Study of the estimation techniques for the Carrier Frequency Offset (CFO) in OFDM systems

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Abstract
Orthogonal frequency division multiplexing (OFDM) has been selected for broadband wireless communication system. OFDM can provide large data rates with sufficient robustness to radio channel impairments. One of the major drawbacks for OFDM system is Carrier frequency offset (CFO). Frequency offset has been recognized as a major disadvantage of OFDM. The OFDM systems are sensitive to the frequency synchronization errors in form of Carrier Frequency Offset (CFO), because it can cause the Inter Carrier Interference (ICI) which can lead to the frequency mismatched in transmitter and receiver oscillator. Lack of the synchronization of the local oscillator signal (L.OSC); for down conversion in the receiver with the carrier signal contained in the received signal can cause to degrade the performance of OFDM. On the other hand the orthogonality of the OFDM relies on the condition that transmitter and receiver operate with exactly the same frequency reference. To compensate the effect of CFO the researchers have proposed various CFO estimation and compensation techniques and algorithms by now. In this paper, the reason of creating CFO and the effects of the CFO on the performance of the OFDM system will study. The major CFO estimation algorithm and techniques will be reviewed and discussed in literature briefly and then our proposed algorithm and technique for estimating and compensation of the effect of CFO will be offered.

Key words:
Carrier frequency offset (CFO), Orthogonal Frequency Division Multiplexing (OFDM), Inter Carrier Interference (ICI), CFO estimation, CFO compensation, OFDM performance and Doppler Effect (DE)

1. Introduction

The orthogonality of the OFDM relies on the condition that transmitter and receiver operate with exactly the same frequency reference. If this is not the case, the perfect orthogonality of the subcarrier will be lost, which can result to subcarrier leakage, this phenomenon is also known as the Inter Carrier Interference (ICI) [1]. In another word, the OFDM systems are sensitive to the frequency synchronization errors in form of CFO. CFO can lead to the Inter Carrier Interference (ICI); therefore CFO plays a key role in Frequency synchronization. Basically for getting a good performance of OFDM, the CFO should be estimated and compensated. Lack of the synchronization of the local oscillator signal (L.OSC); for down conversion in the receiver with the carrier signal contained in the received signal causes Carrier Frequency Offset (CFO) which can create the following factors:

(i) Frequency mismatched in the transmitter and the receiver oscillator
(ii) Inter Carrier Interference (ICI)
(iii) Doppler Effect (DE)

2. Effects of frequency offset on OFDM signals

When CFO happens, it causes the receiver signal to be shifted in frequency (δf); this is illustrated in the figure 1. If the frequency error is an integer multiple I of subcarrier spacing δf, then the received frequency domain subcarriers are shifted by $\delta f \times I$ [2].

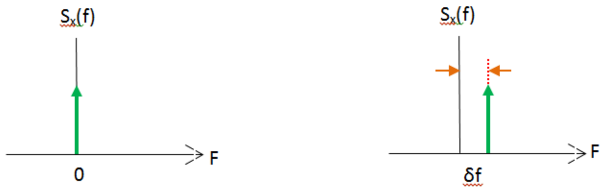

Fig.1 frequency offset (δf)

On the other hand, as we know the subcarriers (SCs) will sample at their peak, and this can only occur when there is no frequency offset, however if there is any frequency offset, the sampling will be done at the offset point, which is not the peak point. This causes to reduce the amplitude of the anticipated subcarriers, which can result to raise the Inter Carrier Interference (ICI) from the adjacent subcarriers (SCs). Figure 2 shows the impact of carrier frequency offset (CFO).
It is necessary to mention that although it is true that the frequency errors typically arise from a mismatch between the reference frequencies of the transmitter and the receiver local oscillators, but this difference is avoidable due to the tolerance that electronics elements have.

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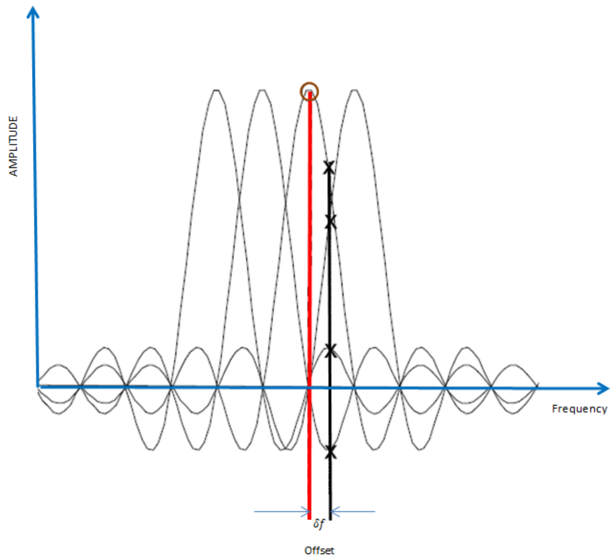

Fig.2 Frequency offset (δf)

Therefore there is always a difference between the carrier frequencies that is generated in the receiver with the one that is generated in transmitter; this difference is called frequency offset f_{offset} i.e.

$$f_{offset} = f_c - f_c'$$

In where f_c is the carrier frequency in the transmitter and f_c' is the carrier frequency in receiver.

3. Carrier Frequency Offset (CFO)

The OFDM systems are very sensitive to the carrier frequency offset (CFO) and timing, therefore, before demodulating the OFDM signals at the receiver side, the receiver must be synchronized to the time frame and carrier frequency which has been transmitted. Of course, In order to help the synchronization, the signals that are transmitted, have the references parameters that are used in receiver for synchronization. However, in order the receiver to be synchronized with the transmitter, it needs to know two important factors:

(i) Prior to the FFT process, where it should start sampling the incoming OFDM symbol from.
(ii) How to estimate and correct any carrier frequency offset (CFO)

After estimating the symbol boundaries in the receiver and when the presence of the symbol is detected the next step is to estimate the frequency offset. Figure 3 shows the block diagram of the OFDM system. At the receiver, the output of the FFT, in figure 3, y[k] is as follows [3]:

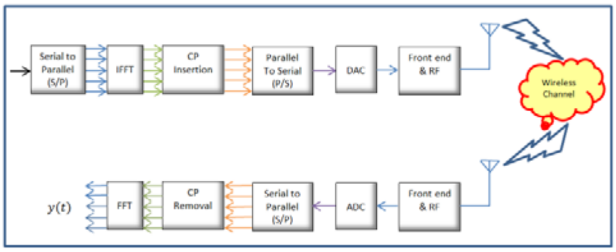

Fig.3 block diagram of the OFDM system

$$y[k] = \frac{1}{N}\sum_{m=0}^{N-1} s[m]H[m]\frac{sin\left(\pi(m-k+\Delta f)\right)}{sin\left(\frac{\pi(m-k+\Delta f)}{N}\right)} * exp\left(j\left(\frac{N-1}{N}\right)(m-k+\Delta f)\right) \quad (1)$$

$$y[k] = \frac{1}{N}s[k]H[k]\left(\frac{\sin(\pi\Delta f)}{\sin\left(\pi\Delta f/_N\right)}\right)exp\left(\frac{j(N-1)\Delta f}{N}\right) + \frac{1}{N}\sum_{\substack{m=0\\m\neq k}}^{N-1} s[m]H[m]\left[\frac{\sin(\pi(m-k+\Delta f))}{sin\left(\frac{\pi(m-k+\Delta f)}{N}\right)}\ exp\left(j\left(\frac{N-1}{N}\right)(m-k+\Delta f\right)\right]$$
(2)

For simplicity, let's set α equal to:

$$\alpha_{m-k} = \left[\frac{\sin(\pi(m-k+\Delta f))}{sin\left(\frac{\pi(m-k+\Delta f)}{N}\right)}\ exp\left(j\left(\frac{N-1}{N}\right)(m-k+\Delta f)\right)\right]$$

$$if\ m = k \quad then$$

$$\alpha_{m-k} = \alpha_0 = \left[\frac{\sin(\pi\Delta f)}{sin\left(\frac{\pi\Delta f}{N}\right)}\ exp\left(j\left(\frac{N-1}{N}\right)(\Delta f)\right)\right] \quad (3)$$

Therefore

$$y[k] = \frac{1}{N}s[k]H[k]\left(\frac{\sin((\pi\Delta f)}{\sin\left(\pi\Delta f/_N\right)}\right)exp\left(\frac{j(N-1)\Delta f}{N}\right) + \frac{1}{N}\sum_{\substack{m=0\\m\neq k}}^{N-1} s[m]H[m]\alpha_0 \quad (4)$$

The result in Eq. 4 indicates in the case of the existence of any frequency offset, the estimation of the output symbol depends on the input values.

On the other side if there is no frequency offset i.e. $\Delta f = 0$ then the received signal is:

$$if\ \ \Delta f = 0 \quad \rightarrow \quad y[k] = \frac{1}{N}s[k]H[k] \quad (5)$$

Due to the frequency mismatched, the performance of an OFDM system can be reduced, this loss of performance can be compensated by estimating the frequency offset in receiver side. Figure 4 shows an OFDM Receiver with frequency synchronization.

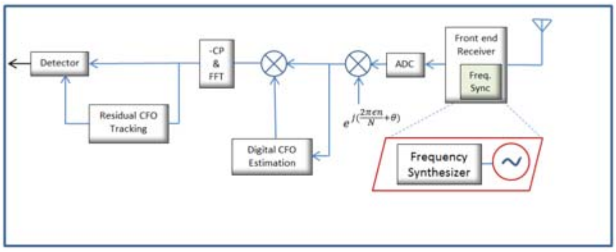

Fig.4 OFDM Receiver with frequency synchronization

Table 1 is a cliff notes for the effect of CFO on transmitted signal in time domain and frequency domain.

	Received signal	Effect of CFO on received signal
Time domain	$y[n]$	$e^{j2\pi\varepsilon/n}x[n]$
Frequency domain	$Y[k]$	$X[k-\varepsilon]$

Table 1: Effect of the CFO on transmitted signal

3.1 Sources of frequency offset

A few other sources can cause frequency offset, such as frequency drifts in transmitter and receiver oscillators, Doppler shift, radio propagation and the tolerance that electronics elements have in local oscillators in transmitter and the receiver. When there is a relative motion between transmitter and receiver the Doppler can happen [4]. It is worth to mention the radio propagation talks about the behavior of radio waves when they are broadcasted from transmitter to receiver. In terms of propagation, the radio waves are generally affected by three phenomena which are: diffraction, scattering and reflection.

3.2 Doppler Effect

The Doppler Effect (DE) defines as follows:

$$f_d = \frac{v.f_c}{c} \tag{6}$$

In where f_d is Doppler frequency, c is the speed of light, and v is the velocity of the moving receiver. (i.e. 100 km/h). The normalized CFO (ε) is defined as follows:

$$\varepsilon = \frac{f_{offset}}{\Delta f} \tag{7}$$

In where Δf is the subcarrier spacing, it is necessary to mention that ε has two parts, one integer (ε_i) and one fractional (ε_f) so we have:

$$\varepsilon = \varepsilon_i + \varepsilon_f \tag{8}$$

In where $\varepsilon_i = \lfloor\varepsilon\rfloor$

4. CFO estimation algorithm and Techniques

CFO can produce Inter Carrier Interference (ICI) which can be much worse than the effect of noise on OFDM systems. That's why various CFO estimation and compensation algorithms have been proposed. For showing the importance of it, it is enough to mention that, by now the researchers have proposed numerous and various CFO estimation and compensation techniques and algorithms, which these methods can generally be categorized into two major branches:

1. Training based algorithm
2. Blind algorithm and Semi-blind algorithm

4.1 Training based algorithm

The training sequence can be designed the way that can limit the number of computation at the receiver side; therefore in general, these algorithms have a low computational complexity. On the other hand, the negative point of training based algorithm is the training sequences that must be transmitted from transmitter during its transmission. This can cause the reduction of the effectiveness of the data throughput.

4.2 Blind and Semi-blind algorithms

Another algorithm that has been used is called Blind CFO estimation algorithm. In these algorithms by using the statistical properties of the received signal, the CFO will be estimated. Since the receiver doesn't have any knowledge of the signal that the transmitter has been sending, therefore the blind algorithms are considered to have a high computational complexity. The high computational complexity is the disadvantage of these algorithms. In compared with training based algorithm, blind algorithms have no need to the training sequences; therefore there is no training overhead for these algorithms.

4.3 Study of the training based and blind algorithms

In this part, we do some discussions on a few proposed training and Blind algorithms.

For the CFO estimation, Paul H. Moose [5] in his paper "The technique for OFMD frequency offset correction" suggested a training based CFO estimation algorithm. His paper is divided into two major sections, in the first section he showed the effect of offset errors on the signal to noise (SN) and in the second part he presented an algorithm to estimate the offset and use it to remove it, prior to the demodulation. He presented the algorithm for maximum likelihood estimate (MLE) of frequency offset using the values of a repeated data symbol. In this algorithm two repetitive OFDM symbol will be sent. This algorithm works on the base of knowing the start point of the OFDM symbol. In this paper, the maximum likelihood estimate (MLE) of CFO is defined as follows [5]:

$$\hat{\varepsilon} = \frac{1}{2\pi}\, tan^{-1}\{(\sum_{k=1}^{M} Im[Y_{2k}Y_{1k}^{*}])\ (\sum_{k=1}^{M} Re[\,Y_{2k}Y_{1k}^{*}]\} \quad (9)$$

Where:

$\hat{\varepsilon}$ is the maximum likelihood estimate (MLE) for CFO, Im is the imaginary part, Re is the real part and * means the complex conjugate.

In this estimation the mean square error is:

$$Mean\ square\ of\ [\hat{\varepsilon}] = \frac{1}{(2\pi)^2 N\mu} \quad (10)$$

Where μ is the ratio of the signal to noise for the received signals, and N is the number of subcarriers (SCs). According to the paper, the limit of accurate estimation (for acquisition range) for this algorithm is $|\varepsilon| \leq 0.5 \Rightarrow (-0.5 \leq \text{Acquisition Range} \leq 0.5)$ therefore the acquisition range for subcarrier spacing is between -0.5 and 0.5, which is smaller than the value that is in the IEEE 802.11a. When acquisition range goes towards the 0.5, $\hat{\varepsilon}$ may due to the noise and the discontinuity of the arctangent, jump to -0.5 when this occurs the estimate is no longer unbiased and in practice, it becomes useless. This estimation for the small values of CFO is conditionally unbiased. However, the big weakness for suggested algorithm in Moose's paper is its dependency to the starting point; therefore the algorithm needs to know the start point of the OFDM symbol.

For the CFO estimation, and in order to overcome the weakness in Moose's algorithm; Timothy M. Schmidl and Donald C. Cox, in their paper "Robust Frequency and Timing Synchronization for OFDM [6]"; to solve the problem for determining the starting point of the OFDM symbol, they suggested the time domain OFDM system based on the two identical halves. In this method, finding the symbol timing for OFDM means finding an estimate of where the symbol starts. Figure 5, shows an example of the timing metric as a window slides past coincidence for the Additive White Gaussian Noise (AWGN) channel for an OFDM signal with 1000 subcarriers, a carrier frequency offset of 12.4 subcarrier spacing, and an signal-to-noise ratio (SNR) of 10 dB. Here the SNR is the total signal (of all the subcarriers) to noise power ratio.

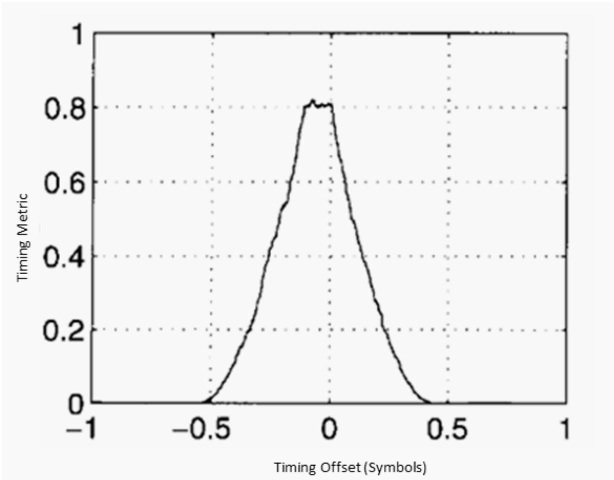

Fig. 5 Example of the timing metric for the AWGN channel (SNR = 10dB) [6]

As it is shown, the timing metric reaches a plateau and since there is no Inter Symbol Interference (ISI) within this plateau to distort the signal, therefore the starting point of OFDM can be chosen at any spot on this plateau. The timing metric is defined as follows [6]:

$$M(d) = \frac{|P(d)|^2}{((R(d))^2} \quad (11)$$

Where

$$P(d) = \sum_{m=0}^{L-1}(r_d^{*} + mr_d + m + L) \quad (12)$$

And

$$R(d) = \sum_{m=0}^{L-1}|r_d + m + L|^2 \quad (13)$$

As you see, the $P(d)$ is an auto correlation function and $R(d)$ is a normalized constant. In this estimation the mean square error is:

$$Mean\ square\ of\ [\hat{\varepsilon}] = \frac{2}{\pi^2 N\mu} \quad (14)$$

The Mean Square Error (MSE) in both method (Moose and Timothy) are near and similar to each other.

By comparing the Moose's method and Timothy's method, we can tell that the advantage of Timothy's method is its simplicity, plus it is not depended to the starting point. In Moose' method the frequency synchronization is done in frequency domain but in the Timothy's method the frequency synchronization is achieved in time domain in which the complexity of Fast Fourier Transform (FFT) in time domain is much less than in frequency domain. By the way both of these methods use the periodic training sequence, one in frequency domain and the other in time domain. It is necessary to mention that the time domain periodic training sequences are the one that has been accepted in various wireless standards. However, there are some limitations for CFO estimation in these methods for SISO-OFDM.

A semi-blind method was proposed for simultaneously estimating the carrier frequency offsets (CFOs) and channels of an uplink for MIMO-OFDM system by Yonghong Zeng, A. Rahim Leyman, and Tung-Sang Ng [7], in this method a pilot OFDM block for each user is exploited for resolving the CFOs and the ambiguity matrix. Two dedicated pilot designs, periodical and consecutive pilots, had been discussed. Based on each pilot design and the estimated shaped channels, two methods were proposed to estimate the CFOs. The algorithm that is used in this method can be summarized as follows [7]:

1. Computing $R_x = E(x_i x_i^{\dagger})$
2. Finding orthogonal eigenvectors of matrix R_x
3. Finding the eigenvalue decomposition (EVD)

After finding the CFOs the ambiguity matrix obtains. In this method when the CFOs have been estimated at the base station, recovery of the signal is still a problem [7], however the complex steps that we have to follow them in semi-blind algorithms, make its implementation difficult. Basically the computational complexity of this algorithm is high for practical implementation.
Other algorithms which are called, Semi-blind algorithm proposed to estimate multiple CFO values. these algorithm have cons and pros. However, most of the proposed algorithms have computational complexity which makes their implementation hard and their complexity grows nonlinearly. On the other hand by using the MIMO instead of SISO, the other requirements both in transmitter side and receiver side come into the boarding table, such as multiple clock signals (distributed or centralized clock signal).

5 The relation between frequency offset and SNR

Degradation caused by frequency offset can be state as follows [8]:

$$D_{freq} \cong \frac{10}{3ln10}(\pi \Delta f T)^2 \frac{E_b}{N_0} \quad (15)$$

D_{freq} , T, E_b and N_0 are: frequency offset, symbol duration, energy per bit (for OFDM signal) and one sided noise power spectrum density (PSD).

The effect of frequency offset is similar to the effect of noise and it causes degradation the Signal-to-Noise Ratio (SNR) where SNR is:

$$SNR = \frac{E_b}{N_0} \quad (16)$$

6 Investigating of the estimation Technique using Cyclic Prefix (CP)

This technique uses the length of Cyclic Prefix (CP) to compensate the effect CFO [9]. Let's consider the signal of the transmitter as follows:

$$S_i(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} b_i(k) e^{j2\pi k/N} \quad (17)$$

Where $-N_g \le n \le N-1$

N is the Inverse Fast Fourier Transform (IFFT) block length and N_g is length of Cyclic Prefix (CP).

The signal received by receiver can be stated as:

$$r(n) = \sum_{i=0}^{I-1} x_i(n - d_i) e^{j(\epsilon_i n + \varphi_i)} + v(n) \quad (18)$$

Where d_i is propagation delay and φ_i is initial phase, ϵ_i is CFO and $v(n)$ is Additive White Gaussian Noise (AWGN)

In receiver side, the OFDM demodulator removes the CP and considers the sample vector as:

$$r(0) = \sum_{i=0}^{I-1} e^{j\varphi_i} E_i(0) H_i(0) S_i(d_i) + V(0) \quad (19)$$

Where $S_i(d_i)$ is a symbol vector, $E_i(0)$ is CFO matrix, and $H_i(0)$ is the channel matrix, by switching the column of $H_i(0)$ we can state the Eq. 19 as:

$$r(0) = \sum_{i=0}^{I-1} e^{j\varphi_i} E_i(0) H_i S_i + V(0) \quad (20)$$

Here the goal is to exploit the redundancy of Cyclic Prefix (CP) on Eq. 20. An equation similar to Eq. 20 can be stated as follows:

$$r(m) = \sum_{i=0}^{I-1} e^{j\varphi_i} E_i(m) H_i S_i + V(m) \quad (21)$$

$E_i(m)$ is:

$$E_i(m) = \begin{bmatrix} 0_{(m|N)\times(N-m|N)} & diag\{e^{j\epsilon_i(-m)}, \dots, e^{j\epsilon_i(-m-1+m|N)}\} \\ diag\{e^{j\epsilon_i(-m+m|N)}, \dots, e^{j\epsilon_i(N-m-1)}\} & 0_{(N-m|N)\times(m|N)} \end{bmatrix} \quad (22)$$

Considering the equations 20 and 21 we can construct the following equation:

$$\begin{bmatrix} r(0) \\ \vdots \\ r(M-1) \end{bmatrix} = \sum_{i=0}^{I-1} e^{j\varphi_i} \begin{bmatrix} E_i(0) \\ \vdots \\ E_i(M-1) \end{bmatrix} H_i S_i + \begin{bmatrix} V(0) \\ \vdots \\ V(M-1) \end{bmatrix} \quad (23)$$

Let's for the sake of simplicity states the above equation as follows:

$$y = \sum_{i=0}^{I-1} e^{j\varphi_i} A_i H_i S_i + u \quad (24)$$

The goal is to design a CFO mitigation matrix X as follows:

$$X A_i = I_N \quad \rightarrow \quad X = I_N E_i(m) \quad (25)$$

In Eq. 22, let's consider $0 \le p \le N-1$ and $0 \le l \le N-1$, therefore this equation can be defined as follows:

$$[E_i(m)]_{p,l} = \begin{cases} e^{j\epsilon_i(p-m)} & if \quad p = (l+m)|N \\ 0 & Otherwise \end{cases} \quad (26)$$

Since all $r(m)$ does not have to be used, we select Q vector among them, where:

$$0 \le m_0 \le m_1 \le \cdots \le m_{Q-1} \le M-1 \quad (27)$$

For $0 \le i \le I-1$, the CFO matrices are $E_i(m_0), \dots, E_i(m_Q - 1))$, in this case the Eq. 25 will be altered to search for a $N \times NQ$ CFO mitigation matrix X as follows:

$$X \begin{bmatrix} E_i(m_0) \\ \vdots \\ E_i(m_Q - 1) \end{bmatrix} = I_N \quad (28)$$

Where $0 \le i \le I-1$

In this technique firstly by using Eq. 27 you choose appropriate parameters, and by using of ϵ_i and Eq. 25 you calculate X and then by using Eq. 23 you can get a sample vector for the OFDM block. Finally by using these results you can compensate the effect of the CFO.

This method has a good efficiency and a linear complexity, since $I \ll N$ and for the whole OFDM blocks we only need to determine X one time. Therefore the computational complexity of this method is linear which makes it an effective technique but the CFO estimation technique using CP, is only and only good when $-0.5 \le \varepsilon \le 0.5$.

6 Proposed Algorithm

Due to the weakness of the CFO estimation using CP, and improving it for the amounts of ε which are greater than the mentioned values, we offer using training symbols. Before we start, firstly we need to select a signal model therefore the first step in our algorithm is: selecting signal model. The general models that have been used for the received signal can be considered as follows:

$$x(n) = e^{j2\pi n\delta} s(n) + \omega(n) \quad (29)$$

In where $\omega(n)$ is Gaussian noise and δ is Carrier Frequency Offset (CFO).

$s(n)$ can be defined as follows:

$$S(n) = \sum_{k=0}^{L-1} h(K) a_{n-k} \qquad n = 0, 1, 2, \dots, N-1 \quad (30)$$

Let's consider an OFDM-MIMO system, like figure 6 with n_t transmitter and n_r receiver antennas as follows:

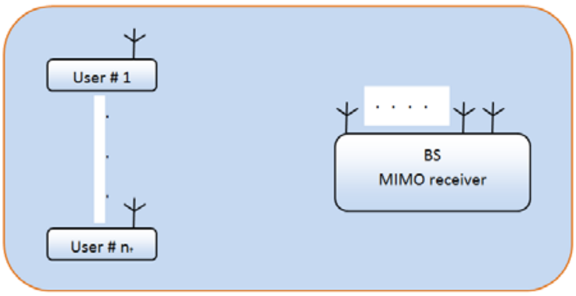

Fig. 6 Multi-user MIMO-OFDM system

Here the channel is $n_r \times n_t$ matrix.

$$\begin{bmatrix} r_1 \\ \vdots \\ r_{n_r} \end{bmatrix} = \begin{bmatrix} H_{1,1} & \cdots & H_{1,n_t} \\ \vdots & \ddots & \vdots \\ H_{n_r,1} & \cdots & H_{n_r,n_t} \end{bmatrix} \begin{bmatrix} s_1 \\ \vdots \\ s_{n_t} \end{bmatrix} + \begin{bmatrix} n_1 \\ \vdots \\ n_{n_r} \end{bmatrix} \quad (31)$$

The received signal vector form can be written as follows:

$$h = [h(0), h(1), h(2), \dots, h(L-1)]^T \quad (32)$$

This is a vector containing $T_s - spaced$ samples; the symbol $[\,]^T$ means vector transpose and the training symbols are a_n with the condition:

$$-L+1 \le n \le -1 \quad (33)$$

The Eq. 29 can be written as:

$$x = \Gamma(\delta)Ah + \omega \quad (34)$$

However this is the general model that they usually consider for investigating. But in our algorithm for simplicity let's consider one transmit antenna for each user, which in general denotes it by n_t , and let's consider the number of antenna for the Base Station by n_r with this condition that n_r always is bigger or equal to n_t ($n_r \ge n_t$). With this assumption, the received signal at any receiver antenna can be stated as:

$$r_i(k) = \sum_{m=1}^{n_t}(e^{j\delta_m k} \sum_{d=0}^{L-1} h_{i,m}(d)\, S_m(k-d)) + n_i(k) \quad (35)$$

Where:

$h_{i,m}(d)$ Channel impulse response
S_m Transmitted signal from m^th user
n_i Additive white Gaussian noise (AWGN)

As the Eq. 35 states, the CFO should be calculated for each of the n_t. After removing the CP in receiver the Eq. 35 can be presented in matrix form as follows:

$$r_i = \sum_{m=1}^{n_t} E(\delta_m) S_m h_{i,m} + n_i \quad (36)$$

So we consider our signal model as Eq. 36 Now the second step in algorithm is using the training symbols. Let's consider the transmitter is sending the training symbols with a repetitive pattern that we can call it D, it is worth to mention that these training symbols can be generated by taking the IFFT of the signal in frequency domain; therefore:

$$X_l[D] = \begin{cases} A_m & if \quad K = D.i \\ 0 & otherwise \end{cases} \quad (37)$$

Here A_m is an M-array symbol. The receiver by using the following equation can estimate the CFO:

$$\hat{\varepsilon} = \frac{D}{2\pi} arg\{\sum_{n=0}^{N/D-1} y_l^*[n] y_l[n+N/D]\} \quad (38)$$

Where N/D is an integer; Here the range is:

$$-\frac{D}{2} \le \varepsilon \le \frac{D}{2} \quad (39)$$

By considering the average; Eq. 39 can be stated as:

$$\hat{\varepsilon} = \frac{D}{2\pi} arg\{\sum_{m=0}^{D-2} \sum_{n=0}^{N/D-1} y_l^*[n+mN/D] y_l[n+(m+1)ND]$$
(40)

As it is obvious from the Eq. 40 by increasing the repetitive pattern (D) the range for (ε) increases; the figure 7 illustrates the result of the simulation of the suggested algorithm.

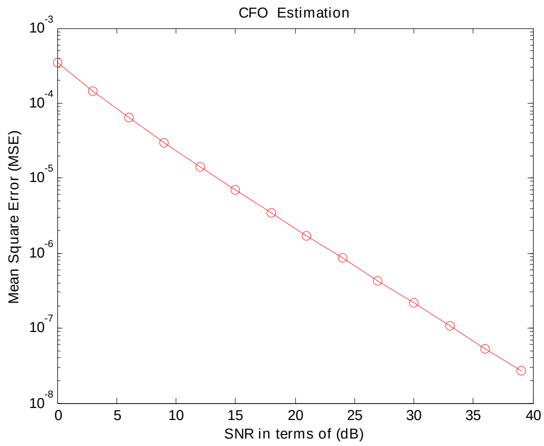

Fig. 7 CFO Estimation

Conclusion

In this paper an information framework for carrier frequency offset (CFO) is provided. The importance of the study of carrier frequency offset estimation in OFDM systems has been covered and then the common algorithms for estimating CFO have been discussed. At the end a CFO technique using training symbols offered and simulated. The result of the simulation confirms the gain in performance. But as it can be seen from the figure 7, although the range of ε increases but it costs for the price of decreasing mean square error (MSE) performance.

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