

A comparative study of image compression between Singular value decomposition, Block truncating coding, Discrete cosine transform and Wavelet

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ABSTRACT

With the growth of multimedia and internet, compression techniques have become the thrust area in the fields of computers. Popularity of multimedia has led to the integration of various types of computer data. Multimedia combines many data types like text, graphics, still images, animation, audio and video. Image compression is a process of efficiently coding digital image to reduce the number of bits required in representing image. Its purpose is to reduce the storage space and transmission cost while maintaining good quality.

Many different image compression techniques currently exist for the compression of different types of images. Image compression is fundamental to the efficient and cost-effective use of digital imaging technology and applications. In this study Image compression was applied to compress and decompress image at various compression ratios. This was then compared with the formal compression standard "Discrete Cosine Transform" DCT, "Singular Value Decomposition" SVD, "Block Truncation Coding" BTC, Wavelet. Histogram analysis, Bar Comparison, Graph comparison was used as a set of criteria to determine the 'acceptability' of image compression. Wavelet methods have been shown to have no significant differences in diagnostic accuracy for compression ratios of up to 30:1. Visual comparison was also made between the original image and compressed image to ascertain if there is any significant image degradation.

The image compression techniques are categorized into two main classifications namely lossy compression techniques and Lossless compression techniques [1]. Lossless compression ratio gives good quality of compressed images, but yields only less compression whereas the lossy compression techniques [2] lead to loss of data with higher compression ratio.

Keywords:

Block truncating coding (BTC), discrete cosine transform(DCT), Image compression, singular value decomposition(SVD) and wavelet

1. INTRODUCTION

The goal of this paper is to investigate the effect of wavelet compression, Block Truncating Compression, Singular Value Decomposition and discrete Cosine Transform compression, also to compare these compression standards

by using Image Histogram, Graphs on various images. This investigation is carried out by calculating the compression ratio, the histogram result and bar comparison ratio for wavelet, DCT, SVD and block truncating for the same JPEG image.

1.1 Image Compression

Image compression means minimizing the size in bytes of a graphics file without degrading the quality of the image to an unacceptable level. The reduction in file size allows more images to be stored in a given amount of disk more memory space. It also reduces the time required for image to be sent over the internet or downloaded from web pages. The recent growth of data intensive multimedia based web application have not only sustained the need for more efficient ways to encode signals and images but have made compression of such signal central to storage and communication technology. In the present research four algorithms (SVD, DCT, Block truncating Coding, and Wavelet) has been used. The aim of this work is to understand different compression techniques and comparison between them.

Interpixel Redundancy [3]

Interpixel redundancy implies that any pixel value can be reasonably predicted by its neighbors. Usually the value of certain pixel in the image can be reasonably predicted from the values of group of other pixels in the image. For example the gray levels of neighboring pixels are roughly the same and by knowing gray level value of one of the neighborhood pixels one has a lot of information about gray levels of other neighborhood pixels. Thus the value of the individual pixel carries relatively small amount of information and much more information about pixel value can be inferred on the basis of its neighbor's values. These dependencies between pixels values in the image are called interpixel redundancy.

In order to reduce the interpixel redundancies in the image, the 2-D pixel array of image values, used for image

visualization should be transformed into another, generally "non-visual" representation. Transformations used to reduce the interpixel redundancies are called mapping. Since in this paper we deal only with lossless compression, the mappings, which will be considered further, will be reversible.

Scaling [4]

The theme of the technique of magnification is to have a closer view by magnifying or zooming the interested part in the imagery. By reduction, we can bring the unmanageable size of data to a manageable limit. For resampling an image Nearest Neighborhood, Linear, or cubic convolution techniques [5] are used.

Magnification

This is usually done to improve the scale of display for visual interpretation or sometimes to match the scale of one image to another. To magnify an image by a factor of 2, each pixel of the original image is replaced by a block of 2x2 pixels, all with the same brightness value as the original pixel.

Reduction

To reduce a digital image to the original data, every mth row and mth column of the original imagery is selected and displayed. Another way of accomplishing the same is by taking the average in 'm x m' block and displaying this average after proper rounding of the resultant value.

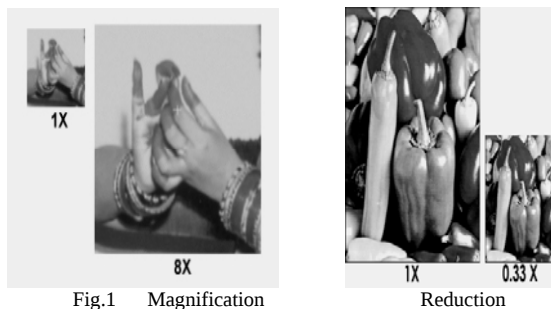


Fig.1 Magnification

Reduction

2. Different Techniques Used:

2.1 Image compression with Singular Value Decomposition (SVD)

The SVD matrix decomposition is extensively used in Mathematics [15]. It appears in fields related directly with algebra, such as least squares problems or the calculus of the matrix rank. Its usefulness in applications concerning image processing has also been evaluated. Among these applications we can mention patron recognition, secret communication of digital images, movement estimation, classification and quantization [6]. The interest of this transformation comes from the fact that by using it we obtain easily the best approximation of a given rank in

terms of the 2-norm. Singular Value Decomposition of a matrix [7] is Calculated By:

$$[u,s,v] = \text{svds}(\text{red}, \text{singvals}); \quad (1)$$

$$\text{Imred} = \text{uint8}(u * s * \text{transpose}(v)); \quad (2)$$

For $M*N*3$ Image

Where $u=N*\text{singvals}$, $s=\text{singvals}*\text{singvals}$,

$$v=M*\text{singvals} \quad (3)$$

singvals is the number of largest singular values (positive integer)

This equation reconstruct matrix for red plane. Similarly green plane and blue plane reconstructed and compressed image reconstructed by combining these three planes R, G, B.

We can see changes in images by changing the singvals values, size increase with increase in signvals value and quality also improve.



Fig.2 Original Image



Fig.3 N=20



Fig.4 N=40



Fig.5 N=80

2.2 Image compression with Block Truncation Coding (BTC)

Block Truncation Coding [8] is a lossy image Compression techniques. BTC is a recent technique used for compression of monochrome image data [9]. It is one-bit adaptive moment-reserving quantizer that preserves certain statistical moments of small blocks of the input image in the quantized output. The original algorithm of BTC preserves the standard mean and the standard deviation [10]. In this technique first empty matrix of same Size of image matrix is created. Then compression is applied on red plane by calculating first number of non overlapping blocks required to cover the entire input image by equation:

$$nbx = \text{size}(\text{dvalue}, 1) / bx; \quad (4)$$

$$nby = \text{size}(\text{dvalue}, 2) / by; \quad (5)$$

Where bx, by are x block size, y block size of compression matrix.

nbx, nby is number of non overlapping blocks, dvalue is reconstructed matrix

Then matrix of bx * by is created in which value of current pixel is stored and average color level of the current block is calculated by

$$m = \text{mean}(\text{mean}(\text{blocco})); \quad (6)$$

Where blocco is current block

The compressed data which correspond to the input image is stored in red plane and then green plane and blue plane created in same way.

Block Truncation Coding (BTC) is a well-known compression scheme proposed in 1979 for the grayscale images. It was also called the moment-preserving block truncation [16] because it preserves the first and second moments of each image block.



Fig.6 8 by 8 block



Fig.7 16 by 16 Block



Fig.8 32 by 32 Block

2.3 Image compression with Discrete Cosine Transform (DCT)

The DCT-based encoder worked by segmentating the image into 8*8 blocks. Each block makes its way through each processing step, and yields output in compressed form into the data stream. As image pixels are highly correlated, the DCT achieves data compression by concentrating most of the signal in the lower spatial frequencies. For a typical 8*8 sample block from a typical source image, most of the spatial frequencies have zero or near-zero amplitude and need not be encoded. In principle, the DCT introduces no loss to the source image samples; it transforms them to a domain in which they can be more efficiently encoded[11]. It first calculate DCT2 of every pixel on RGB plane by equation

$$\text{red_dct}=\text{dct2}(\text{red}); \quad (7)$$

$$\text{green_dct}=\text{dct2}(\text{green}); \quad (8)$$

$$\text{blue_dct}=\text{dct2}(\text{blue}); \quad (9)$$

Where red, green, blue are three planes of image and red_dct, green_dct, blue_dct are corresponding DCT calculated matrix.

Then for M*N image a single column matrix is generated and their square is calculated and store in reconstructed matrix.



Fig.9 DCT Coefficient=5000



Fig.10 DCT Coefficient=9000



Fig.11 DCT Coefficient=15000

2.4 Image compression with Wavelet Transform

Wavelet transform image compression involves the use of a new field of applied mathematics often called 'wavelet theory' or simply "wavelets". Wavelet compression is a subset of a larger class of techniques generally referred to as "transform-based compression". The first step in a transform-based technique typically involves a lossless mathematical transform to provide a sparse representation of an input image. The transformed data are then quantized, in order to achieve the desired level of compression. Transform domain values that are quantized can never be restored to their original accuracy, but such quantization is necessary in order to achieve higher compression ratios. The wavelet uses subband coding to selectively extract different subbands from the given image. These subbands can then be quantized with different quantizers to give better compression. The wavelet filters are specifically designed to satisfy certain constraints called the smoothness constraints. The wavelet filters are designed so that the coefficients in each subband are almost uncorrelated from the coefficients in other subbands [12]. The wavelet transform achieves better energy compaction than the DCT and hence can help in providing better compression for the same Peak Signal to Noise Ratio (PSNR). A lot of research has been done on the performance comparison of the DWT and DCT for image compression. A comparative study of DCT and wavelet based image coding can be found in [13].

In this first noise is inserted to the image.

$$\text{IMNOISE}(I, \text{'speckle'}, V) \quad (10)$$

It adds multiplicative noise to the image I, using the equation $J = I + n \cdot I$, where n is uniformly distributed random noise with mean 0 and variance V. Then decomposition matrix is created by equation

$$[C, S] = \text{WAVEDEC2}(X, N, \text{'wname'}) \quad (11)$$

Returns the wavelet decomposition of the matrix X at level N , using the wavelet named in string 'wname'. Outputs are the decomposition vector C and the corresponding bookkeeping matrix S . N must be a strictly positive integer. Then threshold of wavelet coefficients calculated by:

$$[THR, NKEEP] = WDCBM2(C, S, ALPHA, M) \quad (12)$$

Returns level-dependent thresholds THR and numbers of coefficients to be kept $NKEEP$, for de-noising or compression. THR is obtained using a wavelet coefficients selection rule based on Birge-Massart strategy. Then compressed image is obtained by wavelet packet coefficients thresholding by equation

$$[compressed_image, TREED, comp_ratio, PERFL2] = WPD(ENCMP(thr, 's', n, 'haar', 'threshold', 5, 1); \quad (13)$$

Where 'thr' (2-D) obtained by wavelet packet coefficients thresholding. The additional output argument $TREED$ is the wavelet packet best tree decomposition of $compressed_image$. $PERFL2$ and $PERF0$ are L^2 recovery and compression scores in percentages. Multi-level 2-D wavelet reconstruction.



Fig.12 Wavelet decomposition Level=1



Fig.13 Wavelet decomposition Level=2



Fig.14 Wavelet decomposition Level=3

3. COMPARISON

3.1 MEASUREMENT OF THE DIFFERENCE BETWEEN THE ORIGINAL AND THE RECONSTRUCTED IMAGE

It is natural to raise the question of how much an image can be compressed and still preserve sufficient information for a given application. This section discusses some parameters used to measure the trade-off between image quality and compression ratio. Compression ratio is defined as the nominal bit depth of the original image in bits per pixel (bpp) divided by the bpp necessary to store the compressed image. For each compressed and reconstructed image, an error image was calculated. The maximum absolute error (MAE) is calculated as [14].

$$MAE = \max |f(x, y) - f^*(x, y)|$$

Where $f(x, y)$ is the original image data and $f^*(x, y)$ is the compressed image value.

For a typical RGB image of size 879394 bytes, compressed image is constructed whose size corresponding to different techniques are:

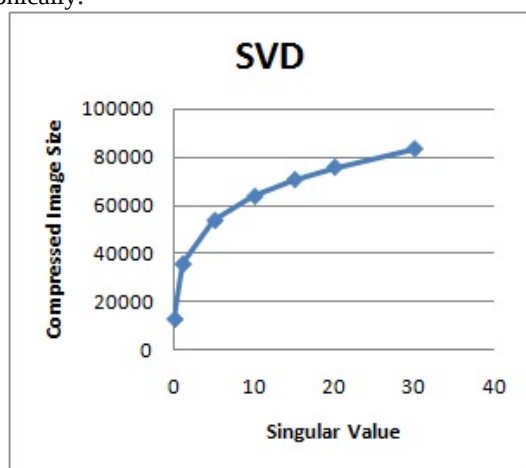
Table 1: results of SVD and BTC compression system

Singular Value Decomposition		Block Truncating	
Singular Value	Size	Block size	Size
0	12915	2*2	117393
1	35744	4*4	112632
5	53833	8*8	111937
10	63849	10*10	120194
15	70746	16*16	114824
20	75913	20*20	121049
25	83547	32*32	115213
		64*64	115237

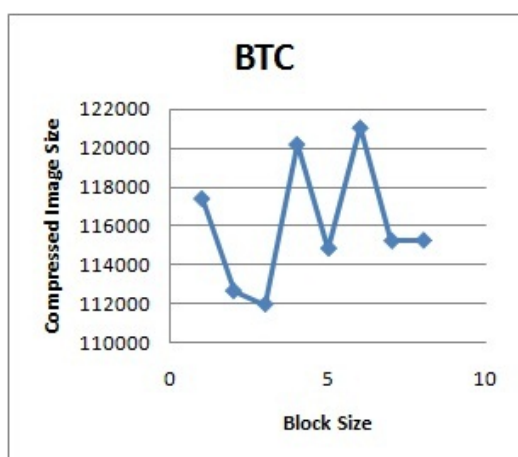
Table 2: results of DCT and Wavelet compression system

<i>DCT compression</i>		<i>Wavelet Compression</i>	
DCT coefficient	Size	Decomposition level	Size
500	50207	1	152791
1000	59550	2	152903
1500	64369	3	153067
3000	72846	4	153018
3500	74921	5	152938
4000	76608	6	153734
5000	79398	7	152908
7000	83688	8	152837

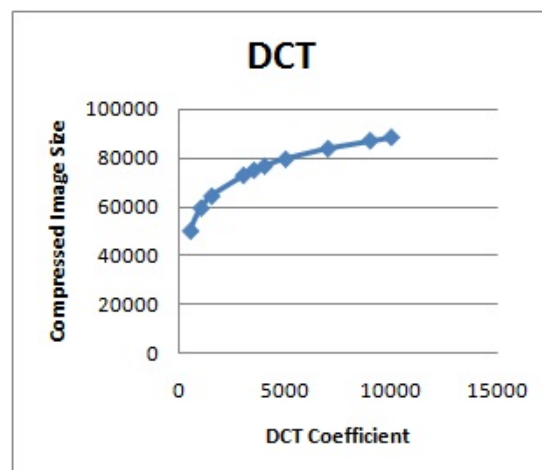
Here size corresponds to compressed image size in bytes. These values of compressed image size can be shown graphically.



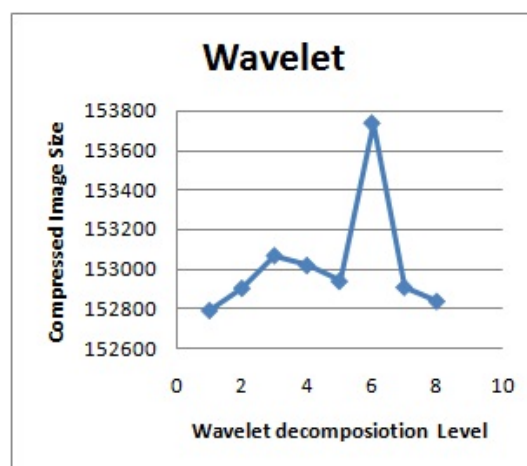
Graph.G1 displaying results of singular value decomposition



Graph.G2 displaying results of block truncation coding



Graph.G3 displaying results of Discrete cosine transform



Graph.G4 displaying results of Wavelet image compression

Graph show corresponding curves w.r.t to image Size with degree of compression. In SVD with increase of value of signalvals size of compressed image also increase and quality of image also increase. In DCT with Increase of coefficients value size will increases. For a grey scale image of size 95048 byte compressed image is constructed whose size corresponding to different techniques are:

Table 3: displaying result of SVD and BTC(grey scale image)

<i>Singular Value Decomposition</i>		<i>Block Truncating</i>	
Singular Value	Size	Block size	Size
0	9548	2*2	94935
1	33114	4*4	95103
5	45438	8*8	95204

10	53064	10*10	97120
15	58123	16*16	95269
20	62825	20*20	97431
25	65979	32*32	92201

Table 4: displaying result of DCT and wavelet(grey scale image)

DCT compression		Wavelet Compression	
DCT coefficient	Size	Decomposition level	Size
500	41143	1	220203
1000	46187	2	220026
1500	48564	3	219894
3000	54598		
3500	56143		
4000	57706		
5000	60635		
7000	66012		

Here size corresponds to compressed image size in bytes. As shown in above tables (Table 3, Table 4) we see that for a grey scale image size of compressed image increases in case of BTC and wavelet compression. So these techniques have no much use for compressing a grey scale image.

4. CONCLUSION

In SVD when singular value increases size of compressed image also increases, But quality of compressed image also improve. In BTC with increase in size of block visual quality of image degrades and there is no much reduce in compression size with increase of block size. In DCT when coefficient value is increase Image quality improve and size of compress image also improve. In wavelet size of compressed image remains same. There is only a little difference in size of compressed image with increasing the value of decomposition level.

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