cdot \beta(i,j) \tag{6}$$

where the shrinking factor has the form:

$$\beta(i,j) = \left(1 - \lambda^2 / S^2(i,j)\right)_+ \tag{7}$$

The sign + at the end of the formula indicates that only the positive values will be retained, while for the rest *beta* will take the value zero. The factor λ is the threshold used by Donoho: $\lambda = \sqrt{2\sigma^2 \log n^2}$, with the specification that n^2 is the size of the neighborhood $B(i,j)$ and not of the entire image.

The modified NeighShrink method proposed here envisages a further shrinking of the Haar coefficients. This can be achieved by decreasing the factor $\beta(i,j)$. Thus the factor $\beta(i,j)$ becomes:

$$\beta(i,j) = \left(1 - c \cdot \lambda^2 / S^2(i,j)\right)_+ \tag{8}$$

where c is a constant.

Noise supression using the NeighShring method brings about an image blurring, but this drawback can be eliminated through a geometric averaging of the image.

4. Experimental Results

In order to obtain comparative results through the three discussed methods we have used in simulations three commonly used images: Lena, Cameraman and Peppers. These images have the same size 256x256 and were affected by Gaussian additive noise of various levels. For the simulations the noise standard deviation had values in the interval [5 - 100]. Image decomposition in the wavelet domain was done on a single level and the size of the neighborhood window in the NeighShrink method is 3x3.
The results obtained through the used methods will be compared based on the peak signal-to-noise ratio (PSNR).

$$PSNR = 20\log 10(255^2 / MSE) \tag{9}$$

The MSE measure represents the mean square error between the image unaltered by noise and the image resulted from the de-noising process:

$$MSE = \frac{1}{M \cdot N}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}\left|I(m,n) - J'(m,n)\right|^2 \tag{10}$$

The first investigations envisaged the improvement of the NeighShrink method. More specifically, we intended to evaluate the influence of the coefficient *c* which occurs in the expression of the shrinking factor *β*. In the figure below we presented the influence of the coefficient *c* variation on the NeighShrink method for different noise levels.

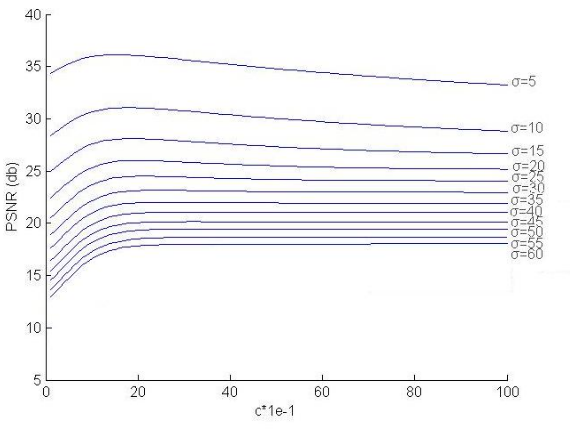

Fig. 2 The contribution of the coefficient c at different noise levels

From the figure an optimum value of the coefficient for all noise levels can be noticed namely for *c*=1,5.
Setting the coefficient to this value and using noise of various levels, for the three methods the results shown in Fig. 3 were obtained:

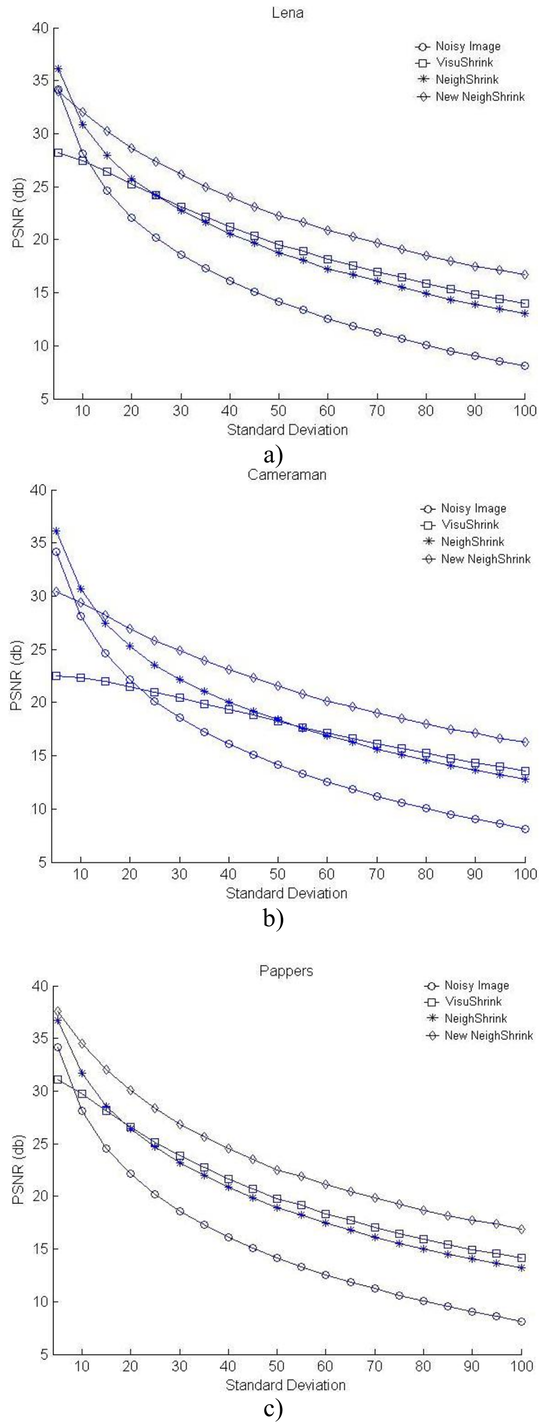

Fig. 3 PSNRs obtained for the VisuShrink, NeighShrink and modified NeighShrink

From these results it can be concluded that the proposed method gives improved results compared to other methods for high noise levels while for low noise levels the results are comparable with those given by the original NeighShrink method, but better than those obtained using the VisuShrink method.

4. Conclusions

In this work we presented an improved version of a de-noising method which takes into account also the wavelet coefficients situated in the neighborhood of the coefficient to be thresholded. As a wavelet transform we used the Discrete Haar Transform which besides being easy to calculate is also very efficient. The results obtained from simulations prove that the proposed de-noising method performs better than the VisuShrink and original NeighShrink methods.

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