# Robust Fault Diagnosis of a Reverse Osmosis Desalination System Modeled by Bond Graph Approach

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**Summary**
This paper proposes a robust diagnosis of a reverse osmosis desalination system modeled by Bond Graph approach. The design is achieved by using graphical methods taking advantage of structural proprieties of the bond graph model. The fault indicators are generated in the presence of parameter uncertainties. Simulation results are used to show the dynamic behavior of system variables and to evaluate the performance of the bond graph for fault diagnosis.

***Key words:***
*Bond graph, Modeling, Desalination, Robust Diagnosis*

## 1. Introduction

To improve the production quality, safety and effectiveness of industrial units, the robust fault diagnosis was the subject of much research. FDI (Fault Detection and Isolation) is a procedure of comparing behavior of the real system and behavior of the referral process. In the literature, two approaches exist troubleshooting: quantitative and qualitative. Among the books published during the years of robust diagnosis using these methods, can be found ref. [1].

An industrial process has a highly complex behavior because of the mutual interaction of several different phenomena in nature, and combining technology components that implement different kind of energy (electrical, mechanical, hydraulic, thermodynamic, chemical, etc.). . The dynamic behavior of this type of system is generally described by nonlinear differential equations. The bond graph tool allows multi-purpose in nature and graphics using a unified language to display explicitly the nature of trade in the power system phenomena such as storage, energy dissipation and processing and highlight the physical nature and location of the state variables

The diagnosis of uncertain systems has been the subject of several research works in recent years [2]. Dauphin-Tangy and al [3] are proposed two methods for modelling uncertainties by using bond graph approach. The first method is based on describing parameter uncertainties as bond graph elements, and the second method introduces the LFT form (Linear Fractional Transformation) for uncertainties modelling.

The innovative interest of the present paper is the use of the bond graph tool for modelling and robust diagnosis, taking into account the parameter uncertainties. In this way, by applying the bond graph methodology using LFT model, it becomes possible to obtain physical knowledge of the systems and to improve their monitoring by deducing residuals fault indicators and consequently, to insure the best safety able to detect and to isolate imperfections.

This paper is organized as follows: Section 2 deals with diagnosis using bond graph approach. Section 3 proposes a robust diagnosis using the BG. Section 4 deals with sensibility residual using bond graph approach. An illustrative example of reverse osmosis desalination system is developed in section 5 and shows the efficiently of the proposed method.

## 2. Diagnosis Using Bond Graph Approach

### 2.1 Bond graph modeling

The bond graph approach was defined in 1961 by Henry Paynter [4] and then developed by Karnopp [5]. This energetic approach serves to emphasize analogies between different fields of physics (mechanics, electricity, hydraulics, thermodynamics, acoustics, etc.) and to represent in uniform multidisciplinary physical systems. Because of its structure and causal properties, the bond graph tool is more and more used for modeling and fault diagnosis. The causal properties of the bond graph tool were initially used for the determination of the origin of the faults.

### *2.2* Diagnosis using bond graph approach

Monitoring system by the bond graph approach can be illustrated in figure 1 [6]. There are basically two parts:

- The first part concerns the transfer of power and energy (formed by the process and all the actuators).

Manuscript received December 5, 2011
Manuscript revised December 20, 2011

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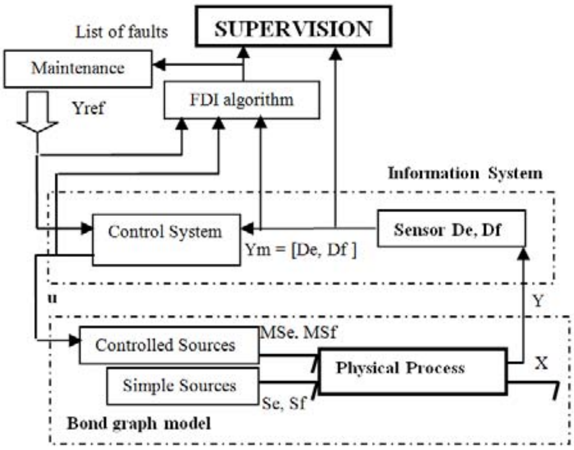



Fig. 1 Monitoring system by the bond graph approach

The bond graph model is the part of the energy system, the process is generally modeled by bond graphs usual elements (R, C, I, and junctions). Sources can be single (Se, Sf) or modulated (MSe, MSf), that is controlled by an external signal provided by a controller or an operator. The sensors and the control system are the information system. In the first system (energy), the power exchanged is represented by a half arrow (link power) led the effort variables and flow in the second system (information system) exchanged power is negligible; it is then represented by an information link (arrow) which is the same used in conventional block diagrams.

Monitoring algorithms (detection and fault isolation FDI) receive online information from the sensors (sensors effort and flow Df) and issue the monitoring system alarms. Information on the status of faulty elements are passed to service

## 3. Robust Diagnosis Using the Bond Graph Model

### 3.1 Construction of a bond graph model

There are two methods proposed by G. Dauphin-Tanguy and C. Sie Kam [3] to construct the parametric uncertainties by BG. The first is to represent the uncertainty of a bond graph element as another element of the same type, causally linked to the element or the rest of the nominal model. These uncertainties are kept in derivative causality when the model is fully causal preferred not to change the order of the model. The second method is the form LFT (Linear Fractional

- The second part shows the signals (the information system and the control system and sensors).

Transformations) on mathematical models introduced by R. Redheffer [7].

### 3.2 LFT representation

The linear fractional transformations (LFT) are generic objects used extensively in system modeling uncertainty. The universality of the LFT is due to the fact that any expression can be written in this from A. Oustaloup [8] and D. Alazard et al. [9]. This form of representation is widely used for the synthesis of control laws for uncertain systems using the principle of the μ-analysis. It is to separate the face of an uncertain model of the part as shown in figure 2.

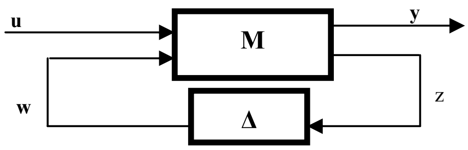



Fig. 2 Representation LFT

The ratings are combined in an augmented matrix denoted M, supposedly clean, and uncertainty regardless of type (parametric uncertainties and unstructured, uncertainty modeling, measurement noise ...) are combined in a matrix structure of $\Delta$ diagonal. In the linear case, the standard form leads to a state representation of the form (19):

$$\begin{cases} \dot{x} = Ax + B_1 w + B_2 u \\ z = C_1 x + D_{11} w + D_{12} u \\ y = C_2 x + D_{21} w + D_{22} u \end{cases} \quad (1)$$

With:

- $x \in R^n$ the state vector of the system;
- $u \in R^m$ the vector grouping the control inputs of the system;
- $y \in R^p$ the vector grouping the measured system output;
- $w \in R^l$ and $z \in R^l$ include respectively the inputs and auxiliary outputs;
- $n$, $m$, $l$ and $p$ are positive integers.

Matrices (A, $B_1$, $B_2$, $C_1$, $C_2$, $D_{11}$, $D_{12}$, $D_{21}$ and $D_{22}$) are matrices of appropriate dimensions.

### 3.3 Modeling elements by BG-LFT

Modeling of linear systems with uncertain parameters has been developed in C. Sie Kam, we invite the reader to consult the corresponding references for details on the modeling of uncertain elements BG (*R*, *I*, *C*, *TF* and *GY*).

Therefore, we limit ourselves in this part to show the two methods of modeling elements BG uncertain and the benefits of BG-LFT for the robust diagnosis.

### 3.3.1 BG elements with additive uncertainty

By introducing uncertainty in an additive manner on such causal element resistance *R* we obtain (2):

$$e_R = (R_n + \Delta R) f_R = R_n f_R + \Delta R_n f_R = e_n + e_{inc} \qquad (2)$$

With:

- $R$: The nominal value of the element R;
- $\Delta R$: additive uncertainty on the parameter;
- $e_R$ and $f_R$: represent the effort and flow in the element $R$;
- $e_n$ and $e_{inc}$: represent the effort made by the parameter nominal stress introduced by the additive uncertainty.

The bond graph model equivalent to the mathematical model of equation (2) is given by figure 3.

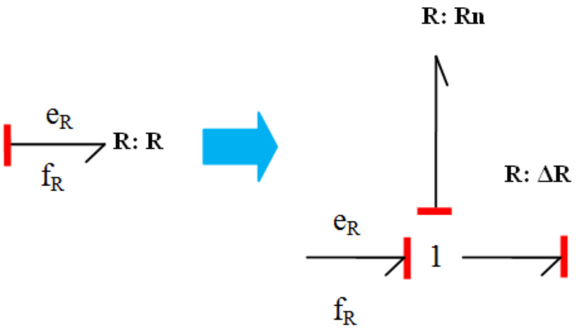



Fig. 3 Model BG-LFT for rresistance with additive uncertainty.

### 3.3.2 BG elements with a multiplicative uncertainty

The introduction of a multiplicative uncertainty on how such causal element resistance *R* given in (3):

$$e_R = R_n (1 + a_R) f_R = R_n f_R + a_R R_n f_R$$
$$= e_n + a_R e_n = e_n + e_{inc} \qquad (3)$$

With:

- $R_n$: The nominal value of the element $R$;
- $a_R$: multiplicative uncertainty on the parameter;
- $e_R$ and $f_R$: represent the effort and flow in the element $R$;
- $e_n$ and $e_{inc}$: represent the effort made by the parameter nominal stress introduced by the multiplicative uncertainty.

The bond graph model equivalent to the mathematical model of equation (3) is given by figure 4.

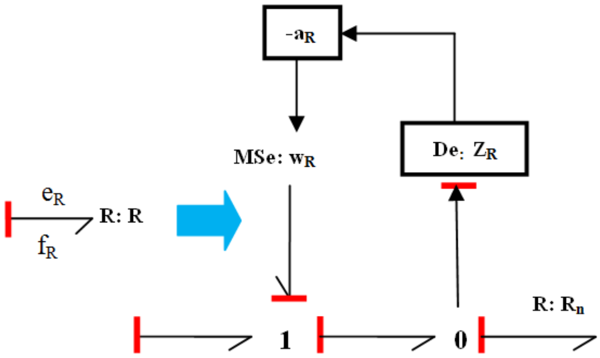



Fig. 4 Model BG-LFT for rresistance with multiplicative uncertainty.

### 3.3.3 BG-LFT models of energy storage elements with a multiplicative uncertainty

*Element I and C in integral causal*

- *Element I in integral causal*

Law characteristic of the uncertain element I in integral causality with an uncertainty multiplicative is given as follows:

$$f_I = \frac{1}{I_n}(1 + a_{\frac{1}{I}}) \int e_I dt + f_{I_0}$$
$$f_I = \frac{1}{I_n}\int e_I dt + a_{\frac{1}{I}}(\frac{1}{I_n}\int e_I dt) + f_{I_0} \qquad (4)$$
$$f_I = \frac{1}{I_n}\int e_I dt - w_{\frac{1}{I}}$$

LFT Bond Graph model I elements in integral causality with a multiplicative uncertainty equivalent equation (4) given in figure 5.

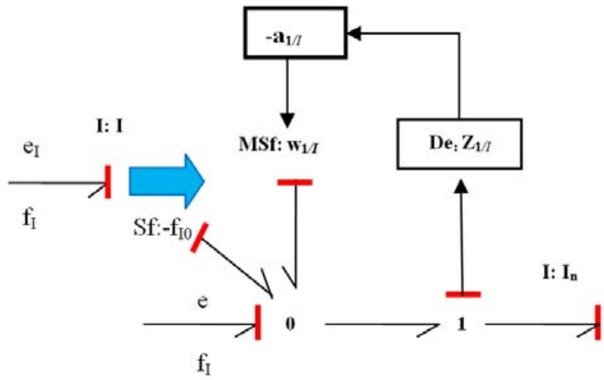



Fig. 5 I element in integral causality with a multiplicative uncertainty

With $a_{\frac{1}{I}} = \dfrac{-\Delta I}{I_n + \Delta I}$

Dummy entry that represents the uncertainty of the flow at the exit of element I. $f_{I0}$ is a constant representing the initial condition.

- *Element C in integral causal*

Law characteristic of the uncertain element I in integral causality with an uncertainty multiplicative is given as follows:

$$
\begin{aligned}
e_C &= \frac{1}{C_n}(1 + a_{\frac{1}{C}})\int f_C dt + e_{C_0} \\
e_C &= \frac{1}{C_n}\int f_C dt + a_{\frac{1}{I}}\left(\frac{1}{C_n}\int f_C dt\right) + e_{C_0} \\
e_C &= \frac{1}{C_n}\int f_C dt - w_{\frac{1}{C}}
\end{aligned}
\qquad (5)
$$

LFT Bond Graph model C element in integral causality with a multiplicative uncertainty equivalent equation (10) given in figure 6.

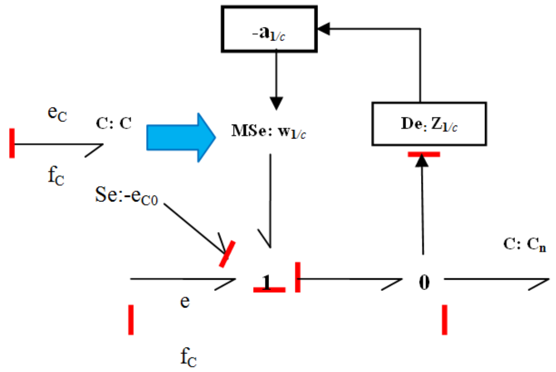



Fig. 6 C element in integral causality with a multiplicative uncertainty

*The model BG-LFT*

The BG-LFT full can then represented by the diagram in figure 7.

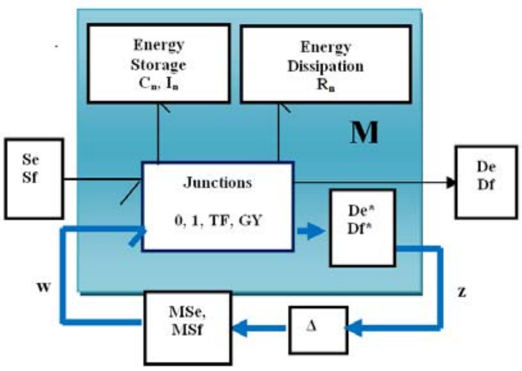



Fig.7 Representation of a BG-LFT

*Robust ARR generation*

The generation of robust analytical redundancy relations from a bond graph model specific, observable and over determined is summarized by the following steps:

*Step 1:* Check the status of the coupling on the bond graph model deterministic causality derived preferentially, if the system is over determined, then continue the following steps;

*Step 2:* The bond graph model is shaped LFT;

*Step 3:* The symbolic expression of the ARRs is derived from the equations at the junctions. This first form will be expressed by:

- *In the case of 0 Junction:*

$$\sum b_i f_{inc} + \sum Sf + \sum w_i \qquad (6)$$

- *In the case of 1 Junction*:

$$\sum b_i e_{inc} + \sum Se + \sum w_i \qquad (7)$$

With the sum of flow sources associated with the junction *0*, the sum of flow sources related to a junction, $b = \pm 1$ depending on whether the half-arrow entering or leaving the junction and ein and purpose are unknown variables.

*Step 4:* The unknown variables are eliminated by browsing through the causal paths between sources and detectors or the unknown variables;

*Step 5:* After removal of the unknown variables, the ARRs are uncertain form (6):

$$ARR : \Phi\left(\sum Se, \sum Sf, De, Df, \tilde{D}e, \tilde{D}f, \sum w_i, R_n, I_n, C_n, TF_n, GY_n\right) \qquad (8)$$

Where:

- $TF_n$ and $GY_n$ are respectively the nominal values of the modules and components *TF* and *GY*,
- $R_n$, $C_n$ and $I_n$ are the nominal values of elements R, C and I. They are the sum of modulated inputs corresponding to uncertainties on the elements related to the junction.

## 4. Residual Sensibility

### 4.1 Generation of performance indices

#### 4.1.1 Normalized sensitivity index parametric (SI)

The energy supplied will be evaluate to the residual of uncertainty about each parameter by comparing the total energy contributed by all the uncertainties

$$SI_{ai} = \frac{|a_i|}{d}\frac{\partial d}{\partial |a_i|} = \frac{|w_i|}{d} \quad (9)$$

$a_i$: Uncertainty on the ith parameter

$i \in \{R, C, I, RS, TF, GY\}$

$w_i$: The input modulated corresponding to the ith parameter uncertainty.

### 4.1.2 The index of detectability of default (ID)

The index of detectability default is the difference between the effort (or flux) provided by the defects in absolute and that granted by all the uncertainties in absolute value.

- Junction 1: $ID = |Y_i||e_{in}| + |Y_s| - d$ (10)
- Junction 0: $ID = |Y_i||e_{in}| + |Y_s| - d$ (11)

*Proposal:* Condition detectability of faults

- Undetected faults: $ID \leq 0$
- Detected faults: $ID > 0$

*Fault detection rate parametric*

- Junction 1: $|Y_i| > \frac{d}{|e_{in}|}$ (12)
- Junction 0: $|Y_i| > \frac{d}{|f_{in}|}$ (13)

*Detectable values of a structural fault*

$$|Y_s| > d \quad (14)$$

## 5. Application

### 5.1 Bond graph model of reverse osmosis

Consider the reel system of reverse system and its bond graph model given in figure 8. The most important parameters that must be controlled are the permeate conductivity and flow rate [10].

Fig. 8. Photo of the reverse osmosis desalination system

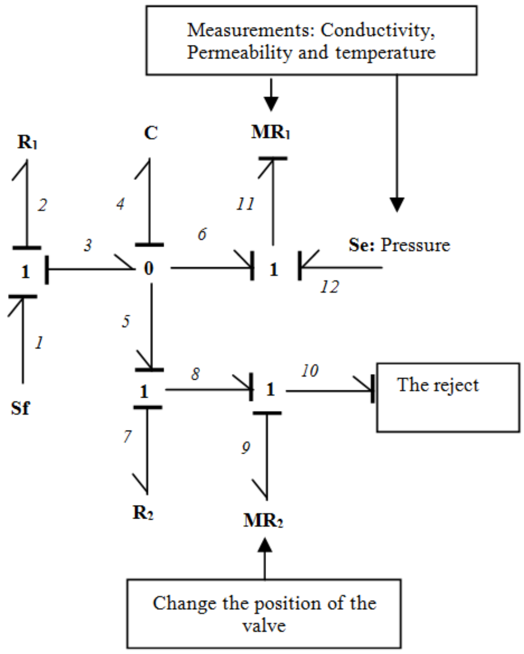



Fig. 9 Bond graph model of reverse osmosis

The bond graph model of reverse osmosis shown in figure 9 above is illustrated as follows:

- The RO desalination system model is equivalent to a storage element with a hydraulic inlet (supply) and two outputs (permeate conductivity and flow rate). It will therefore be represented in bond graph model with a storage element (C) and also by a resistive pressure drop ($R_2$).
- The losses in the supply line are represented by a resistive element ($R_1$).
- The tangential flow of the water after a pressure drop across the membrane and represented by a variable resistor element according to the hydraulic characteristics of the membrane ($MR_1$).
- The control valve is represented by a resistive element modulated ($MR_2$).
- The sum of the osmotic pressure of the water pressure drops and permeate pressure is represented by only source of effort (Se)

To measure the salinity of the raw water supply of reverse osmosis module can use a conductivity meter.

### 5.2 ARRs based on BG-LFT model

Figure 10 shows the BG-LFT model of the system reverse osmosis.

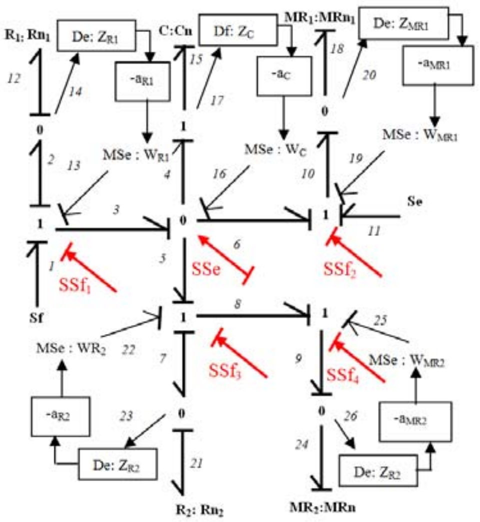


Fig. 10 BG-LFT model of reverse osmosis

From BG model (figure 10), we can deduce the ARRs:

- $ARR_1 : e_1 - e_2 - e_3 - e_{13} = 0$

$$Sf - Rn_1 SSf_1 - Cn \frac{dSSe}{dt} + w_{Rn1} = 0 \qquad (15)$$

Eq. (15) is composed by two parts, the first is related to normal residual and the second to the uncertainty parameter.

$$\begin{cases} RRA_1 = r_1 + d_1 \\ r_1 = Sf - Rn_1 SSf_1 - Cn \dfrac{dSSe}{dt} \\ d_1 = \left| w_{Rn\,1} \right| \end{cases}$$

- $ARR_2 : e_3 - e_4 - e_5 - e_6 + e_{16} = 0$

$$R_{n1} SSf_1 - C_n \frac{dSSe}{dt} - R_{n2} SSf_2 - MR_1 SSf_3 + w_{Cn} = 0$$

$$\begin{cases} ARR_2 = r_2 + d_2 \\ r_2 = R_{n1} SSf_1 - C_n \dfrac{dSSe}{dt} - R_{n2} SSf_2 - MR_1 SSf_3 \\ d_2 = \left| w_{Cn} \right| \end{cases} \qquad (16)$$

- $ARR_3 : e_6 - e_{10} + e_{11} + e_{19} = 0$

$$C_n \frac{dSSe}{dt} - MR_{n1} SSf_3 + Se + w_{Cn} = 0 \qquad (17)$$

$$\begin{cases} ARR_3 = r_3 + d_3 \\ r_3 = C_n \dfrac{dSSe}{dt} - MR_{n1} SSf_3 + Se \\ d_3 = \left| w_{MRn\,1} \right| \end{cases}$$

- $ARR_4 : e_5 - e_7 + e_8 + e_{22} = 0$ (18)

$$C_n \frac{dSSe}{dt} - R_{n2} SSf_4 - MR_{n2} SSf_5 + w_{Rn\,2} = 0$$

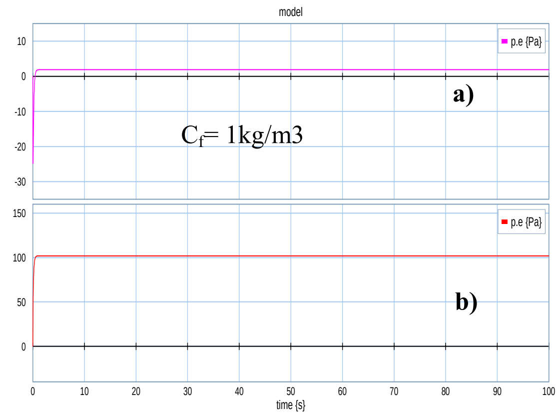
$$\begin{cases} ARR_4 = r_4 + d_4 \\ r_4 = C_n \dfrac{dSSe}{dt} - R_{n2} SSf_4 - MR_{n2} SSf_5 \\ d_4 = \left| w_{Rn\,2} \right| \end{cases}$$

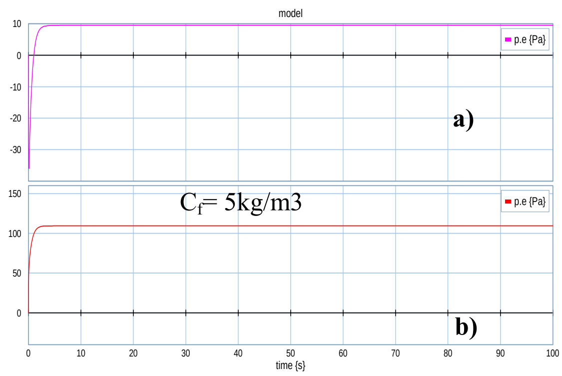
- $ARR_5 : e_8 - e_9 + e_{25} = 0$

$$R_{n2} SSf_4 - MR_{n2} SSf_5 + w_{MRn\,2} = 0 \qquad (19)$$

$$\begin{cases} ARR_5 = r_5 + d_5 \\ r_5 = R_{n2} SSf_4 - MR_{n2} SSf_5 \\ d_4 = \left| w_{MRn\,2} \right| \end{cases}$$

## 5.3. Simulation results



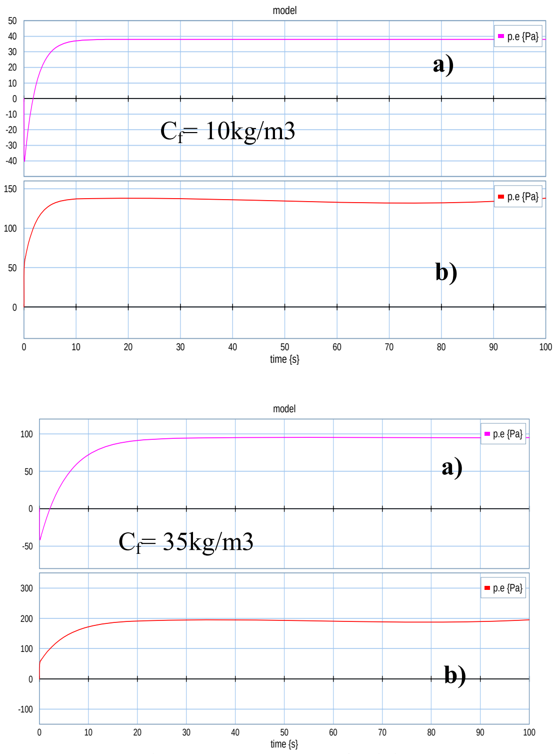



Fig. 11 a) Permeate Conductivity
b) Permeate flow rate

Figures **(a)** show the change in salinity of water produced for inlet concentration of 1 kg / m$^3$. Note that the salinity of water output decreases and stabilized with small fluctuations around 35kg / m$^3$ for a short time. Figures **(b)** show that the permeate flow evolves exponentially with little decreasing when the feed salinity is around 35kg / m$^3$.

## 6. Conclusion

In this paper, a fault detection and isolation (FDI) using on BG modeling is proposed. The bond graph tool is a graphical method of multidisciplinary dynamic systems modeling. It has the main advantage of the link between various components of diverse systems governed by nonlinear equations and sometimes difficult to model. The fault indicators are generated in the presence of parameter uncertainties. As perspective of this work we'll determine an observer by bond graph for location sensor fault or actuator fault (DOS and GOS). New techniques will be developed for the fault isolation and location.

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