

Modeling Security Policies with Recommendations

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Summary

Classical security policies are generally expressed through permissions, prohibitions and obligations. Deontic logic is commonly used for modeling such security rules. We recently emphasize the need of the recommendation modality and we tried to formally specify this new notion by extending the Deontic logic. In this paper we first develop further our Recommendation Specification Language. Then, in order to be able to reason on the security policy and to derive new rules, we give more details about our new recommendation-based axiomatic. Finally, we prove that our new formal system is semantically complete and sound.

Key words:

Information systems security, security policies, access control models, Deontic logic.

1. Introduction

Over the last two decades, information and communication systems have continually been connected, open, pervasive, powerful and complex. At the same time it has become increasingly difficult to achieve an acceptable level of security, especially for sensitive and critical systems. Consequently, several key issues need to be investigated in this priority field: Information systems security.

Dealing with this issue every security study should identify an “organizational security policy”: a set of security rules, procedures, or guidelines imposed (or presumed to be imposed) now and/or in the future by an actual or hypothetical organization in the operational environment [1]. Such an organizational security policy usually relies on an access control policy, which defines who has access to what, when and in which conditions. However, a security policy does not guarantee a secure functioning of the system; more important, it could be badly designed, inconsistent or incomplete, especially for huge systems that are congested and under stress. Therefore, The fundamental objective, now is to provide a continuous monitoring of the policy to verify its coherence and detect possible conflicting situations (e.g., situation where a certain user has the recommendation (or the permission) and the prohibition to carry out a certain action on the same object); and also guarantee that all the security objectives are covered by the security mechanisms implementing the policy; etc. In this context, an access control model is often used to rigorously specify and

reason on the access control policy (permissions, obligations and prohibitions rules that control the actions performed by subjects on objects).

Moreover, even if most of existing access control models, tools and mechanisms only deal with permissions and prohibitions, it is equally clear that obligations and recommendations become more and more present in current systems, and thus, should be modeled and enforced. To manage that situation, actually many several works was intended to model obligations. As we know, obligations can be useful to impose some internal or external, manual or automatic actions that should be respected or carried out by users or by the system itself.

So, For example, in the OrBAC model [17], security rules have the form Access Modality (org, r, v, a, c); while Access Modality is a Permission, Obligation or a Prohibition. This rule means: in the context c, organization org grants role r the permission or the obligation or the prohibition to perform activity a on view v.

Bettini et al. distinguish between provisions and obligations [3]. Provisions are conditions that need to be satisfied or actions that must be performed before a decision is rendered, whereas obligations are actions that must be fulfilled by either the users or the system after the decision.

However, these access modalities do not consider the interaction between system and user: system recommend user to do something-user must assume his decision if he refuses the advice of system.

For example, in the General Security Referential document [2] issued by the National Security Agency information systems in France, we find recommendations for the selection and design of cryptographic mechanisms: “It is recommended for any application, to use RSA public exponents strictly greater than 65.536”. In this respect, when a user authenticates himself with a RSA public key based certificate (e.g., to access a protected website), it is recommended to use RSA public exponents strictly greater than 65.536.

An other example is about Recommendations on collective cross-border management of copyright and related rights for legitimate on line music services published by the Commission of the European Communities [26]. This document specifies several recommendations like:

- *Commercial users should inform collective right managers*
- *of the different features of the services for which they want*
- *to acquire on line rights.*
-
- *where a right-holder has transferred the management of*
- *an on line right to another collective rights manager, without prejudice to other forms of cooperation*
- *among rights managers, all collective rights managers*
- *concerned should ensure that those on line rights are*
- *withdrawn from any existing reciprocal representation*
- *agreement concluded amongst them.*
-

We can cite many other examples, but due to space limitation we can clearly state that recommendations are present in many current and emergent applications.

We recently emphasize the need of recommendations and we considered them as a Deontic logic modality[5]. However, our statements were neither justified nor enough mature. To overcome this limitations, Section 2 of this paper presents our new logical-based framework for modeling recommendations. In particular, we precisely and formally give the syntax, semantic and axiomatization of our Recommendation Language. It is worth to note that this paper is the first one that gives a sounded and complete logical framework with mathematical proofs and justification, which is very important in our context. Afterward, Section 3 shows how using our formalism, for example for querying the security policy and verifying its consistency and coherence. Finally, Section 4 draws conclusions and perspectives.

2. Recommendation Specification Language

A formal language for specifying a security policy should be expressive enough to cover all the requirements of the targeted application. Basically, in order to specify security policies with the different access modalities (permissions, prohibitions, obligations and recommendation), we need first to express norms, e.g., rules which say what must be the case, must not be the case, may be the case or may not be the case. Actually, these norms was already addressed by several logical models such as deontic logic. The latter can be seen as an extension of modal logic that considers modal operators such as obligations, permissions and

prohibitions. Note that researches in deontic reasoning within a modal logic point of view has already been done by several works such as by Aqvist [20] and Prior [21]. Moreover, within the context of computer security, several authors like Bieber and Cuppens [22], Glasgow et al. [23], Prakken and Sergot [24], etc. have used deontic logic. In the rest of the following sub-sections, we extend the modal logic in order to model the notions of "recommendation" and "inadvisabilities". We particularly present the axiomatic, semantic and axiomatization as well as all the proofs related to our language.

2.1 Syntax

Let PV be a countable set of propositional variables, with typical members denoted p, q , etc. By means of the Boolean operators \neg ("not . . .") and \vee (" . . . or . . .") of classical logic and the modal operator O ("it is obligatory that . . .") of modal logic, we combine these variables so as to build up the set of formulas of deontic logic given by the rule:

$$\bullet \quad ::= p \mid \neg \mid (\quad \vee \quad) \mid O \quad .$$

We make use of the standard abbreviations for the other Boolean operators. We supplement the language by the modal operators F, P , and E expressing "it is forbidden that . . .", "it is permitted that . . .", and "it is elective that . . .": $F = O \neg$, $P = \neg O \neg$, $E = \neg O$. Basically, the specific characteristic of a norm is the consistency of the set of all obligations that make it up. This characteristic corresponds to the formula $\neg(O \wedge O \neg)$. Seeing that the "obligatory that" is the "forbidden that not" and the "forbidden that" is the "obligatory that not"; this characteristic also corresponds to the formulas:

$$\neg(F \wedge F \neg) \text{ and } \neg(O \wedge F).$$

Furthermore, using the equivalences $\neg O \neg = E$ and $\neg F \neg = P$, we can deduce that $O \wedge E$ and $F \wedge P$. The modal operators P ("it is permitted that . . .") and E ("it is elective that . . .") keep up similar relations: the "permission that" is the "elective that not" and the "elective that" is the "permission that not". Hence, we can deduce the following formulas:

$$O \wedge P \text{ and } F \wedge E.$$

However, none of the previous modalities is able to directly capture the notion of "recommendation". Subsequently, we introduce the modal operator R ("it is recommended that . . .") and we use it to extend the previous set of deontic logic formulas. In fact, let us now consider the set of formulas given by the rule:

$$\bullet \quad ::= p \mid \neg \mid (\quad \vee \quad) \mid O \mid R \quad .$$

Let us take a simple example. If we assume that (Read,

Bob, UserGuide) is a formula expressing the fact that Bob reads the user guide, in our language we can express formulas such as $R(\text{Read}, \text{Bob}, \text{UserGuide})$; meaning that: it is recommended that Bob reads the user guide.

Moreover, to be able to express rules / sentences such as “it is inadvisable that . . .”, we supplement the language by the modal operator $I: I \phi = R \neg \phi$. E.g., the formula $I(\text{Execute}, \text{Bob}, \text{OldVersion})$ means that executing the old version of the program is inadvisable; i.e., it is recommended to not execute the old version.

In this respect, our new set of formulas allows us to give an account of the consistency of a set of recommendations by means of the formula $\neg(R \phi \wedge R \neg \phi)$. In fact, seeing that the “recommended that” is the “inadvisable that not” and the “inadvisable that” is the “recommended that not”, this formula corresponds to the following formulas:

$\neg(I \phi \wedge I \neg \phi)$ and $\neg(R \phi \wedge I \neg \phi)$, it is not possible that something being both recommended and inadvisable.

The question that arises now is: what are the relations between the “obligatory that”, the “recommended that” and the “permitted that” on one hand, and the “forbidden that”, the “inadvisable that” and the “elective that”, on the other hand. The semantics and the axiomatics of the two next subsections will allow us to show, among others, that the formulas $O \supset R$, $R \supset P$, $F \supset I$ and $I \supset E$ express indisputable obvious deontic facts.

2.2 Semantics

The most elementary model of obligations is composed of a non-empty set W of states and a relation R on W . Therefore, a deontic frame will be an ordered pair $F = (W, R)$ where W is a nonempty set of states and R is a binary relation on W called accessibility relation: for all states x , the states y such that xRy are those states in which all the obligations in x are satisfied. For this reason, we may also consider that for all states x , the set $R(x) = \{y: xRy\}$ characterizes the set of all permissions in x . Actually, the formulas of deontic logic are valued at states. The valuation of the formula $O \phi$ at state x depends on the valuation of ϕ at states y such that xRy . In this respect, a deontic model is an ordered triple $M = (W, R, V)$ where $F = (W, R)$ is a deontic frame and V is a valuation on W , i.e., a function assigning to each state x in W a subset $V(x)$ of the set PV of all propositional variables. $V(x)$ can thus be considered as the set of propositional variables that x verifies. Subsequently, in the deontic model M , the function V can be extended to the function $\neg V$ defined as follows:

- $p \in V(x)$ iff $p \in \neg V(x)$; and $\neg \in V(x)$ iff $\neg \notin \neg V(x)$;
- $\neg \in V(x)$ iff $\neg \in \neg V(x)$ or

$\neg \in \neg V(x)$;

- $O \phi \in V(x)$ iff for all states y such that xRy , $\phi \in V(y)$.

Furthermore, according to the relationships between obligations, permissions, prohibitions, it is a simple matter to check that:

- $F \phi \in V(x)$ iff for all states y such that xRy , $\phi \notin V(y)$,
- $P \phi \in V(x)$ iff for some state y with xRy , $\phi \in V(y)$,
- $E \phi \in V(x)$ iff for some state y with xRy , $\phi \notin V(y)$.

Let us now define the notions of “satisfiability” and “validity” in our model. Let ϕ be any formula. We say that ϕ is valid in the model $M = (W, R, V)$ iff

$\phi \in V(x)$ for all states x ; whereas ϕ is said to be valid in the frame $F = (W, R)$ iff ϕ is valid in every model $M = (W, R, V)$ based on F . Furthermore, we say that ϕ is satisfiable in $M = (W, R, V)$ iff $\neg \phi$ is not valid in $M = (W, R, V)$; whereas ϕ is said to be satisfiable in frame $F = (W, R)$ iff ϕ is satisfiable in some model $M = (W, R, V)$ based on F .

Actually, the definitions of satisfiability and validity come from the semantics for modal logic. The reader may easily verify that in all models $M = (W, R, V)$:

- $O \phi \in V(x)$ iff $R(x) \cap \{y: \phi \in V(y)\} = R(x)$, i.e. $\{y: \phi \in V(y)\}$ entirely covers $R(x)$
- $P \phi \in V(x)$ iff $R(x) \cap \{y: \phi \in V(y)\} \neq \emptyset$, i.e. $\{y: \phi \in V(y)\}$ partially covers $R(x)$.

Seeing that we would like the formulas $O \supset R$ and $R \supset P$ to be valid, the interpretation of the recommendation modal operator R in a model $M = (W, R, V)$ should actually be halfway between the interpretations of O and P , i.e. it should correspond to the following interpretation:

- $R \phi \in V(x)$ iff $\{y: \phi \in V(y)\}$ covers a large part of $R(x)$.

In this respect, the interpretation of I in $M = (W, R, V)$ should correspond to $I \phi \in V(x)$ iff $\{y: \phi \in V(y)\}$ covers a small part of $R(x)$. Note that the notions entirely cover (obligations), partially cover (permissions) and cover a large part (recommendations) perfectly reflect that recommendations are stronger than permissions but not as restricting as obligations. Following our reasoning, we consider that a frame for recommendation is an ordered triple $F = (W, R, N)$ where (W, R) is a deontic frame and N is a neighborhood function on W , i.e. a function assigning to

each state x in W a set $N(x)$ of subsets of $R(x)$. For all states x , we will think of $N(x)$ as the set of large subsets of $R(x)$. Such large subsets will characterize the set of all recommendations in x .

Now, with the recommendation notion, our model is an ordered 4-tuple $M = (W, R, N, V)$ where $F = (W, R, N)$ is a frame for recommendation and V is a valuation on W . In this respect, the function V can be extended (in M) to the function V as follows:

- $R \models V(x)$ iff $R(x) \subseteq \{y: V(y)\} \cap N(x)$.

We will say that a frame $F = (W, R, N)$ is \cap -stable iff for all states x in W , the set $N(x)$ of all large subsets of $R(x)$ is closed for the set-theoretical operation of intersection. We will say that a \cap -stable frame $F = (W, R, N)$ is filtered iff for all states x in W , the set $N(x)$ of all large subsets of $R(x)$ is closed upward, i.e.: for all subsets S, T of $R(x)$, if S is in $N(x)$ and $S \subseteq T$ then T is in $N(x)$ too.

In the sequel, we always consider that frames of recommendation are fitted out with a serial relation, (for all states x , there exists a state y such that xRy) and with a neighborhood function N such that for all states x , $R(x)$ is closed for the set-theoretical operation of intersection and closed upward.

Note that correspondence theory in modal logic teaches us the ways the validity of the modal formula $\Box(O \Box O \Box)$ considered above is related to the condition of seriality.

Proof. Assume that the formula $\Box(O \Box O \Box)$ is not valid, then there exists a model M and world x in all possible worlds of M such that $M, x \not\models \Box(O \Box O \Box)$, so $(O \Box O \Box) \in V(x)$. In otherwise, the condition of seriality means that there exists $y \in W$ such that xRy then $V(y)$, therefore $\Box(O \Box O \Box) \in V(x)$ we conclude that $O \Box O \Box \in V(x)$

Let see that the validity of $\Box(R \Box R \Box)$ considered above is related to the condition saying that a neighborhood function N is such that for all states x , $N(x)$ of all large subsets of $R(x)$ is closed for the set-theoretical operation of intersection and $N(x)$.

Proof. Assume that the formula $\Box(R \Box R \Box)$ is not valid then there exists a model $M = (W, R, V)$ and world x in all possible worlds of M such that $M, x \not\models \Box(R \Box R \Box)$, so $(R \Box R \Box) \in V(x)$, then $R(x) \subseteq \{y: V(y)\} \cap N(x)$ and $R(x) \subseteq \{y: \Box V(y)\} \cap N(x)$. As $N(x)$ is closed for intersection, $R(x) \subseteq \{y: V(y)\} \cap \{y: \Box V(y)\} \cap N(x)$, so

$R(x) \subseteq \{y: (\Box V(y)) \cap V(y)\} \cap N(x)$, as $N(x)$, we conclude that $\exists z \in W$ such that $(\Box V(z)) \cap V(z)$ absurd by definition of V .

2.3 Axiomatization/completeness

The previous section presents the semantics of our specification and representation language for obligations and recommendations. This is certainly a first step in building a global and robust logical framework; but it remains not sufficient as we need a mean to derive new informations and to reason (e.g., by verification) on our language. Moreover, it seems necessary to give axioms and rules that define the relationships between the different access modalities (obligations, recommendations and permissions) and to give proofs of our axiomatization. To achieve these tasks, we define in this section the axiomatic system LR of our Logic of Recommendation. In addition to the classical axioms of propositional logic, we define the following axioms of LR:

- $O(\Box \phi) \rightarrow (O \Box \phi)$,
- $O \Box \phi \rightarrow \Box O \phi$,
- $O(\Box \phi) \rightarrow (R \Box \phi)$,
- $O \Box \phi \rightarrow \Box O \phi$,
- $R \Box \phi \rightarrow \Box R \phi$.
- $R \Box \phi \rightarrow R(\Box \phi)$.
- $O(\Box \phi) \rightarrow (R \Box \phi)$
- $R(\Box \phi) \rightarrow R \Box \phi$
- The axiom $O(\Box \phi) \rightarrow (O \Box \phi)$ is called axiom (K). It intuitively corresponds to the fact that the modal operator O is interpreted in models by means of a binary relation.

Proof. Let x be a world in all possible worlds of model M . We must show that at least one of the following conditions is satisfied:

- (C1) $M, x \not\models O(\Box \phi)$
- (C2) $M, x \models (O \Box \phi)$

Suppose that neither (C1) or (C2) is satisfied in x . We then have $M, x \models O(\Box \phi)$, so for all y in W such that xRy , $M, y \models (\Box \phi)$. And $M, x \models O\phi$ and $M, x \models O \Box \phi$ (C2 not satisfied), so for all $y \in W$ such that xRy , $M, y \models \phi$ and there exists at least one y_0 such that xRy_0 and $M, y_0 \not\models \phi$ then there exists y_0 such that xRy_0 , $M, y_0 \models \phi$ and $M, y_0 \not\models \phi$ we conclude that such y_0 satisfies both $(\Box \phi)$, ϕ and $\neg \phi$ which is absurd.

- The axiom $O \rightarrow P$ (axiom D) intuitively corresponds to the fact that in every frame $F = (W, R, N)$, R is such that for all states x , there exists a state y such that xRy .

Proof. Let x be a world in all possible worlds of model M . We Suppose that $M, x \models O$, so for all y such that xRy we have $M, y \models P$. As R is serial then there exists $z \in W$ such that xRz , so $M, z \models P$ then $M, x \models P$.

- The axiom $O(\Box \phi) \rightarrow (R \Box \phi)$ intuitively corresponds to the fact that the modal operator R is interpreted in models by means of a neighborhood function.

Proof. Let x be a world in all possible worlds of model M .

Suppose that $M, x \models O(\Box \phi)$, so for all worlds y such that xRy , we have $M, y \models (\Box \phi)$, which means that for all y such that xRy , we have $M, y \models \phi$ iff $M, y \models \Box \phi$. Then in particular, for the set in $N(x)$ contained y such that xRy , we have $M, y \models \phi$ iff $M, y \models \Box \phi$. We conclude that $M, x \models (R \Box \phi)$.

- The axiom $O \rightarrow R$ intuitively corresponds to the fact that for all states x , $R(x) \subseteq N(x)$.

Proof. Let x be a world in all possible worlds of model M . Suppose that $M, x \models O$ so we have for all y such that xRy , $M, y \models \Box \phi$. As $R(x) \subseteq N(x)$, then the set of all y such that xRy and $M, y \models \Box \phi$ is in $N(x)$. We conclude that $M, x \models R \Box \phi$.

- The axiom $R \Box \phi \rightarrow P$ intuitively corresponds to the fact that for all states x , $N(x) \subseteq P$.

Proof. Let x be a world in all possible worlds of model M . Suppose that $M, x \models R \Box \phi$ then the set of all y such that xRy and $M, y \models \Box \phi$ is in $N(x)$ and it is not empty as $N(x) \subseteq P$. Then there exists y such that xRy and $M, y \models \Box \phi$. We conclude that $M, x \models P$.

- The axiom $(R \Box \phi \rightarrow R \Box \phi) \rightarrow (R \Box \phi)$ intuitively corresponds to the fact that for all states x , $N(x)$ of all large subsets of $R(x)$ is closed for the set-theoretical operation of intersection.

Proof. Let x be a world in all possible worlds of model M . We Suppose that $M, x \models (R \Box \phi \rightarrow R \Box \phi)$, so the set of all y in W such that xRy and $M, y \models \Box \phi$, is in $N(x)$ and the set of all y in W such that xRy and $M, y \models \Box \phi$ is in $N(x)$. As $N(x)$ is closed for the

set-theoretical operation of intersection, then the set of all y in W such that xRy and $M, y \models \Box \phi$ and $M, y \models \Box \phi$ is in $N(x)$.

Then the set of all y in W such that xRy and $M, y \models \Box \phi$ is in $N(x)$. We conclude that $M, x \models R \Box \phi$.

- The axiom $O(\Box \phi) \rightarrow (R \Box \phi)$ intuitively corresponds to the fact that for all states x , $N(x)$ of all large subsets of $R(x)$ is closed upward.

Proof. Let x be a world in all possible worlds of model. We Suppose that $M, x \models O(\Box \phi)$ and $M, x \models R \Box \phi$.

and show that $M, x \models R \Box \phi$.

$M, x \models R \Box \phi$ means that the set of all y such that xRy and $M, y \models \Box \phi$ is in $N(x)$. And $M, x \models O(\Box \phi)$

means that for all y such that xRy , $M, y \models \Box \phi$, then the set of y such that xRy and $M, y \models \Box \phi$ is included in the set of y such that xRy and $M, y \models \Box \phi$ is in $N(x)$ is closed upward, we deduce that the set of y such that xRy and $M, y \models \Box \phi$ is in $N(x)$ which means $M, x \models R \Box \phi$.

- The axiom $R(\Box \phi) \rightarrow R \Box \phi$ intuitively corresponds to the fact that for all states x , $N(x)$ of all large subsets of $R(x)$ is closed upward.

Proof. Let x be a world in all possible worlds of model.

We Suppose that $M, x \models R(\Box \phi)$, so the set of y such that xRy and $M, y \models \Box \phi$ is in $N(x)$. Then the intersection of the set of y such that xRy and $M, y \models \Box \phi$ and the set of y such that xRy and $M, y \models \Box \phi$ is in $N(x)$, this intersection is included both in the set of y such that xRy and $M, y \models \Box \phi$ and in the set of y such that xRy and $M, y \models \Box \phi$.

As the frame $F = (W, R, N)$ is filtered, we deduce that the set of y such that xRy and $M, y \models \Box \phi$ is in $N(x)$ and the set of y such that xRy and $M, y \models \Box \phi$ is in $N(x)$. We deduce that $M, x \models (R \Box \phi)$.

Besides that, in addition to the classical inference rules of propositional logic, the inference rules of LR is: “from ϕ, ψ infer $O \psi$ ”.

Now, let us give proofs of the most important formulas derivable from axioms and inference rules of LR. Each line of proof is either an instance of an axiom schema, or the application of the inference necessitation rule or the inference Modus Ponens rule

- $O \rightarrow O\psi$

- $O(A \rightarrow B),$

Proof. First show that $\frac{(A \wedge B) \rightarrow C}{(OA \wedge OB) \rightarrow OC}$

$A \rightarrow B \rightarrow C$
 $A \rightarrow (B \rightarrow C)$ (propositional language)
 $O(A \rightarrow (B \rightarrow C))$ (LR inference rule)
 $OA \rightarrow O(B \rightarrow C)$ (axiom K)
 $OA \rightarrow (OB \rightarrow OC)$ (axiom K)
 $OA \rightarrow OB \rightarrow OC$ (propositional language)

Consequently, from $O(A \rightarrow B),$ we can deduce $O(A \rightarrow B \rightarrow C)$ by applying the inference rule deduced above.

- $O(A \rightarrow B), R\psi \rightarrow R(A \rightarrow B),$

Proof. First show that: $\frac{(A \wedge B) \rightarrow C}{(OA \wedge RB) \rightarrow RC}$

$A \rightarrow B \rightarrow C$
 $A \rightarrow (B \rightarrow C)$ (propositional language)
 $O(A \rightarrow (B \rightarrow C))$ (LR inference rule)
 $OA \rightarrow O(B \rightarrow C)$ (axiom K)
 $OA \rightarrow (RB \rightarrow RC)$ (LR axiom)
 $OA \rightarrow RB \rightarrow RC$ (propositional language)

As $O(A \rightarrow B)$ conclusion from $O(A \rightarrow B \rightarrow C)$ we can deduce $O(A \rightarrow B \rightarrow C \rightarrow R\psi \rightarrow R(A \rightarrow B))$ by applying the inference rule demonstrated above.

- $O(A \rightarrow B), P(A \rightarrow B) \rightarrow P(A \rightarrow B).$

Proof. First show that $O(B \rightarrow C) \rightarrow PB \rightarrow PC$

$O(\neg C \rightarrow \neg B) \rightarrow \neg C \rightarrow \neg B$
 (axiom K)

$O(B \rightarrow C) \rightarrow \neg PC \rightarrow \neg PB$ ($PA = \neg O \neg$
 A by definition)
 $O(B \rightarrow C) \rightarrow PB \rightarrow PC$

Let show that $\frac{(A \wedge B) \rightarrow C}{(OA \wedge PB) \rightarrow PC}$

$A \rightarrow B \rightarrow C$
 $A \rightarrow (B \rightarrow C)$ (propositional language)
 $O(A \rightarrow (B \rightarrow C))$ (LR inference rule)

$OA \rightarrow O(B \rightarrow C)$ (axiom K)
 $OA \rightarrow (PB \rightarrow PC)$ (The formula just demonstrated above)
 $OA \rightarrow PB \rightarrow PC$ (propositional language)

As $O(A \rightarrow B)$ conclusion from $O(A \rightarrow B \rightarrow C)$ we can deduce $O(A \rightarrow B \rightarrow C \rightarrow P\psi \rightarrow P(A \rightarrow B))$ by applying the inference rule demonstrated above.

Let us now proof that our logical system is sound and complete.

Proposition 1. (soundness property) All formulas derivable from the axioms and inference rules of LR are valid in all frames.

Proof. It is actually sufficient to prove that the axioms are tautologies and the inference rules are valid. Afterwards, we can use proof by induction on the length of the derivation of ψ in LR to proof that "if ψ is derivable in LR then ψ is valid in all frames".

Note that we have already shown that the axioms are valid. Now, let us see that the rule " $\psi \rightarrow \phi$ infers $O\psi \rightarrow O\phi$ " is also valid. Let x be a world in all possible worlds of model M . If $M, x \models \psi$, then for all $y \in W$ such that xRy , $M, y \models \phi$. By definition of the validity of the obligation, we can deduce that $M, x \models O\psi$ and then $M, x \models O\phi$.

Proposition 2. (completeness property) All formulas valid in all frames are derivable from the axioms and inference rules of LR.

Proof. The proof is done by means of a canonical model construction. Let $M = (W, R, N, V)$ be the model defined as follows:

- W is the set of all maximal LR-consistent sets of formulas,
- R is the binary relation on W , for all x, y in W , xRy iff $\{ \phi : O\phi \in x \} \subseteq y$,
- N is the neighborhood function such that for all x in W and for all subsets S of $R(x)$, S is in $N(x)$ iff there exists a formula ϕ such that $R \subseteq x$ and $S = \{y \in W : xRy \text{ and } \phi \in y\}$,
- V is the valuation function such that for all x in W , $V(x) = \{p : p \in x\}$.

Let prove that in our canonical model, R is serial, for all states x in W , $R(x) \cap N(x) \neq \emptyset$, $N(x)$ is closed for intersection and closed upward.

• **R is serial.**

First show that any set of consisting formulas is contained in some set of formulas maximum-consistent.

Let Σ be a set of consistent formulas. Consider an enumeration, starting with 1, of formulas of the language. We denote the k -th formula according to this enumeration, A_k . We construct a series $(\Sigma_k)_{k \in \mathbb{N}}$ of set of formulas as follows:

1. $\Sigma_1 = \Sigma$
 2. $\Sigma_{k+1} = \Sigma_k \cup \{A_{k+1}\}$ if $\Sigma_k \cup \{A_{k+1}\}$ is consistent, and $\Sigma_{k+1} = \Sigma_k \cup \{\neg A_{k+1}\}$ if not.
- Σ_k , if $\Sigma_k \cup \{A_{k+1}\}$ is not consistent, then $\Sigma_k \cup \{\neg A_{k+1}\}$ is as well. Consequently, Σ_k , if Σ_k is consistent, then Σ_{k+1} is as well. As $\Sigma_k = \Sigma_{k-1} \cup \{A_k\}$ is consistent hypothetically, all the Σ_k are consistent.

Let G the union of all Σ_k .
 $G = \bigcup_{k \in \mathbb{N}} \Sigma_k$ by construction, G is consistent (if it is not the case, one of Σ_k should be inconsistent) and G is maximal by construction.

Now let prove that R is serial.

Let $x \in W$.
 If $\{ : O \ x\}$, then $O \ , O \neg \ x$
 As x is LR-consistent, we conclude that $\{ : O \ x\}$ is LR-consistent tow. So there exists y LR-consistent maximal such that $\{ : O \ x\} \cup y$

• **R(x) N(x)**

Let $x \in W$, x is not empty, so there exists x , and then $O \ x$ wish means that for all $y \in W$ such that xRy , y , then $R(x) = \{y \in W \text{ such that } xRy, y\}$, therefor there exists y such that $O \ x$ and $R(x) = \{y \in W \text{ such that } xRy, y\}$ and then there exists such that $R \ x$ and $R(x) = \{y \in W \text{ such that } xRy, y\}$ as x is LR-consistent maximal and $O \ R \ W$ We conclude that $R(x) \subseteq N(x)$.

• **N(x)**

We assume that $N(x)$, then there exists y such that $R \ x$ and $\{y \text{ such that } xRy \text{ and } y\} = \emptyset$ so there exists y such that $R \ x$ and for all y such that xRy , y or $R(x) = \emptyset$. Or R is serial, so there exists y such that $R \ x$ and for all y such that xRy , $\neg y$. We conclude that there exists y such that $R \ x$ and $O \neg \ x$, absurd because x is LR-consistent.

• **N(x) is closed for intersection.**

First show that: $(R \ R) \ x \ R(\) \ x$
 We suppose that $R \ x, R \ x$ and $R(\) \ x$
 x
 As x is LR maximal, we have $\neg R(\) \ x$
 we have $R \ R \ R(\)$, so $R(\)$ belongs to the closure of x , which is absurd because x is consistent.

Let now see that $N(x)$ is closed for intersection.

Let $S1, S2 \subseteq N(x)$, then there exists I such that $R \ I$ and $S1 = \{y \in W, xRy\}$ and $\{y\}$ and there exists such that $R \ x$ and $S2 = \{y \in W, xRy\}$ and $y\}$, so there exists y , such that $R \ x$ and $S = S1 \cap S2 = \{y \in W, xRy\}$ and $y\}$, then there exists y , such that $R(\) \ x$ and $S = S1 \cap S2 = \{y \in W, xRy\}$ and $y\}$. We conclude that $S1 \cap S2 \subseteq N(x)$.

• **N(x) is closed upward.**

Let $S, T \subseteq R(x)$ such that $S \subseteq T$ and $S \subseteq N(x)$.
 $S \subseteq N(x)$ then there exists y such that $R \ x$ and $\{y : xRy, y\} \subseteq N(x)$.
 Suppose that $T \subseteq N(x)$, then there exists $z \in T$ such that xRz and $\neg z$ as z is LR maximal.
 As infer O and z is LR maximal consistent, $O(\neg z)$, then $O(\neg z) \subseteq V(z)$, so for all $y \in W$ such that zRy , $\neg V(y)$, and then for all $y \in W$ such that zRy , $\neg y$.
 If $R \ x$ then $O(R \) \ x$ and then for all y such that $xRy, R \ y$, so $R \ z$, then $R \ V(z)$, so there exists $y0$ such that $zRy0$ and $y0$.
 Absurd because $y0$ is LR consistent. We deduce that $T \subseteq N(x)$ and then $N(x)$ is closed upward.

• **Let now see that, by using a proof by induction on the complexity of the formula ϕ , for all states $x \in W$, x iff $V(x)$**

Let ϕ a formula of length 0 (atomic)
 We have x iff $V(x)$.
 We suppose that x iff $V(x)$ is true for all formula of length smaller or equal than a given number n and let ϕ a formula of length $n+1$. We have the following cases:

1. ϕ has the form $\neg \psi$ with ψ is of length n
2. ϕ has the form $\psi \wedge \theta$ with ψ and θ are of length smaller or equal than n
3. ϕ has the form $O \ \psi$ with ψ is of length n
4. ϕ has the form $R \ \psi$ with ψ is of length n

The first two cases are obvious by definition of \forall

For the third case:

$O \rightarrow x$ means that for all y such that xRy , $\forall y, \text{ then } O$
for all y such that xRy , $\forall(y)$ and then O

$\forall(x)$.

we suppose that $O \rightarrow x$ and $O \rightarrow \forall(x)$.

so $\neg O \rightarrow x$ and for all y such that xRy ,
 $\forall(y)$.

So $P \rightarrow x$ and for all y such that xRy , $\forall y$. Then

there exists z such that xRz and $\neg z$ and $\neg z$.

absurd because z is LR-consistent. So $O \rightarrow x$ iff O

$\forall(x)$

For the last case:

$R \rightarrow x$ iff $\{y \text{ such that } xRy, \forall y\} \rightarrow N(x)$
iff $\{y \text{ such that } xRy, \forall(y)\} \rightarrow N(x)$
iff $R \rightarrow \forall(x)$.

As a result, if \neg is a formula not derivable in LR (\neg does not belong to the closure of \neg), then \neg is LR-consistent and there is $x \rightarrow W$ such that $\neg x$. Therefore, x and $\forall(x)$. It follows that \neg is not valid in all frames.

3. Using our formalism

3.1 Specification of the security policy

The axiomatic system defined in the last section, coupled with classical logic axioms could be used for several aims. In this section, two of the possible uses are explained:

- (1) query a given policy in order to know which rules apply to a given situation; and
- (2) Check the security policy consistency.

To achieve these tasks, it is first necessary to specify the operational rules, the security policy, and the security objectives. In our view, operational rules can be described by means of the propositional logic operators (non modal). For example, to specify that users play roles in their organizations, we can introduce the play predicate between the constant symbols: organizations, users and roles. An instance of this predicate could be for instance Play(Hospital1, Alice, Physician).

Besides that, we suggest expressing security objectives by using modal operators. For example, the $R(\text{Nurse, Read, notice})$ security objective means that it is recommended that the Nurse reads the notice. Finally, we propose expressing security rules using modal formula with at least a non-modal clause (e.g., $f \rightarrow Rq$). It describes the link between the permissions, prohibitions, obligations, or

recommendations and the state of the system. For example, the security rule: "if the patient is minor, it is inadvisable that he/she read its medical file" can be specified by: $\text{Age}(p) < 18 \rightarrow \neg I(p, \text{read, MedicalFile}(p))$. In this rule we have considered that p is a variable of type "patient"; Age (resp. MedicalFile) is a function that returns the age (resp. the medical file) of a certain patient).

3.2 Querying the security policy

Once we have specified the operational rules, the security policy, and the security objectives of the studied application, we can use our axiomatic to develop a tool which enables a user to query the security policy. For instance, let us assume that security administrator wants to know who is recommended to read a notice? This query is translated in the following logical formula: " $\forall n, \text{Notice}(n) \rightarrow R(x, \text{Read, } n)$ ".

Note that there are two ways to program this formula in logical-based languages such as PROLOG. The first one lists the persons who are actually recommended to read a notice; while the second method answers by a formula which corresponds to a sufficient condition that satisfies the query. This second technique of query answering is called intentional answer in [25].

3.3 Checking the security policy consistency

Different techniques can be used to check the security policy consistency, in particular, we can use:

- Axiom-based methods, called Frege-Hilbert methods. The idea is to derive new rules by applying the inference rules to the set of axioms until demonstrating the intended property.
- Natural deduction methods: these techniques are closed to the reasoning used by mathematicians to demonstrate their theorems. In this kind of calculus, every derivation starts by some hypothesis and assumptions.

In our context, it is important to choose the method that (1) gives enough information about the reasons of success or failure while demonstrating a certain security property, (2) identifies the system state that is responsible (3) identifies some resident vulnerabilities in the system or a certain weakness in the security policy specification. This will greatly enhance the system security and rigorously help to refine the security objectives. For these reasons, we suggest using a constructive verification technique such as the "Tableau method" or its variant "Gentzen sequence calculus". In order to prove a certain formula \neg , the main idea is to assume that \neg is true and to derive a contradiction by successively splitting up \neg in each of its derived sub-formulas, until obtaining a state satisfying a formula and its negation.

Actually, in this method, we draw a graph where the initial node contains an initial secure state (e.g., a state where certain security objectives are true/satisfied). Then, we progressively apply some derivation rules (specific to this method). At each state we also apply one of the security rules (rules that specify how the system can, must or should evolve). The demonstration is ended when attending a non-secure state (a state where a contradiction is detected). The "Tableau method" can also be used to detect conflicting situations, e.g., if, from a secure state, and by applying the security rules as well as the derivation rules, we reach a state where a certain user has the permission/obligation/recommendation and the prohibition to carry out a certain action on the same object); This problem comes to draw our graph and to look for nodes where one of the following formulas are true: $R_p \quad F_q$ or $P_p \quad F_q$ or $O_p \quad F_q$ or $I_p \quad F_q$ or R_q or $I_p \quad O_q$.

4. Conclusion

A security policy specifies, usually in a textual form, who has access to what, when and in which conditions? Nevertheless, the security policy does not guarantee a secure and correct functioning of the system. Consequently, it is important to associate a model to: abstract the policy and handle its complexity; represent the secure states of a system as well as the way in which the system may evolve; verify the coherence of the security policy and detect possible conflicting situations; guarantee that all the security objectives are covered by the security mechanisms implementing the policy; etc. Deontic logic is a good candidate to model several security properties and modalities. However, none of the existing works have studied the recommendation and inadvisable access modalities, while these concepts are unavoidable in many applications. Several regulations are in fact in the form of recommendations and directives, and these regulations should be reflected in security policies. Modeling recommendations is thus a new challenge in the security policies and models field.

In this paper we first develop further our Recommendation Specification Language. Then, in order to be able to reason on the security policy and to derive new rules, we give more details about our new recommendation-based axiomatic. Finally, we prove that our new formal system is semantically complete and sound. Currently, we are developing mechanisms to integrate our recommendation access modality in existing tools and languages such as Prolog. We also hope extending this work to distributed systems by considering several authorities claiming different kind of statements. Finally, we also expect applying our work to a representative case study.

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