The Optimization Algorithm of Image Mosaic Based on Wavelet Analysis and Mutual Information

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Summary

Image mosaic technique can be used to acquire wide view scene without reducing the resolution. According to the characteristics of the wavelet analysis and mutual information, a new optimization algorithm of image mosaic was put forward in this paper. The optimized mosaic was realized by following means, that is, narrowing the registering region by image segmentation, optimizing calculation through wavelet decomposition and optimizing searching path by Powell algorithm. The experimental results showed that the algorithm possessed high computing efficiency and good effect of mosaic.

Key words:
image mosaic; wavelet analysis ; mutual information; Powell optimization

1. Introduction

At present, the wide view and high resolution images or videos are more and more important in photogrammetry, panoramic view structure, panoramic video coding and video monitoring, but the wide view images acquired by the ordinary cameras have the lower resolution, so the size and the resolution of the images are a pair of contradictory. In order to acquire wide view scene without reducing the resolution, the image mosaic technology is used to solve this difficult problem. Image mosaic is a kind of technology which mosaics a group of overlapped images into seamless high resolution images of wider view or panoramic images, and the essence of the technology is to carry on the image registration and image fusion at the overlapping part of the images. The computational complexity of image registration is normally bigger, so the development of the image mosaic technology largely depends on the innovation of the image registration technology. The image mosaic technology can be divided into two categories according to image registration method, that is, image mosaic algorithm based on characteristic and image mosaic algorithm based on region. The requirement of image content is higher for image mosaic algorithm based on the characteristic, that is, the content need possessing apparent angular points or contour features, therefore, when processing the image with smoothing transition and flat scenery, this kind of algorithm has lower ability to realize edge extracting. For the algorithm based on region, because the similarity of region are calculated through point by point means, the image mosaic algorithm has large calculated amount; However, it has little specific requirement for image content and fits for more broad range.

According to the characteristics of the wavelet analysis and mutual information, a new optimization algorithm of image mosaic were put forward in this paper. The main idea of the optimal algorithm were described as follow. First, the initiative optimization were made in this step. The registration region were narrowed through intercepting the adjacent area between two pre-registering image, after this process, the pre-registering area were only a part of the original registration image. Second, calculation were optimized through wavelet decomposition. In this step, the relationship between the approximate son-image and original image were used for reference, the son-image of the pre-registering region were selected as the son-registration region ,and the registration between the son-registration region were utilized to realize the original image registration. Third, searching path were optimized. When calculated the mutual information, Powell algorithm were adopted to reduce the calculated amount through iteration based on above registering results until the original registering region were registered. Finally, the obtained registration parameters were used to transform the pre-registering images and the weighted average fusion method were made at the registered images for acquiring the mosaic image.

2. The optimization algorithm of image mosaic

The optimization algorithm were divided into following several processes, that is, image preprocessing, wavelet decomposition, mutual information registration and image mosaic. The processing methods and steps were introduced as follows.

2.1 Image preprocessing

The pre-mosaic images are high resolution images, if the whole images were used to carry out registration and mosaic, the calculating complexity are very high. Therefore, in order to reduce the computational

Manuscript received February 5, 2012
Manuscript revised February 20, 2012

complexity, the registration image A and B were pre-processed separately. The pre-mosaic image A and image B should possess the neighboring relationship at the space location, according to this judge, the four adjacent section were set, that is, up and down, down and up, left and right, right and left, and the area of each section is a quarter of the original image. The results were shown in following figure.

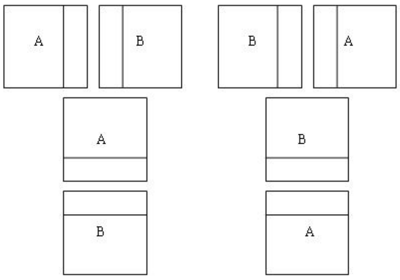

Fig 1 neighbor section location

In addition, the mosaic location need be judged before registration, which can be determined by two ways. One was in manual means, such as left and right, up and down, etc. The other was determined automatically, which calculated the feature information (means or variances, etc) of gray histogram for each section, and set the similar threshold to compare and judge the pre-matching region. This paper set the left and right registration as the example to go on the image registering processes.

2.2 Wavelet decomposition and mutual information registration for image

$f(x,y)$ is the two-dimensional image, ψ is the wavelet function, $f,\psi \in L^2(R^2)$,the wavelet transform can be described as:

$$W_s f(u,v) = \int_R \int_R f(x,y)\frac{1}{s^2}\times\overline{\psi(\frac{u-x}{s},\frac{v-y}{s})}dxdy \quad (1)$$

For 2D orthogonal wavelet transform, the Mallat algorithm is its fast algorithm. Mallat algorithm converts the calculating problem of wavelet transform into transformed coefficient calculating problem. During the operation, the discrete sampling value $\left|f^{M+1}(m,n)\right|$ at M+1 scale layer were given to calculate the wavelet transform coefficients at M scale layer. Let $H=\{h_n\}, G=\{g_n\}$, they are the low-pass and high-pass filters of wavelet decomposition.

Then the coefficient calculating processes of tensor product wavelet decomposition were as follows.

$$\begin{cases} c^{M+1}(m,n) = f(m,n) \\ c^M(m,n) = \sum_{k,l} h(k-2m)h(l-2n)c^{M+1}(k,l) \\ \alpha^M(m,n) = \sum_{k,l} h(k-2m)g(l-2n)c^{M+1}(k,l) \\ \beta^M(m,n) = \sum_{k,l} g(k-2m)h(l-2n)c^{M+1}(k,l) \\ \gamma^M(m,n) = \sum_{k,l} g(k-2m)g(l-2n)c^{M+1}(k,l) \end{cases} \quad (2)$$

Among them, $c^{M+1}(m,n)$ are the wavelet coefficients at M + 1 scale layer, which are the original image data; $c^M(m,n)$ are the image data at M scale, which are the low frequency component data of the image data $c^{M+1}(m,n)$; $\{c^M(m,n)\}$ are the general view of $\{c^{M+1}(m,n)\}$, they also are the breviary express of $\{c^{M+1}(m,n)\}$ and similar with $\{c^{M+1}(m,n)\}$ at outline. $\{\alpha^M(m,n)\}$ are the general views at x direction and high frequency details at y direction of $\{c^{M+1}(m,n)\}$ after wavelet decomposition. $\{\beta^M(m,n)\}$ are the general views at y direction and high frequency details at x direction of $\{c^{M+1}(m,n)\}$ after wavelet decomposition. $\{\gamma^M(m,n)\}$ are the high frequency details at x direction and at y direction of $\{c^{M+1}(m,n)\}$ after wavelet decomposition, they show the detail information at the diagonal direction.

Because the wavelet filtering process adopted the principle of 2 interval sampling, the sampled image was supposed as $\{f(2m,2n)\}$, the standard image was supposed as $\{f(2m',2n')\}$, the coefficient images after low-pass filter were recorded as $\{c^M(m,n)\}$ and $\{c^M(m',n')\}$. Here $Tm=m', Tn=n'$ .T was the affine transformation. The coordinate change after stretching, rotating and translating by wavelet filter were studied as follows.

(1)Stretching transform.

if T is stretching transform, it transformed the image $\{c^{M+1}(2m,2n)\}$ into $\{c^{M+1}(\alpha\times 2m,\alpha\times 2n)\}$, after wavelet low-pass filtering, following formula were obtained.

$$\begin{aligned} &\sum_{k,l} h[\alpha\times(k-2m)]h[\alpha\times(l-2n)]\times c^{M+1}(\alpha\times k,\alpha\times l) \\ &= \sum_{k,l} h(\alpha k-\alpha\times 2m)h(\alpha l-\alpha\times 2n)\times c^{M+1}(\alpha k,\alpha l) \quad (3) \\ &= \sum_{p,q} h(p-\alpha\times 2m)h(q-\alpha\times 2n)\times c^{M+1}(p,q) \\ &= c^M(\alpha\times m,\alpha\times m) \end{aligned}$$

Among them, $p=\alpha k, q=\alpha l$.

From above formula, the stretching registration of two images can be converted into approximate component registration of original images after wavelet decomposition. And the stretching coefficients of original images were equal to the stretching coefficients of approximate component images after decomposition.

(2)Rotating transform

if T is rotating transform, it transformed the image $\{c^{M+1}(2m,2n)\}$ into $\{c^{M+1}(2m\cos\theta+2n\sin\theta,-2m\sin\theta+2n\cos\theta)\}$, after wavelet low-pass filtering, following formula were obtained.

$$\sum_{k,l}\{h[(k-2m)\cos\theta+(l-2n)\sin\theta]\times h[-(k-2m)\sin\theta+(l-2n)\cos\theta]$$
$$\times c^{M+1}(k\cos\theta+l\sin\theta,-k\sin\theta+l\cos\theta)\}$$
$$=\sum_{k,l}\{h[(k\cos\theta+l\sin\theta)-2(m\cos\theta+n\sin\theta)]\times h[(-k\sin\theta+l\cos\theta)-$$
$$2(-m\sin\theta+n\cos\theta)]\times c^{M+1}(k\cos\theta+l\sin\theta,-k\sin\theta+l\cos\theta)\}$$
$$=\sum_{k,l}\{h[p-2(m\cos\theta+n\sin\theta)]\times h[q-2(-m\sin\theta+n\cos\theta)]\times c^{M+1}(p,q)$$
$$=c^{M}(m\cos\theta+n\sin\theta,-m\sin\theta+n\cos\theta) \tag{4}$$

Among them, $p=k\cos\theta+l\sin\theta, q=-k\sin\theta+l\cos\theta$.

From above formula, the rotating registration of two images can be converted into approximate component registration of original images after wavelet decomposition. And the rotating angles of original images were equal to the rotating angles of approximate component images after decomposition.

(3) Translating transform.

If T is translating transform, it transformed the image $\{c^{M+1}(2m,2n)\}$ into $\{c^{M+1}(2m+2\Delta x,2n+2\Delta y)\}$. In order to get the relationship between the filtered image and its approximate component, the following transform were made.

Let $p=k-2m, q=l-2n$, then

$$\sum_{k,l}h(k-2m)h(l-2n)c^{M+1}(k,l)=\sum_{p,q}h(p)h(q)c^{M+1}(2m+p,2n+q) \tag{5}$$

Therefore, after wavelet low-pass filtering, $\{c^{M+1}(2m+2\Delta x,2n+2\Delta y)\}$ changed as

$$\sum_{p,q}h(p)h(q)c^{M+1}(2m+2\Delta x+p,2n+2\Delta y+q)$$
$$=\sum_{p,q}h(p)h(q)c^{M+1}[(2(m+\Delta x)+p,2(n+\Delta y)+q] \tag{6}$$
$$=c^{M}(m+\Delta x,n+\Delta y)$$

From above formula, the translating registration of two images can be converted into approximate component translating registration of original images after wavelet decomposition. If the translating amount of original images were $(2\Delta x,2\Delta y)$, the translating amount of approximate component images were $(\Delta x,\Delta y)$.

Through above discussion, the registering problem for affine transformation $T(f(x,y))=f(x',y')$ of original image $f(x,y)$ can be attributed to the coefficient image registration of original images and transformed images which filtered by wavelet. Therefore, the registration of original images were converted into corresponding approximate component image registration after wavelet decomposition, and the stretching parameters and rotating parameters of approximate component were immovable, the translating parameters were half of the original image registration parameters. The advantage of the processing were saving the computational cost of registration. If the original image size was N×N, the computational amount of the registration parameters was K, then the size of low frequency image after wavelet decomposition was N/2×N/2, and the computational amount of the registration parameters was K/4.

3) Mutual information calculation

Mutual information was a conception based on entropy , which was a measure of two random variables. So it can express the inclusive degree of information between two images. The size of the pre-registering images A and image B were Supposed as M×N, and the gray level is 0-255. The calculating procedure of the mutual information I (A, B) is described as follows.

(1) Calculating gray probability distribution function $p_{AB}(i,j)$ of image of A and B .

Define a matrix $H[i,j]$ of 256×256, which the element $h(i,j)$ corresponding to pixel number of the gray pair (i,j), and all elements were initialized as zero.

a) Image A and B were traversed at corresponding pixel position, the value of $h(i,j)$ $(0<i,j<255)$ were accumulated and calculated.

b)Let $P[i,j]=\frac{1}{M\times N}H[i,j]$,among them, $p(i,j)=\frac{1}{M\times N}h(i,j)$.

(2) Using the formula (7) to calculate the combined entropy of image A and B.

$$H(A,B)=-\sum_i\sum_j p_{AB}(i,j)\log p_{AB}(i,j)(0<i,j<255) \tag{7}$$

(3) Using the formula (9) to calculate the fringe entropy of image A and B.

$$p_A(i)=\frac{h_A(i)}{M\times N}, p_B(j)=\frac{h_B(j)}{M\times N} \tag{8}$$

$$H(A)=-\sum_i p_A(i)\log p_A(i)$$
$$H(B)=-\sum_j p_B(j)\log p_B(j) \quad (0<i,j<255) \tag{9}$$

(4) Computing mutual information I（A，B）of image A and B.

From
$$p_A(i)=\sum_j p_{AB}(i,j)$$
$$p_B(j)=\sum_i p_{AB}(i,j) \tag{10}$$

$$I(A,B) = H(A) + H(B) - H(A,B) \quad (11)$$

Obtained

$$I(A,B) = \sum_{i,j} p_{AB}(i,j) \log \frac{p_{AB}(i,j)}{p_A(i)\, p_B(j)} \quad (12)$$

4) The Powell optimal algorithm for solving parameter optimization

Powell algorithm were called the accelerated direction method, and it belonged to a kind of conjugate direction method without calculating derivative. The one dimensional optimized process of multivariable function were carried out at conjugate direction to approach the optimum point, so the algorithm used less time and possessed fast convergence speed.

For a n-dimension problem which owned n parameters, the iterative computation process of Powell algorithm were as follows.

Each round search started from above round optimum point, and along n sequential linear independent direction $S_1^{(k)}, S_2^{(k)}, .., S_n^{(k)}$. Each round search can start from any point, namely $X_0^{(1)} = X^{(0)}$, and the direction can select *n* coordinate direction, that is,

$$S_1^{(1)} = e_1, S_2^{(1)} = e_2, \ldots, S_n^{(1)} = e_n$$

$$e_1 = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, e_2 = \begin{bmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix}, \cdots, e_n = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} \quad (13)$$

Now the K round search were given.

$$f_1 = f(X_0^{(k)}),\ f_2 = f(X_n^{(k)})$$

$$\Delta_m^{(k)} = \max_{i=1,2\ldots n} \left| f(X_{i+1}^{(k)}) - f(X_i^{(k)}) \right| \quad (14)$$

Among them, $X_0^{(k)}$ represented the starting point of K round search, $X_n^{(k)}$ represented the minimum point of K round at Nth time.

$$X_{n+1}^{(k)} = 2X_n^{(k)} - X_0^{(k)} \quad (15)$$

In the K round iteration, if the condition were satisfied as follows.

$$f_3 < f_1$$

$$(f_1 + f_3 - 2f_2)(f_1 - f_2 - \Delta_m^{(k)})^2 < 0.5\Delta_m^{(k)}(f_1 - f_2)^2 \quad (16)$$

Then, a new direction S_{n+1}^k were chosen, and in the K + 1 round search, the corresponding direction S_n^k at the functional value maximal descending $\Delta_m^{(k)}$ were replaced by using the new direction .Otherwise, the original direction group were still adopted for K + 1 round search.

2.3 Registration process

A is supposed as the standard location image, B is supposed as the pre-registering image, according to above principles, the registering processes between A and B were described as follows.

Step1: The image A and B were decomposed by two-dimensional wavelet decomposition algorithm(this paper carried out three layer decomposition). LL_{Aj} and LL_{Bj} were the approximate components at *j* layer of image A and B separately, among them, $0 \le j \le N$.

Step2: Approximate components were registered at the top layer, namely LL_{AN} and LL_{BN} .The size of the LL_{AN} and LL_{BN} was assumed as $m \times n$ .Then, $[-\theta, \theta]$ and $[-m/2, m/2] \times [-n/2, n/2]$ were made as the interval, $\{\Delta\theta_N、\Delta t_{x_N}、\Delta t_{y_N}\}$ were set as the search precision, LL_{BN} were rotated and translated, and the mutual information MI at each LL_{AN} and LL_{BN} were calculated. Finally, the various parameters $\{\theta_N、t_{x_N}、t_{y_N}\}$ when the MI value reached maximum were chosen, which were the registration results at N layer.

Step3: The *N* layer registration parameters were used to register *N*-1 layer approximate component $LL_{A(N-1)}$ and $LL_{B(N-1)}$,also the Powel optimization algorithm were used to get *N*-1 layer registration parameters $\{\theta_{N-1}、t_{x_{N-1}}、t_{y_{N-1}}\}$.

Step4: Iteration were made based on the above registration result, until the registration of original images were finished and registration result $\{\theta, t_x, t_y\}$ were gotten.

The registration algorithm based on wavelet transform and mutual information were depicted as figure 2.

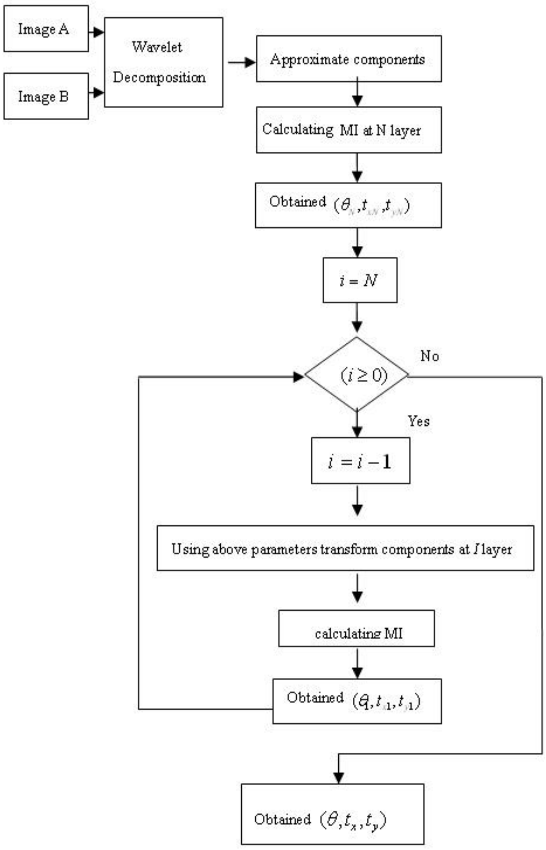

Fig.2 Flowchart of registration algorithm based on wavelet transform and mutual information.

2.4 Image mosaic

After registration, two pre-mosaic image may show the lighting and shading intension and deforming degree difference at overlap position. In order to remove the juncture, the weighted average method were used to carry out the image smooth transition. f_1 and f_2 were supposed as two pre-mosaic image. Overlayed the two images f_1 and f_2 in space, the pixels of mosaic image f can be expressed as:

$$f(x,y) = \begin{cases} f_1(x,y) \quad (x,y) \in f_1 \\ d_1 \cdot f_1(x,y) + d_2 \cdot f_2(x,y) \quad (x,y) \in (f_1 \cap f_2) \\ f_2(x,y) \quad (x,y) \in f_2 \end{cases} \quad (17)$$

In the formula, d_1 and d_2 were the weight, and $d_1 + d_2 = 1$, $0 \leq d_1, d_2 \leq 1$.

In the overlapping area, the values were gradual changed from 1 to 0 and from 0 to 1. Through this way, the smooth transition from f_1 to f_2 at the overlay area were realized.

3. The experiment and analysis

In order to test the image mosaic algorithm based on wavelet transform and mutual information, two Landscape image were selected to carry out experiment. The data were gained at the Windows XP system, and the experimental tool is the Matlab 7.0 image processing and wavelet toolbox. In the experiment, the db orthogonal wavelet filter(possessing compactly supported set) were adopted to pre-process the pre-mosaic image, that is ,three layer wavelet decomposition. After wavelet transform, the top layer reference region and pre-registering region were carried out the mutual information calculation and the parameters were obtained. Then, the parameters from above layer were used to carry out Powell optimization to get the lower layer registration parameters, until the final registration parameters were output. Finally, the image mosaic were realized according to registration parameters. The experimental results were shown as follows.


a) pre-mosaic image A b) pre-mosaic image B

c) mosaic image

Fig.3 Image mosaic based on wavelet transform and mutual information

Through the comparison of mosaic image, the algorithm can realize the image mosaic very well. At the same time, the calculated amount for mosaic were reduced greatly. The mutual information calculated times(supposing the size of the registering region is 512*512) were described as follows.

MI times of only using mutual information :

512*512*20=5242880(the searching region is $\left[0^\circ, 20^\circ\right]$,searching accuracy is $1^\circ \times 1 \times 1$)

MI times of using wavelet and mutual information: **1734+405+405+405=2949**(the initial searching region is $\left[0^\circ, 20^\circ\right]$,searching accuracy is 4, following region and accuracy were adjusted by corresponding registration results of above Layers).

From the experimental result , the mosaic image possessing good visual effect. All above proved that the algorithm were stable and credible.

4. Conclusions

According to the characteristics of the wavelet analysis and mutual information, a new optimization algorithm of image mosaics were put forward in this paper. The optimized mosaic were realized by following means, that is, narrowing the registration region, optimizing calculation through wavelet decomposition and optimizing searching path by Powell algorithm. The experimental images and calculated amount statistics showed that the algorithm used muti-step optimization and layering search strategy which can effectively reduce the mosaic errors and improve the accuracy and efficiency of the mosaic. Besides, the algorithm has less manual intervention, it only depends on the information belongs to the pre-mosaic image, so it is a kind of effective and steady mosaic algorithm.

Acknowledgments

I would like to take this chance to express my sincere gratitude to my Ph.D. tutor Prof. Jianping Li and my senior Dr.Senhua Wang , who offered valuable help in my scientific research.

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