Application of Wang's Recurrent Neural Network to solve the Transportation problem

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Summary
This paper presents a procedure that uses an extension of Wang's Recurrent Neural Network to solve the Transportation problem. The choice of Neural Network parameters is similar to Traveling Salesman and Assignment problems, and shown optimal or near-optimal solutions in almost all cases randomly generated. An algorithm similar to Bisection method and Binary Search is presented in this paper to set the value of a parameter of Neural Network. The advantages of this technique are the easy computational implementation, the low computational complexity and good solutions obtained.
Keywords:
Recurrent neural network, Transportation problem, Assignment problem, Operations Research.

1. Introduction

The Transportation problem is a classical problem of combinatorial optimization of Operations Research's area. Its applications are found in problems of industrial supplies distribution, and a particular case is the Assignment problem. For example, a company produces at the units situated at various places, called origins, and supplies them to depots, called destinations [17].
The delivery of vehicles to dealers' problem is showed in [16] with an integer programming formulation of the Auto-Carrier Transportation problem trough of a three-step heuristic procedure based on the formulation. In [18] an alternative genetic algorithm is used to solve the linear transportation problem through of relationship between representation structures and genetic operators for constrained problems. A hybrid technique is used in [10] to solve linear transportation problem: a genetic algorithm find a feasible solution and this solution is used as a starting point in the Revised Simplex Method to find an improved solution.
The extension of linear transportation problem called fixed charge transportation problem is solved by two genetic algorithms in [3], a tabu search approach in [15], and a branch and bound procedure with two subproblems in [11]. In [1] the special condition of transportation problem is showed when the transportation capacity is often poor and the mileages from some sources to some destinations are no definite. An alternative formulation to the assignment problem is showed in [5] as a special case of the fixed charge transportation problem.
Transportation problems whose cost functions satisfy Monge properties appear in [2]. The transportation problem was solved by a permutation of variables with a greedy algorithm in [12]. A neural network approach for multicriteria solid transportation problems appear in [8], with an adaptation of the original multicriteria problem. A variant of Vogel's Approximation Method with total opportunity cost and an alternative allocation costs is used in [7] to solve linear Transportation problem.
The Wang's Recurrent Neural Network can be applied to solve Assignment problem [6] and Traveling Salesman problem [13]. Some changes in matrix formulation and in choice of parameters allow the use of the same Recurrent Neural Network to solve the Transportation problem. The choice of parameters is similar to Traveling Salesman and Assignment problems, and shown optimal or near-optimal solutions in almost all cases randomly generated.
An algorithm similar to the Bisection method and Binary Search is presented in this paper to find a parameter of Neural Network. The architecture of this neural network had n^2 neurons and the advantages of this technique are: easy computational implementation; low computational complexity of $O(n^2)$ [20]; the problem of tuning of parameters is corrected by Bisection method; and good solutions obtained always near of optimal solution.
This paper is divided into four sections, including this introduction. In Section 2, the Recurrent Neural Network to solve Transportation problem is presented. In Section 3 some results of proposed technique and a methodology to define parameters of neural network are showed. Section 4 presents the paper's conclusions.

2. Formulation of the problem

The transportation problem is one of the simplest combinatorial problems of Operations Research. In this problem a shipper with supplies at various places must ship to the depots, each with a given demand with minimum cost of transportation [9].
The transportation problem can be mathematically formulated as follow [17]:

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$$\text{Minimize } z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \quad (1)$$

$$\text{Subject to } \sum_{j=1}^{n} x_{ij} = a_i \text{ , } i = 1, ..., m \quad (2)$$

$$\sum_{i=1}^{m} x_{ij} = b_j \text{ , } j = 1, ..., n \quad (3)$$

$$x_{ij} \geq 0 \text{ , } i = 1, ..., m; \ j = 1, ..., n \quad (4)$$

where c_{ij} and x_{ij} are, respectively, the costs and the decision variables associated to the amount of supplies to be shipped from source i to destination j.

The objective function (1) represents the total cost to be minimized. Constraint set (2) ensures that the shipments from a source cannot exceed its supply, while constraint set (3) guarantees that each destination must satisfy its demand with the sum of shipments.

When $m < n$ the slack variables x_{kl} are added, with sources $a_k = 0$ to $k = m + 1, ..., n$, and $l = 1, ..., n$. In cases where $m > n$ the slack variables x_{kl} are added, with destinations $b_l = 0$ to $k = 1, ..., m$, and $l = n + 1, ..., m$.

Let $p = \max\{m, n\}$. How we use the sigmoid function, the solution belongs to interval [0, 1]. Thus, the vectors a and b should have values in same interval of x, and the formulation of the Transportation problem becomes:

$$\text{Minimize } \bar{z} = \sum_{i=1}^{p}\sum_{j=1}^{p} c_{ij}\bar{x}_{ij} \quad (5)$$

$$\text{Subject to } \sum_{j=1}^{p} \bar{x}_{ij} = \bar{a}_i \text{ , } i = 1, ..., p \quad (6)$$

$$\sum_{i=1}^{p} \bar{x}_{ij} = \bar{b}_j \text{ , } j = 1, ..., p \quad (7)$$

$$\bar{x}_{ij} \in [0, 1], \ i, j = 1, ..., p; \quad (8)$$

where $q = \max\{a_i, b_i : i = 1, 2, ..., p\}$, $x = q\bar{x}$, $a = q\bar{a}$ and $b = q\bar{b}$.

The matrix form of the problem described in (5)–(8) is the following [6], [13]:

$$\text{Minimize } \bar{z} = c^T\bar{x}$$

$$\text{Subject to } A\bar{x} = d$$

$$\sum_{i=1}^{p} \bar{x}_{ij} = \bar{b}_j \text{ , } j = 1, ..., p$$

$$\bar{x} \in [0, 1]$$

where vectors c^{T} and $\bar{x}$ contains, respectively, all the rows from the cost matrix c and from the matrix with decision elements x_{ij}.

Matrix A has dimension $3p \times p^2$ and the following form:

$$A = \begin{pmatrix} B_1 & B_2 & \cdots & B_p \\ I & I & \cdots & I \\ 0 & 0 & \cdots & 0 \end{pmatrix}$$

where 0 is null matrix, I is identity matrix and B_i has 0 in all lines with exception of line i that contain 1 in all positions.

Vector d has dimension $3p$ and the following form:

$$d = \begin{pmatrix} \bar{a} \\ \bar{b} \\ 0 \end{pmatrix}$$

where the vector $\bar{a}$ had the p amounts of supplies of p origins and vector $\bar{b}$ has the amount of supplies of demand to p destinations.

Multiplying the set of constraints by A^{T} we have:

$$A^{\mathrm{T}}Ax = A^{\mathrm{T}}d$$
$$\therefore A^{\mathrm{T}}Ax - A^{\mathrm{T}}d = 0.$$

The matrix form of $A^{\mathrm{T}}A$ is:

$$A^{\mathrm{T}}A = W = \begin{pmatrix} P & I & \cdots & I \\ I & P & \cdots & I \\ \vdots & \vdots & \ddots & \vdots \\ I & I & \cdots & P \end{pmatrix},$$

where $P = I + 1$ and vector $A^{\mathrm{T}}d$ is:

$$A^{\mathrm{T}}d = \theta = \begin{pmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_{p^2} \end{pmatrix} \Leftrightarrow \begin{pmatrix} \theta_1 & \theta_2 & \cdots & \theta_p \\ \theta_{p+1} & \theta_{p+2} & \cdots & \theta_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ \theta_{p(p-1)+1} & \theta_{p(p-1)+2} & \cdots & \theta_{p^2} \end{pmatrix}$$

$$\begin{pmatrix} \theta_1 & \theta_2 & \cdots & \theta_p \\ \theta_{p+1} & \theta_{p+2} & \cdots & \theta_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ \theta_{p(p-1)+1} & \theta_{p(p-1)+2} & \cdots & \theta_{p^2} \end{pmatrix} \Leftrightarrow \begin{pmatrix} (B_1 \ \ I \ \ 0)d \\ (B_2 \ \ I \ \ 0)d \\ \vdots \\ (B_p \ \ I \ \ 0)d \end{pmatrix}$$

and $\theta_{kl} = \bar{a}_k + \bar{b}_l$.

The Recurrent Neural Network of Wang is defined by the following differential equation [6]:

$$\frac{du_{ij}(t)}{dt} = -\eta\sum_{k=1}^{p}\bar{x}_{ik}(t) - \eta\sum_{l=1}^{p}\bar{x}_{lj}(t) + \eta\theta_{ij} - \lambda c_{ij}e^{-\frac{t}{\tau}}$$

where $\bar{x}_{ij} = g(u_{ij}(t))$, this neural network's balance state is a feasible solution for the Transportation problem and g is the sigmoid function with parameter β:

$$\bar{x}_{ij}(t) = \frac{1}{1+e^{-\beta u_{ij}(t)}}$$

The matrix form of the Wang's Neural Network is:

$$\frac{du(t)}{dt} = -\eta\left(W\bar{x}(t)-\theta\right) - \lambda c e^{-\frac{t}{\tau}}$$

3. Results

When $W\bar{x} - \theta \cong 0$ the constraints of supply and demand of the Transportation problem have already been satisfied, and the solution $\bar{x}$ is feasible.
Figures 1, 2 and 3 shown sequences of solutions of Transportation problem randomly generated with dimensions 207 × 223, 177 × 165 and 122 × 98 respectively. After a certain number of iterations the neural network is stabilized, and degree of infeasibility converge to zero as the mean errors. To find the solution x just do $x = q\bar{x}$.

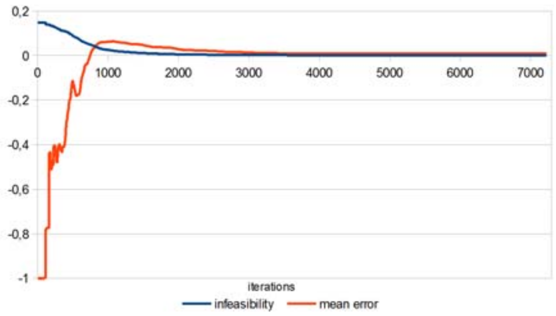

Fig. 1 Infeasibility and mean error of a 207 × 223 transportation problem.

3.1 Parameters of neural network

Parameters β, η and λ are fixed [14]: $\beta = 2.5$, $\eta = 1$ and $\lambda = c_{max}$, where $c_{max} = \max\{c_{ij}: i, j = 1, 2, ..., p\}$.
Parameter τ produces different solutions to Transportation problem, acoording shown Figure 4 with five 9 × 16 randomly generated problems with $\tau \in [100, 1500]$.

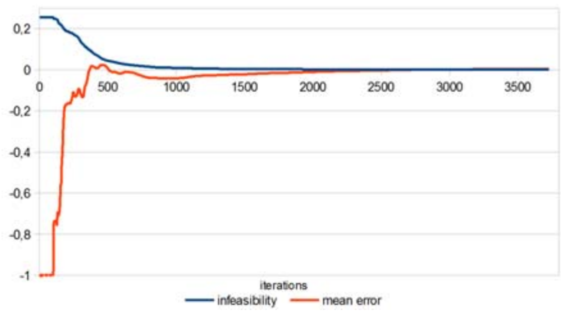

Fig. 2 Infeasibility and mean error of a 177 × 165 transportation problem.

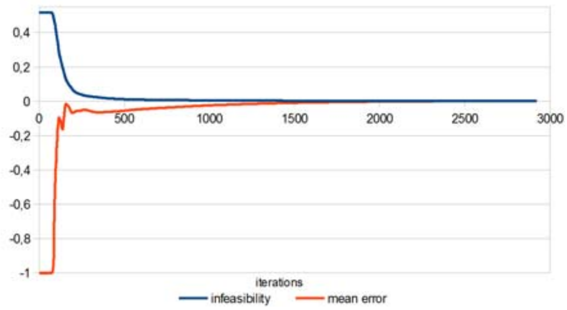

Fig. 3 Infeasibility and mean error of a 122 × 98 transportation problem.

From a certain τ value, each problem stabilizes at a near optimal solution. To determine best value of τ we can use an algorithm similar to Bisection method [19], used to find the root of a function, and Binary Search [4], used to find a value in an ordered vector.

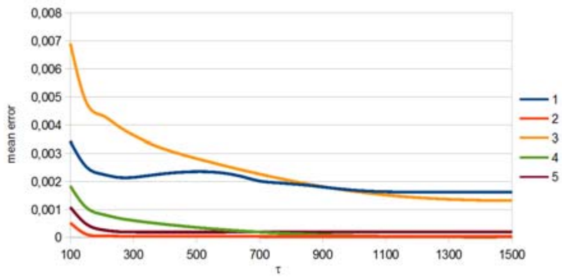

Fig. 4 Mean errors with 5 transportation problems with dimension 9 × 16.

3.2 Algorithm

The algorithm used to determine the parameter is defined by the following steps:

1. Define the interval of parameter τ:
   $\tau \in [\tau_{min}, \tau_{max}]$.
   Define the initial values of:
   $\tau_1 = \tau_{min}$, $\tau_2 = \tau_{max}$, $\delta \cong 0$ and $\varepsilon \cong 0$.
2. Do while $W\bar{x} - \theta > \delta$

$$\frac{du(t)}{dt} = -\eta\left(W\bar{x}(t) - \theta\right) - \lambda c e^{-\frac{t}{\tau_1}}$$

$$\bar{x}(t) = \frac{1}{1 + e^{-\beta u(t)}}$$

Loop
Find $z_1 = c\bar{x}(t)$.
Do while $W\bar{x} - \theta > \delta$

$$\frac{du(t)}{dt} = -\eta\left(W\bar{x}(t) - \theta\right) - \lambda c e^{-\frac{t}{\tau_2}}$$

$$\bar{x}(t) = \frac{1}{1 + e^{-\beta u(t)}}$$

Loop
Find $z_2 = c\bar{x}(t)$.

3. If $z_1 > z_2$ then
   $\tau_1 = (\tau_1 + \tau_2)/2$.
   If $z_1 < z_2$ then
   $\tau_2 = (\tau_1 + \tau_2)/2$.
4. If $z_1 - z_2 \le \varepsilon$ or $\tau_1 - \tau_2 \le \varepsilon$ then
   Stop: the best value of τ was founded.
   Else
   Go to Step 2.
   End If

3.3 Numerical example

Consider the 9 × 6 transportation problem given below, with costs matrix *c*, demand vector *a* and destination vector *b*:

c_{ij}	1	2	3	4	5	6	*a*
1	181	4	263	366	111	93	131
2	256	259	147	16	206	369	296
3	217	266	81	130	81	39	538
4	412	149	428	11	363	91	252
5	311	194	418	243	341	342	41
6	199	67	15	96	160	4	482
7	298	399	40	10	271	355	1017
8	63	106	188	422	341	327	963
9	350	317	165	133	3	231	39
b	688	719	631	264	591	866	3759

Matrix θ verifies if the constraints are satisfied.

$$\theta = \begin{pmatrix} a_1 + b_1 & a_1 + b_2 & \cdots & a_1 + b_6 & 0 & 0 & 0 \\ a_2 + b_1 & a_2 + b_2 & \cdots & a_2 + b_6 & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ a_9 + b_1 & a_9 + b_2 & \cdots & a_9 + b_6 & 0 & 0 & 0 \end{pmatrix}$$

When parameter $\tau = 100$, the solution founded have cost $z = 270543.5665$, and mean error 0.07123%. The matrix of values *x* is:

x_{ij}	1	2	3	4	5	6	*a*
1	0	131	0	0	0	0	131
2	0	0.24	0	0	295.8	0	295.9
3	0	0	0	0	134.4	403.6	538
4	0	216.3	0	0	0	35.7	252
5	0	41	0	0	0	0	41
6	0	55.4	0	0	0	426.6	482
7	0.13	0	631	264	121.9	0	1017
8	687.9	275.1	0	0	0	0	963
9	0	0	0	0	38.9	0	38.9
b	687.9	718.9	631	264	591	865.9	3759

When parameter $\tau = 150$, the solution has cost $z = 270398.1887$, and mean error 0.01745%:

x_{ij}	1	2	3	4	5	6	*a*
1	0	131	0	0	0	0	131
2	0	0	0	0	295.9	0	295.9
3	0	0	0	0	134	403.9	537.9
4	0	242.6	0	0	0	9.43	252
5	0	41	0	0	0	0	41
6	0	29.4	0	0	0	452.6	482
7	0	0	631	264	121.9	0	1017
8	687.9	275	0	0	0	0	963
9	0	0	0	0	38.9	0	38.9
b	687.9	718.9	631	264	591	865.9	3758.9

Parameter $\tau = 300$ define a solution with cost $z = 270351.4897$, and mean error 0.00018%:

x_{ij}	1	2	3	4	5	6	*a*
1	0	131	0	0	0	0	131
2	0	0	0	0	295.9	0	295.9
3	0	0	0	0	134	403.9	537.9
4	0	251.9	0	0	0	0.017	252
5	0	41	0	0	0	0	41
6	0	20	0	0	0	461.9	481.9
7	0	0	631	264	121.9	0	1016.9
8	687.9	275	0	0	0	0	963
9	0	0	0	0	38.9	0	38.
b	687.9	718.9	631	264	591	866	3759

Finally, the solution obtained with parameter $\tau = 450$ have cost $z = 270351.1871$, and mean error 0.00007%:

x_{ij}	1	2	3	4	5	6	*a*
1	0	131	0	0	0	0	131
2	0	0	0	0	295.9	0	295.9
3	0	0	0	0	134	403.9	537.9
4	0	252	0	0	0	0	252
5	0	41	0	0	0	0	41
6	0	19.9	0	0	0	462	481.9
7	0	0	631	264	121.9	0	1016.9
8	688	275	0	0	0	0	963
9	0	0	0	0	38.9	0	38.9
b	688	718.9	631	264	591	866	3759

Ten problems were randomly generated with the following types: 30 × 30, 30 × 25, 30 × 20, 30 × 15, 30 × 10 and 30 × 5. Tables 1 and 2 shows the results of these problems, including the average number of iterations for each type of problem. Most of these problems generated have optimal or near-optimal solutions.

Table 1: Results of 10 problems randomly generated of 3 types

Problem	***30 × 5***	***30 × 10***	***30 × 15***
Mean error	0.0108%	0.0105%	0.0359%
Iterations	6871.1	7460.9	7622.7

Table 2: Results of 10 problems randomly generated of 3 types

Problem	***30 × 20***	***30 × 25***	***30 × 30***
Mean error	0.0143%	0.0000%	0.0254%
Iterations	8762.6	9908.6	11205.4

4. Conclusions

The results shown in demonstrate the efficiency of Wang's Recurrent Neural Network to solve the Transportation problem. The choice of parameter τ was made with the proposed algorithm, and show optimal or near-optimal solutions to all types of problems randomly generated.

The advantages of presented technique in this paper are the low computational complexity of $O(n^2)$, easy implementation and good results to randomly generated problems. This technique can be adapted to solve other variations of Transportation problems.

References

[1] Bai, G., Mao, J., Lu, G., Grey transportation problem, *Kybernetes*, Vol. 33, No. 2, pp. 219-224, 2004.

[2] Burkard, R. E., Monge properties, discrete convexity and applications, *European Journal of Operational Research*, Vol. 176, No. 1, pp. 1-14, 2007.

[3] Gottlieb, J., Paulmann, L., Genetic Algorithms for the Fixed Charge Transportation Problem, *IEEE World Congress on Computational Intelligence*, *IEEE Press*, pp. 330-335, 1998.

[4] Hamerly, G., Perelman, E., Lau, J., Calder, B., SimPoint 3.0: Faster and More Flexible Program Phase Analysis, *Journal of Instruction-Level Parallelism*, Vol. 7, pp. 1-28, 2005.

[5] Hultberg, T. H., Cardoso, D. M., The teacher assignment problem: A special case of the fixed charge transportation problem, *European Journal of Operational Research*, Vol. 101, No. 3, pp. 463-473, 1997.

[6] Hung, D. L., Wang, J., Digital hardware realization of a recurrent neural network for solving the assignment problem, *Neurocomputing*, Vol. 51, pp. 447-461, 2003.

[7] Korukoğlu, S., Ballı, S., An improved Vogel's approximation method for the transportation problem, *Mathematical and Computational Applications*, Vol. 16 No. 2, pp. 370-381, 2011.

[8] Li, Y., Ida, K., Gen, M., Kobuchi, R., Neural Network Approach for Multicriteria Solid Transportation Problem, *Computers & Industrial Engineering*, Vol. 33, No. 3-4, pp. 465-468, 1997.

[9] Murty, K. G., *Linear Programming*, John Wiley & Sons, New York, 1983.

[10] Ramadan, S. Z., Ramadan, I. Z, Hybrid Two-Stage Algorithm for Solving Transportation Problem, Modern Applied Science, Vol. 6, No. 4, pp. 12-22, 2012.

[11] Schaffer, J. R., O'Leary, D. E., Use of penalties in a branch and bound procedure for the fixed charge transportation problem, *European Journal of Operational Research*, Vol. 43, No. 3, pp. 305-312, 1989.

[12] Shamir, R., A fast algorithm for constructing Monge sequences in transportation problems with forbidden arcs, *Discrete Mathematics*, Vol. 114, No. 1-3, pp. 435-444, 1993.

[13] Siqueira, P. H., Scheer, S., Steiner, M. T. A., A new approach to solve the Traveling Salesman Problem, *Neurocomputing*, Vol. 70, No. 4-6, pp. 1013-1021, 2007.

[14] Siqueira, P. H., Steiner, M. T. A., Scheer, S., Recurrent Neural Network with Soft 'Winner Takes All' principle for the TSP. In *ICNC 2010, International Conference on Neural Computation*, SciTePress, pp. 265-270, 2010.

[15] Sun, M., Aronson, J. E., McKeown, P. G., Drinka, D., A tabu search heuristic procedure for the fixed charge transportation problem, *European Journal of Operational Research*, Vol. 106, No. 2-3, pp. 441-456, 1998.

[16] Tadei, R., Perboli, G., Della Croce, F., A Heuristic Algorithm for the Auto-Carrier Transportation Problem, *Transportation Science*, Vol. 36, No. 1, pp. 55–62, 2002.

[17] Taha, H. A., *Operations Research: an introduction*, Pearson Education Inc., New Jersey, 2007.

[18] Vignaux, G. A., Michalewicz, Z. A., Genetic Algorithm for the Linear Transportation Problem. *IEEE Transactions on Systems, Man and Cybernetics*, Vol. 21, No. 2, pp. 445-452, 1991.

[19] Vrahatis, M. N., Perdiou, A. E., Kalantonis, V. S., Perdios, E. A., Papadakis, K., Prosmiti, R., Farantos, S. C., Application of the Characteristic Bisection Method for locating and computing periodic orbits in molecular systems, *Computer Physics Communications*, Vol. 138, pp. 53–68, 2001.

[20] Wang, J., Primal and dual assignment networks, *IEEE Transactions on Neural Networks*, Vol. 8, No. 3, pp. 784-790, 1997.

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