Reconstructing an image by blocks with Legendre Moments

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Abstract
Moment functions are widely use in image analysis as feature descriptors for pattern recognition. In this work, we propose a method to recognition problem using Legendre moments. The proposed approach is based on the decomposition of the original image into block images. The optimal number of moment used to represent original image is deduced from the measure of the error between the original image and its reconstructed. Servo image is used to demonstrate the performance of the proposed method.

Key words:
Orthogonal Legendre moments, Zernike Moments, Moment Invariants

1. Introduction

The mathematical concept of moments and function moments has been around for many years and has been utilized in many fields ranging from mechanics and statistics to pattern recognition, detection of pathology and scene analysis [1-4].
Historically, the first significant work considering moments for pattern recognition was performed by Hu [5]. Teague [6] has suggested the notion of orthogonal of moments to cover the image from moments based on the theory of continuous orthogonal polynomials, and has introduced both Zernike and Legendre moments. Many works have focused on the reconstruction aspect of orthogonal moments and have shown that the image can be reconstructed easily from a set of orthogonal moments: Moment Invariants (MI) [5], Zernike Moments (ZM) [7][8] and Legendre Moments (LM)[9]. MI is poor in the representation of image shape due to its non-orthogonality. LM and ZM can be used to represent an image with minimum amount of redundancy of information. ZM has superior performance both as region-based and shape-based descriptor but is computationally complex when compared to LM [10]. To compute the ZM of an image the centre of the image is taken as the origin and the pixel coordinates are mapped to the range of unit circle. Those pixels that fall outside unit circle are not used in the computation [11]. This has motivated us to use LM to represent the image in this work.
For selecting an optimal number of moments from the digital images, Teh and Chin [12] have considered the mean square error between an image and its reconstructed version as a good measure of image representation ability. Liao and Pawlak [13] suggested a statistic cross-validation methodology. However, all methods depend on the unknown original image function or difficult in its implementation.
In this work, the Legendre moments representation and reconstruction method by block processing is proposed. The optimal order of reconstruction is automatic selection by using the local error of each block image.
The paper is organized as follows: in the section 1,

2. Image reconstruction from Orthogonal Moments

The orthogonal functions and their moments have been utilized as features in much image processing application pattern recognition, scenes analysis, localisation of pathology and target identification. Moments of image are treated as region-based shape descriptor.

2.1 Legendre Polynomial

The nth-order Legendre polynomial is defined by

$$p_n(x) = \sum_{i=0}^{n} a_{ni} x^i = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n \tag{1}$$

The Legendre polynomials have the generating function

$$\frac{1}{\sqrt{1-2rx+r^2}} = \sum_{i=0}^{\infty} r^i p_i(x) \qquad r < 1 \tag{2}$$

From the generating function_ the recurrent formula of the Legendre polynomials can be acquired straightforwardly:

$$\frac{d}{dr}\left(\frac{1}{\sqrt{1-2rx+r^2}}\right) = \frac{d}{dr} \sum_{i=0}^{\infty} r^i p_i(x)$$

$$\frac{x-r}{(1-2rx+r^2)^{3/2}} = \sum_{i=0}^{\infty} i r^{i-1} p_i(x) \tag{3}$$

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$$(x-r)\sum_{i=0}^{n} r^i p_i(x) = (1-2rx+r^2)\sum_{i=0}^{\infty} ir^{i-1} p_i(x)$$

Then we have

$$xp_k(x) - p_{k-1}(x) = (k+1)p_{k+1}(x) - 2xkp_k(x) + (k-1)p_{k-1}(x) \quad (4)$$

Or the recurrent formula of the Legendre polynomials:

$$p_{n+1}(x) = \frac{2n+1}{n+1} xp_n(x) - \frac{n}{n+1} p_{n-1}(x) \quad (5)$$

The Legendre polynomials $p_n(x)$ are a complete orthogonal basis set on the interval $[-1 \quad 1]$:

$$\int_{-1}^{1} p_m(x)p_n(x)dx = \frac{2}{2m+1}\delta_{mn} \quad (6)$$

Where δ_{mn} is the Kronecker function, that is:

$$\delta_{mn} = \begin{cases} 1 & if\ m=n \\ 0 & otherwise \end{cases} \quad (7)$$

2.2 Legendre Moments

Legendre moments belong to the class of orthogonal moments and they were used in several pattern recognition applications. They can be used to attain a near zero value of redundancy measure in a set of moment functions so that the moments correspond to independent characteristics of the image. The definition of Legendre moments has a form of projection of the image intensity function into Legendre polynomials.
The two-dimensional Legendre moments of order ($p+q$), with image intensity function f(x,y), are defined as:

$$L_{mn} = \frac{(2m+1)(2n+1)}{4} \int_{-1}^{1}\int_{-1}^{1} p_m(x)p_n(y)f(x,y)dxdy \quad (8)$$

with $(x,y) \in [-1,1]^2$

The recurrence relation of Legendre polynomial $p_m(x)$, is given as follows;

$$p_m(x) = \frac{(2m-1)p_{m-1}(x) - (m-1)p_{m-1}(x)}{m} \quad (9)$$

Where $p_0(x)=1,\ p_1(x)=x$ and $m>1$. Since the region of definition of Legendre polynomials is the interior of $[-1 \quad 1]$, a square image of $N \times M$ pixels with intensity function $f(i,j)$, ($0 \le i < N$ and $1 \le j < M$), is scaled in the region of $-1 \le x,y \le 1$, as a result of this, equation (8) can now be expressed in discrete form as:

$$L_{mn} = \beta_{mn} \sum_{i=0}^{N-1}\sum_{j=0}^{M-1} p_m(x_i)p_n(y_j)f(i,j) \quad (10\ A1)$$

Where β_{mn} is the normalizing constance:

$$\beta_{mn} = \frac{(2m+1)(2n+1)}{NxM} \quad (11)$$

x_i and y_j denote the normalized pixel coordinates in the range of $[-1 \quad 1]$, which are given by

$$x_i = \frac{2i}{N-1} - 1 \qquad y_j = \frac{2j}{M-1} - 1 \quad (12)$$

If only Legendre moments of order $\le Mx$ are given, the function $f(i,j)$ can be approximated by a truncated series:

$$f(i,j) \cong f_{Mx}(i,j) = \sum_{m=0}^{Mx}\sum_{n=0}^{m} L_{mn} p_{m-n}(i)p_n(j) \quad (13)$$

3. Image Reconstruction from Legendre Moments

The input image, which is described by the function $f(x,y)$, is partitioned into square blocks of pixels of size (k, l), a thing that produces a number of sub-images which will be reconstructed separately.
Let $N \times M$ be the image size by pixels and let (k, l) represent the block size. The number of image blocks is given by $\frac{N}{K} \times \frac{M}{l}$.
The image function f(x,y) can be expressed by image blocks as follows:

$$f(x,y) = \bigcup_{a\in[1,N/K]} \bigcup_{b\in[1,M/l]} f^{a,b}(x,y) \quad (14)$$

Where $f^{a,b}(x,y)$ is the sub-image associated to block {a,b} .
The equation (A1) can rewrite each image block as

$$L_{mn}^{a,b} = \beta'_{mn} \sum_{i=a}^{a+k-1}\sum_{j=b}^{b+l-1} p_m(x_i)p_n(y_j)f(i,j) \quad (15)$$

With $\beta'_{mn} = \dfrac{(2m+1)(2n+1)}{kxl}$

The block image function reconstructed from $L_{mn}^{a,b}$ to a given order θ can intuitively be defined as:

$$\tilde{f}^{a,b}(x,y) = \sum_{m=0}^{R}\sum_{n=0}^{m-n} p_{m-n}(x)p_n(y)L_{m-n,n}^{a,b} \quad (16)$$

4. Optimal-order moments

To measure the error between the original image and its reconstructed version is given by:

$$error(\widetilde{f}_{Mx}, f) = \int_{-1}^{1}\int_{-1}^{1} (f(x,y) - \widetilde{f}_{Mx}(x,y))^2 \, dxdy \quad (17)$$

wherein Mx is the highest moment order involved in reconstruction, and $\widetilde{f}_{Mx}(x,y)$ represents the reconstructed image from $f(x,y)$ and $-1 \le x, y \le 1$.

The normalized mean square error between the original image $f(x,y)$ and the reconstructed image $\widetilde{f}_{Mx}(x,y)$ is defined by:

$$e_{Mx} = \sqrt{\frac{error(\widetilde{f}_{Mx}, f)}{\int_{-1}^{1}\int_{-1}^{1} (f(x,y))^2 \, dxdy}} \quad (18)$$

The local error between the original block image and its reconstructed version block can be approximated by the given expression:

$$Localerror(\widetilde{f}^{a,b}_{Mx}, f^{a,b}) = \frac{\sum_{i=a}^{a+k}\sum_{j=b}^{b+l}(f(i,j) - \widetilde{f}_{Mx}(i,j))^2}{\sum_{i=a}^{a+k}\sum_{j=b}^{b+l}(f(i,j))^2}$$

(19)

and the global error is computed from the whole local error:

$$\overline{error}_{Mx,i} = \sum_{\substack{0<a\le N/k \\ 0<b\le M/l}} Localerror(\widetilde{f}^{a,b}_{Mx}, f^{a,b}) \quad (20)$$

With i is the order of moment.

To measure the performance of resemblance between the block images $\widetilde{f}_{Mx,i}$ and $\widetilde{f}_{Mx,i-1}$ can be expressed by:

$$Derror_{Mx} = \overline{error}_{Mx,i} - \overline{error}_{Mx,i-1} \quad (21)$$

5. Experimental results

In this section, simulation results are provided to validate the framework developed in the previous section. The image which is shown in figure (1) is used as the test image (Medical Image).

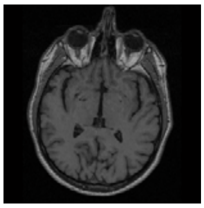
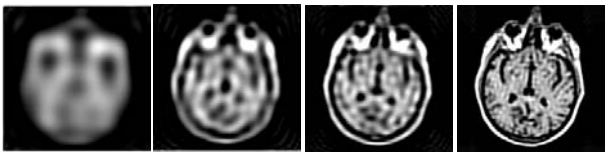


Figure 1: The original image

The original image is reconstructed by blocks using the Legendre moment which is shown in figure (2).



(a) (b) (c) (d)

Figure 2: The reconstructed image using Legendre moments with the block size (4 × 4) at the orders 20(a), 25(b), 34(c) and 42 (d)

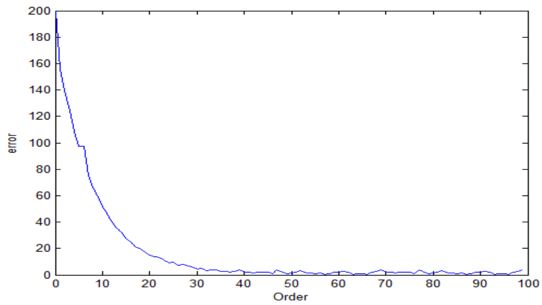


Figure (3) shows the error (Eq 20) values from the reconstructed image from order 1 up to order 90. It should be noted that the error decreases monotonically after order 30.

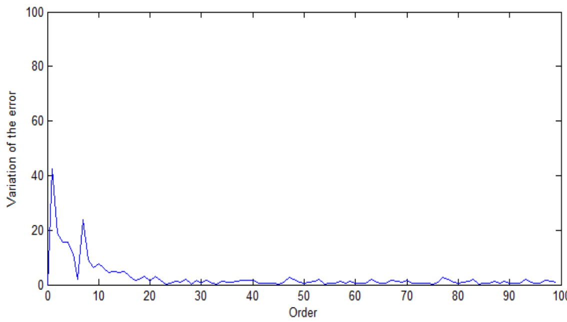


Figure (4) show the variation of error (Eq 21).

Conclusion

In this work a brief historical survey of the development of concept of moments, first introduced by Hu and improved with Teague, Zernike and Legendre, was provided. After a demonstration of image reconstruction from orthogonal moments, the Legendre Moments technique (LM) was proposed. Being of an orthogonal nature, this method proved to be with high performance as it is used to attain a nearly zero value of redundancy in a set of moments functions – by contrast to the Zernike Moments method (ZM) whose the only flaw is that it is computationally more complex.

Reference

[1] F. L. Alt, “Digital pattern recognition by moments,” Journal of the Association for Computing Machinery, vol. 9, no. 2, pp. 240–258, 1962.
[2] M. K. Hu, “Pattern recognition by moment invariants,” Proc. IRE, vol. 49, pp. 1428, September 1961.
[3] R. J. Prokop and A. P. Reeves, “A survey of moment-based techniques for un-occluded object representation and recognition,” Graphical Models and Image Processing, vol. 54, no. 5, pp. 438–460, 1992.
[4] C. -H. Teh and R. T. Chin, “On digital approximation of moment invariants,” Computer Vision, Graphics, and Image Processing, vol. 33, no. 3, pp. 318–326, 1986.
[5] M. K.Hu, “Visual pattern recognition by moment invariants,” IRE Transactions on Information Theory, vol. 8, no. 2, pp. 179– 187, 1962.
[6] M.R. Teague, Image analysis via the general theory of moments, J. of Opt. Soc. Am., vol.70, no.8, pp.920-930, 1980
[7] M. Zhenjiang, “Zernike moment – based image shape analysis and its application”, Pattern recognition Letters, Vol. 21, 169-177, 2000.
[8] A. Khotanzad and Y. H. Hong, “Invariant image recognition by Zernike moments,” IEEE Trans. on Pattern Analysis and Machine Intelligence, vol. 12, no. 5, pp. 489–497, 1990.
[9] J.D. Zhou, H. Z. Shu, L.M. Luo and W. X. Yu, “two new algorithms for efficient computation of legendre moments,” Pattern recognition, vol. 35, no. 5, pp. 1143–1152, 2002.
[10] Y. S. Kim and W. Y. Kim, “Content based trademark retrieval system using a visually salient features,” Image and Vision Computing, vol. 16, no. 3, pp. 931–939, 1998.
[11] W. Y. Kim and Y. S. Kim, “A region based shape descriptor using Zernike moments,” Signal Processing: Image Communication, vol. 16, pp. 95–102, 2000.
[12] C.-H. Teh and R. T. Chin, “On image analysis by the methods of moments,” IEEE Trans. on Pattern Analysis and Machine Intelligence, vol. 10, no. 4, pp. 496–513, 1988.
[13] S. X. Liao and M. Pawlak, “On image analysis by moments,” IEEE Trans. on Pattern Analysis and Machine Intelligence, vol. 18, no. 3, pp. 254–266, 1996.