Throughput Gain using Threshold-based Multiuser Scheduling in WiMAX OFDMA

Ahmed Iyanda Sulyman

King Saud University, Riyadh, Saudi Arabia

Summary
This paper presents the analysis of the throughput enhancements using threshold-based PUSC in WiMAX OFDMA system. We consider a point-to-multipoint (PMP) WiMAX network where base station (BS) schedules downlink packets for simultaneous transmissions to multiple users. Using the OFDMA option in the WiMAX standard, users are assigned different subchannels for reception of their data using the same OFDM symbol. In the proposed threshold-based scheduling scheme, the BS scheduler selects at any time instant users whose channel gains in the available subchannels equal or exceed a pre-determined energy threshold. Thus, only users who can maximize BS throughput on the available subchannels are assigned data transmission opportunities. We quantify the throughput enhancement of the system, and illustrate its benefits by numerical simulations.

Key words:
WiMAX OFDMA, Threshold-based selection, PUSC, scheduling.

1. Introduction

The IEEE 802.16 standard-based WiMAX network specifies OFDMA (orthogonal frequency division multiplexing access) as multiuser access method, where a base station (BS), in a point-to-multipoint (PMP) mode, communicates with multiple users simultaneously on different time-frequency resources [5, 6, 8, 10, 11]. Each subchannel in the OFDMA option of the WiMAX system comprises a set of OFDM subcarriers which may be mapped on to the frequency spectrum either sequentially or in a pseudo-random manner. In the randomly mapped system known as full usage of subcarriers (FUSC), the subcarriers in a subchannel are taken from different portions of the spectrum in a pseudo-random manner, while in the sequentially mapped system known as partial usage of subcarriers (PUSC), only subcarriers adjacent in the frequency spectrum are included in a subchannel.

In this paper, we consider the use of threshold-based multiuser scheduling in WiMAX OFDMA. Motivation for this consideration is the fact that in OFDMA systems, user channels experience deep fades frequently, therefore the regular WiMAX OFDMA scheduler based on PUSC system would be forced to assign subchannels to users when their channels experience deep fades, degrading the BS throughput significantly. To optimize BS throughput in such case, we examine the use of threshold-based PUSC system, where users first undergo threshold test before the regular PUSC scheduling policy of the BS is applied.

The threshold-based selection method was proposed by Sulyman and Kousa in [1] for the diversity combining problem in a single-user transmission system, and has been widely studied in the literature [2-4]. In the context of multiuser scheduling in WiMAX network, we consider here the use of a threshold-based PUSC system, where a BS scheduler uses the energy threshold criterion to select the users to be scheduled for downlink transmission at any time instant. The advantage of this scheduling strategy is that at any time instant, only users whose channels are strong enough to sustain the network operator's target data rate are scheduled. This allows operators to maximize system throughput and is more useful for non-real-time traffic, which are delay tolerant. Scheduling of data transmissions to users with temporarily weak channels can wait until their channel conditions improve. Efficient utilization of the resources for non-real-time traffic as proposed in this work frees up bandwidth resources for real-time traffic and optimizes overall network resource utilizations. In this paper we define a performance metric called the throughput gain, and analyze this metric. We show that the throughput gain in the threshold-based PUSC scheme is enhanced as the threshold level is increased.

2. System Model and Analysis

2.1 System model for threshold-based PUSC

Consider a threshold-based downlink scheduling scheme in OFDMA system where A BS scheduler schedules n users for downlink transmission, out of total of L users, whose SNR meet or exceed a predetermined energy threshold, γ_{th}. The available subchannels in the OFDMA system, Nc, are distributed among the n users (tagged here active users) whose SNRs passed the threshold test, using the regular BS scheduling policy. The number of users, n, satisfying the threshold requirement at any time instant is

Manuscript received July 5, 2012
Manuscript revised July 20, 2012

not fixed, but variable in correspondence to user channel statistics. The specific realization of n could take any value from the set $\{1,2,\cdots,L\}$, at each scheduling period. Let $\{\gamma_1,\gamma_2,\cdots,\gamma_L\}$, denote the instantaneous SNRs of the L users feed back to the BS. At any time instant, the BS scheduler schedules the users whose SNR γ_j satisfy $\gamma_j \geq \gamma_{th}$, for downlink transmission, as illustrated in Fig 1.

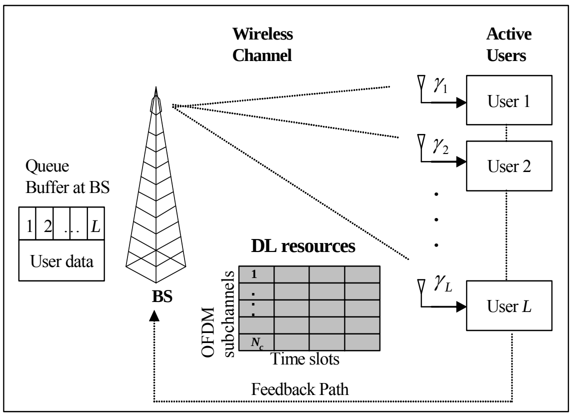

Figure 1: Down link scheduling in WiMAX OFDMA networks

As proposed in [1], we define the threshold as

$$\gamma_{th} = \mu \cdot \max\{\gamma_1,\gamma_2,\cdots,\gamma_L\} \quad (1)$$

where $0 \leq \mu \leq 1$. This threshold definition is tagged the normalized threshold [3], and it insures that in the worst case scenario at least one user will be scheduled for service while in cases when the fading are not severe such that all users meet or exceed the threshold, they are all scheduled for service. Thus, only users with good SNRs, γ_j, to sustain a desired data rate on the subchannels are scheduled at any time instant [2]. Network operators can therefore use the threshold definition to guarantee a desired data rate on the overall network, optimizing the system throughput. Threshold-based multiuser scheduling for $\mu = 1$ reduces to opportunistic scheduling, and as μ is reduced, in the range $0 < \mu < 1$, more users are scheduled per channel use, introducing some fairness. The case $\mu = 0$ corresponds to the regular scheduling policy of the BS.

2.2 Throughput Gain Analysis

In this analysis, we focus on the WiMAX PUSC system, and examine the throughput gain achieved using threshold-based selection in the PUSC system. Assuming a burst of length n OFDM symbols. At any time instant, the users feedback to the BS their SNR in each subchannel, obtained using the assigned pilots in the subchannels. The threshold-based PUSC scheduler then conducts threshold test to select the n_i users whose SNRs, $\gamma_1, \gamma_2, \cdots, \gamma_{n_i}$, are above threshold in the i^{th} subchannel and schedules them inturn for service on that subchannel for n successive OFDM symbols transmitted in a burst, where n = max{ $n_1, n_2, \cdots n_{N_c}$ }. For the case $n_i < n$ for a given subchannel, we assume that the base station fills the remaining time-frequency transmission slots opportunistically by allocating them to the user with the best SNR in that subchannel, as illustrated in Fig 2.

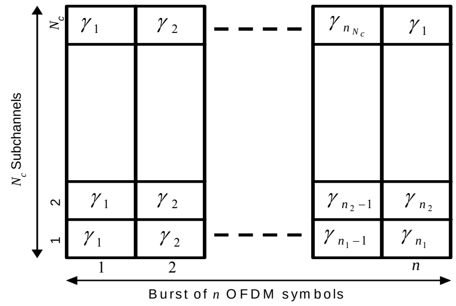

Figure 2: Threshold-based PUSC in WiMAX OFDMA

For M-QAM transmissions over the subchannels in an OFDM-based transmission, it is known that the achievable data rate (upper-bound on the throughput) is given by [7]:

$$r = \sum_{k=1}^{N_c} \log_2\left(1 + \alpha_k P_k\right), \quad (2)$$

where α_k denotes the subchannel gain-to-noise ratio at the receiver, and P_k denotes the transmitted power. The system throughput is proportional to the per subchannel SNR of service, given by $\gamma_k = \alpha_k P_k$, at any time instant. Thus, the BS can enhance its throughput by scheduling only to users with γ_k above certain threshold in the k^{th} subchannel. Define the throughput enhancement on each subchannel due to threshold-based multi-user scheduling as

$$\lambda_{Gain} = \frac{E[\text{SNR of service}]}{E[\text{SNR of one user}]} \quad (3)$$

Let $\{\gamma_{l:L}\}_{l=1}^{L}$ be the order statistics obtained by arranging the set of user SNRs $\{\gamma_j\}_{j=1}^{L}$ in decreasing order of magnitude, (i.e., $\gamma_{1:L} \geq \gamma_{2:L} \geq \cdots \geq \gamma_{n:L} \geq \gamma_{n+1:L} \geq \cdots \geq \gamma_{L:L}$). We assume that the set $\{\gamma_j\}_{j=1}^{L}$ are *iid*. Therefore, the joint probability distribution function (*pdf*), $f_{\gamma_{1:L},\cdots,\gamma_{L:L}}(\gamma_{1:L},\cdots,\gamma_{L:L})$, of $\{\gamma_{l:L}\}_{l=1}^{L}$ is given by [9]

$$f_{\gamma_{1:L},\cdots,\gamma_{L:L}}(\gamma_{1:L},\cdots,\gamma_{L:L}) = L!\prod_{i=1}^{L} f_\gamma(\gamma_{i:L})$$

$$\infty > \gamma_{1:L} \ge \gamma_{2:L} \ge \cdots \ge \gamma_{L:L} > 0 \qquad (4)$$

where $f_\gamma(\gamma)$ denotes the *pdf* of the random variables γ.

Consider the subset $\{\gamma_{l:L}\}_{l=1}^{n}$ designating the *n* largest γ_j's (corresponding to the *n* users with the best SNRs scheduled for downlink transmission per spectrum access, $n \le L$). Then the joint *pdf*, $f_{\gamma_{1:L},\cdots,\gamma_{n:L}}(\gamma_{1:L},\cdots,\gamma_{n:L})$, of $\{\gamma_{l:L}\}_{l=1}^{n}$, $n \le L$, can be obtained as [9]

$$f_{\gamma_{1:L},\gamma_{2:L},\cdots,\gamma_{n:L}}(\gamma_{1:L},\gamma_{2:L},\cdots,\gamma_{n:L})$$

$$= \int_0^{\gamma_{n:L}} \cdots \int_{\gamma_{n+2:L}}^{\gamma_{n:L}} f_{\gamma_{1:L},\gamma_{2:L},\cdots,\gamma_{L:L}}(\gamma_{1:L},\gamma_{2:L}\cdots,\gamma_{L:L}) \quad d\gamma_{n+1:L},\cdots,d\gamma_{L:L},$$

$$= n!\binom{L}{n}\left[F_\gamma(\gamma_{n:L})\right]^{L-n}\prod_{l=1}^{n} f_\gamma(\gamma_{l:L}), \quad \gamma_{1:L} \ge \gamma_{2:L} \ge \cdots \ge \gamma_{n:L}. \qquad (5)$$

where $\binom{L}{n} = (L!/n!(L-n)!)$ denotes the binomial coefficient, and $F_\gamma(\gamma) = \int_0^{\gamma} f_\gamma(\gamma)\ d\gamma$ is the cumulative distribution function (*cdf*) of the random variables γ_j's.

We consider that the underlying user channels experience Rayleigh fading, therefore the $\{\gamma_j\}_{j=1}^{L}$ are exponentially distributed, with pdf, $f_X(x)$, and cdf, $F_\gamma(\gamma)$, given by

$$f_\gamma(\gamma) = a\exp(-a\gamma), \qquad (6)$$

$$F_\gamma(\gamma) = 1 - \exp(-a\gamma), \qquad (7)$$

where $a = 1/E[\gamma_j] = 1/\bar{\gamma}$. (8)

To compute the throughput enhancement for the threshold-based scheduling, we first condition on a fixed *n* and write an expression for the average SNR of service given *n*, as

$$E\left[\text{SNR of service}\ |n\right]$$

$$= E_{\gamma_{1:L},\gamma_{2:L},\cdots,\gamma_{n:L}}\left[\frac{1}{n}\left(\sum_{l=1}^{n}\gamma_{l:L}\right)\Big| n\right] = \varphi(n). \qquad (9)$$

Thus

$$E[\text{SNR of service}] = \sum_{n=1}^{L}\varphi(n)\cdot\Pr(\mathbf{n}=n), \qquad (10)$$

where $\Pr(\mathbf{n}=n)$ denotes the probability that there are *n* users whose SNR equal or exceed $\gamma_{th} = \mu\gamma_{1:L}$.

To compute $\Pr(\mathbf{n}=n)$, we first observe that the event that the random variable $\mathbf{n}=n$ occurs when the following conditions are simultaneously satisfied [2]-[4]:

$\mu\gamma_{1:L} \le \gamma_{2:L} \le \gamma_{1:L}$, $\mu\gamma_{1:L} \le \gamma_{3:L} \le \gamma_{2:L}$, $\cdots$, $\mu\gamma_{1:L} \le \gamma_{n:L} \le \gamma_{n-1:L}$, and $0 \le \gamma_{n+1:L} \le \mu\gamma_{1:L}$, $0 \le \gamma_{n+2:L} \le \gamma_{n+1:L}, \cdots,\ 0 \le \gamma_{L:L} \le \gamma_{L-1:L}$ (11)

Using Eq. (4) and (11), we compute $\Pr(\mathbf{n}=n)$ as

$$\Pr(\mathbf{n}=n) = L!\int_0^{\infty} f_\gamma(\gamma_{1:L})\,d\gamma_{1:L}\int_{\mu\gamma_{1:L}}^{\gamma_{1:L}} f_\gamma(\gamma_{2:L})\,d\gamma_{2:L}$$

$$\times\int_{\mu\gamma_{1:L}}^{\gamma_{2:L}} f_\gamma(\gamma_{3:L})\,d\gamma_{3:L}\cdots\int_{\mu\gamma_{1:L}}^{\gamma_{n-1:L}} f_\gamma(\gamma_{n:L})\,d\gamma_{n:L}$$

$$\times\int_0^{\mu\gamma_{1:L}} f_\gamma(\gamma_{n+1:L})\,d\gamma_{n+1:L}\int_0^{\gamma_{n+1:L}} f_\gamma(\gamma_{n+2:L})\,d\gamma_{n+2:L}$$

$$\cdots\int_0^{\gamma_{L-1:L}} f_\gamma(\gamma_{L:L})\,d\gamma_{L:L}$$

$$= n\binom{L}{n}\sum_{i=0}^{L-n}(-1)^i\binom{L-n}{i}\sum_{j=0}^{n-1}(-1)^j\binom{n-1}{j}$$

$$\cdot\frac{1}{\left[1+j+\mu(n-1-j+i)\right]} \qquad (12)$$

Also using Eq. (5), we compute $\varphi(n)$ as

$$\varphi(n) = \frac{1}{n}\int_0^{\infty}\int_{\gamma_{n:L}}^{\infty}\cdots\int_{\gamma_{2:L}}^{\infty}\left(\sum_{l=1}^{n}\gamma_{l:L}\right)$$

$$\times f_{\gamma_{1:L},\gamma_{2:L},\cdots,\gamma_{n:L}}(\gamma_{1:L},\gamma_{2:L},\cdots,\gamma_{n:L})\ d\gamma_{1:L}\cdots d\gamma_{n-1:L}d\gamma_{n:L}$$

$$= \frac{1}{n}\int_0^{\infty}\int_{\gamma_{n:L}}^{\infty}\cdots\int_{\gamma_{2:L}}^{\infty}\left(\sum_{l=1}^{n}\gamma_{l:L}\right)n!\binom{L}{n}\left[1-\exp(-a\gamma_{n:L})\right]^{L-n}$$

$$\cdot\prod_{l=1}^{n}a\exp(-a\gamma_{l:L})\,d\gamma_{1:L}\cdots d\gamma_{n:L}. \qquad (13)$$

To solve Eq. (13), we consider the transformation of the random variables $\{\gamma_{l:L}\}_{l=1}^{n}$ obtained by defining the spacings [9]

$Y_1 = X_{1:L} - X_{2:L}$, $Y_2 = X_{2:L} - X_{3:L}$, $\cdots$, $Y_{n-1} = X_{n-1:L} - X_{n:L}$, $Y_n = X_{n:L}$. (14)

It can be shown that the random variables $Y_1, Y_2, \cdots Y_n$ are all statistically independent, with *pdf* given by

$$f_{Y_l}(y_l) = al \exp(-aly_l), \qquad (15)$$

where $y_l \geq 0$, $l = 1, \cdots, n$.

Using Eqs. (14) and (15), Eq. (13) can be expressed as

$$\varphi(n) = \frac{1}{n}\int_0^\infty \cdots \int_0^\infty \left(\sum_{l=1}^{n} ly_l\right) n!\binom{L}{n}\left[1-\exp(-ay_n)\right]^{L-n} \cdot \prod_{l=1}^{n}\exp(-aly_l)\,day_1\cdots day_n$$

$$= \frac{1}{n} n!(a)^n \binom{L}{n} \sum_{k=1}^{n-1}\left[\left(\int_0^\infty ky_k \exp(-aky_k)dy_k\right) \cdot \prod_{l=1,l\neq k}^{n-1}\left(\int_0^\infty \exp(-aly_l)dy_l\right) \cdot \left(\int_0^\infty \exp(-any_n)\left[1-\exp(-ay_n)\right]^{L-n}dy_n\right)\right]$$

$$+\frac{1}{n} n!(a)^n\binom{L}{n} \cdot \left(\int_0^\infty ny_n \exp(-any_n)\left[1-\exp(-ay_n)\right]^{L-n}dy_n\right) \cdot \prod_{l=1}^{n-1}\left(\int_0^\infty \exp(-aly_l)dy_l\right) \qquad (16)$$

Solving the integrals in Eq. (16), we arrive at the following final closed-form results, after some algebra

$$\varphi(n) = (n-1)!(a)^n\binom{L}{n}\sum_{k=1}^{n-1}\left[\left(\frac{1}{a}\frac{1}{ak}\right) \cdot \prod_{l=1,l\neq k}^{n-1}(1/al)\left(\frac{1}{an\binom{L}{n}}\right)\right] + (n-1)!(a)^n\binom{L}{n} \cdot \left(\frac{n}{a^2}\sum_{k=0}^{L-n}(-1)^{L-n+k}\binom{L-n}{k}\frac{1}{(L-k)^2}\right) \cdot \prod_{l=1}^{n-1}(1/al). \qquad (17)$$

Using Eqs. (1), (8), (10), (12), and (17), we obtain an expression for the throughput gain using threshold-based scheduling as

$$\lambda_{Gain} = \sum_{n=1}^{L}\left((n-1)!\left(\frac{1}{\bar{\gamma}}\right)^{n+1}\binom{L}{n}\sum_{k=1}^{n-1}\left[\left(\frac{\bar{\gamma}^2}{k}\right) \cdot \prod_{l=1,l\neq k}^{n-1}(\bar{\gamma}/l)\left(\frac{\bar{\gamma}}{n\binom{L}{n}}\right)\right] \cdot \Pr(n) + (n-1)!\left(\frac{1}{\bar{\gamma}}\right)^{n+1}\binom{L}{n} \cdot \left(\bar{\gamma}^2 n\sum_{k=0}^{L-n}(-1)^{L-n+k}\binom{L-n}{k}\frac{1}{(L-k)^2}\right) \cdot \prod_{l=1}^{n-1}(\bar{\gamma}/l)\cdot \Pr(n)\right), \qquad (18)$$

where $\Pr(n)$ is given by Eq. (12).

3. Simulation Results

In Fig 3, we plot the analytical results for the per subchannel throughput gain of threshold-based multiuser scheduling schemes in OFDMA systems, using Eqs. (12) and (18). Simulation results are also included in this figure for reference. Both the analytical and simulation results agree closely and indicate that a multiuser scheduling system where users SNRs, γ_j, undergo threshold test before scheduling, would enhance system throughput significantly as the threshold level is increased in the range $0 < \mu \leq 1$ (0% - 100% threshold). The enhancement becomes very significant when the number of users serviced per base station sector is large, taking advantage of the randomness of the user channel statistics. For example for the 16-user system in this figure, at least 2.5 dB throughput gain per OFDM subchannel can be achieved for moderate threshold level such as $\mu = 0.25$, while over 5 dB gain per subchannel can be achieved with high threshold level such as $\mu = 0.9$. These gains can be very significant in systems with large number of subchannels per OFDM symbol (e.g. IEEE 802.16e OFDMA option with 32 subchannels [11]). Notice that at low threshold level, more numbers of users are scheduled per OFDM symbol, allowing the BS to exhibit more fairness to the users in the scheduling policy, while at high threshold level less numbers of users are scheduled per OFDM symbol, allowing the BS to exhibit less fairness to the users. There is therefore an important trade-off between fairness and throughput enhancement in the proposed scheme. We conduct an optimization search for this trade-off and the results are as summarized in Fig 3. In this figure, we plot both the average number of users scheduled in the proposed scheme, $E[n]$, as well as the average throughput gain, λ_{Gain}, for threshold range $0 \leq \mu \leq 1$, in a 16-user system. It is observed that while the throughput enhancements increase with μ, the average number of users scheduled decreases with it. An optimum value of μ given by the intersection of these two curves, thus exist that allows the BS scheduler to achieve reasonable throughput enhancements while maintaining good level of fairness. The optimum value for the case depicted in this graph is $\mu = 0.25$, yielding $\lambda_{Gain} = 2.8$ (about 2.8dB throughput enhancements), with $E[n] = 7.5$ (i.e. approximately 50% of the users are scheduled per OFDM symbol transmitted by the BS). Going higher above this threshold level achieves more throughput enhancements but at the expense of fairness, since much less number of users will be scheduled per OFDM symbol transmitted by the BS.

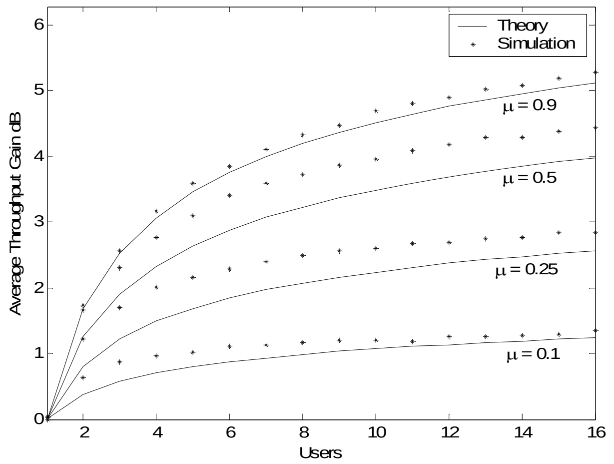

Figure 3: Throughput enhancement using threshold-based multiuser scheduling in WiMAX OFDMA

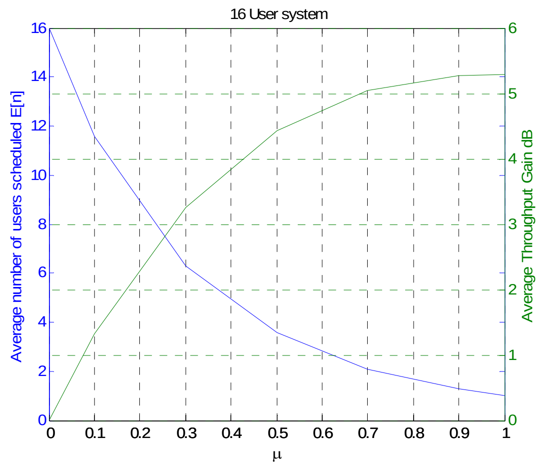

Figure 4: Threshold optimization for throughput gain and average number of user scheduled per transmission in threshold-based multiuser scheduling scheme

4. Conclusions

This paper presents the analysis of a threshold-based multiuser scheduling scheme for use in the downlink transmission in WiMAX OFDMA systems. We consider a point-to-multipoint (PMP) WiMAX network where BS schedules downlink packets for simultaneous transmissions to multiple users using the OFDMA option in the WiMAX standard. In the threshold-based scheduling scheme, the BS scheduler selects at any time instant users whose channel gains in the available subchannels equals or exceed a pre-determined energy threshold. Thus only users who can maximize data rate on the available subchannels are scheduled, enhancing the BS throughput. We quantify analytically the throughput gain per subchannel, provided by the proposed scheme and also present simulation results to verify the analysis. Both analysis and simulations indicate significant enhancements in system throughput when large number of users are serviced per BS scheduler. We also found that throughput enhancements in the proposed scheme increases with the threshold while average number of users scheduled per transmission (fairness criterion) decreases with it. Therefore we illustrate an optimum threshold that achieves a reasonable balance on this trade-off.

Acknowledgments

Author acknowledges support from the Deanship of Scientific Research, King Saud University, Saudi Arabia.

References

[1] A. I. Sulyman, and M. Kousa, "Bit Error rate performance of a generalized selection diversity combining scheme in Nakagami fading channels," Proc., IEEE-WCNC 2000, Vol. 3, pp. 1080-1085, Sept. 2000.

[2] A. I. Sulyman and M. Kousa, "Bit Error rate performance analysis of a threshold-based generalized selection combining scheme in Nakagami fading channels," EURASIP J. Wireless Commun. Networking, Vol. 2005, No. 2, pp. 242-248, Apr. 2005.

[3] M. K. Simon and M. -S. Alouini, "Performance Analysis of generalized selection combining with threshold test per branch (TGSC)," IEEE Trans. Veh. Technol., Vol. 51, No. 5, pp. 1018-1029, Sept. 2002.

[4] L. Cao and N. C. Beaulieu, "Effects of channel estimation errors on absolute threshold-generalized selection diversity combining", ICC 2005, Vol. 4, pp. 2200-2206 May 2005.

[5] M. Guizani, M. Hamdi. P. Lorenz, and M. Ma, "Wireless Brodband Access: WiMAX and Beyond," IEEE Commun. Magazine, vol. 45, Issue 12, pp. 60-61, Dec. 2007.

[6] Z. Abichar, Y. Peng, and J. M. Chang, "WiMAX: The Emergence of Wireless Broadband," IEEE IT Pro., pp. 44-48, Aug. 2006.

[7] K. B. Letaief and Y. J. Zhang, “Dynamic Multiuser Resource Allocation and Adaptation for Wireless Systems,” IEEE Wireless Communications, pp. 38-47, Aug. 2006.
[8] M. Mehrjoo, M. Dianati, X. Shen, and K. Naik, “Opportunistic Fair Scheduling for the Downlink of IEEE 802.16 Wireless Metropolitan Area Networks,” Proc. 3rd ACM International Conference on Quality of Service in heterogeneous wired/wireless networks, Waterloo, Canada, ACM Press, NY, Aug. 2006.
[9] N. Balakrishnan, and A. C. Cohen, “Order Statistics and Inference,” Academic Press, Inc., 1991.
[10] M. Einhaus, O. Klein, “Performance Evaluation of a basic OFDMA Scheduling Algorithm for Packet Data Transmissions,” Proc. IEEE Symposium on Computers and Commun. (ISCC’06), 2006.
[11] IEEE Standard for Local and Metropolitan Area Networks – Parts 16: Air Interface for Fixed and Mobile Broadband Wireless Access System Amendment2: Physical and Medium Access Control Layers for Combined Fixed and Mobile Operation in Licensed Bands, Feb. 2006.

Ahmed Iyanda Sulyman received the PhD degree in Electrical Engineering from Queen’s University, Canada in 2006. He was a Teaching Fellow at the Department of Electrical Engineering, Queen’s University, 2004-2006, a Post-Doctoral Fellow at the School of Computing Queen’s University, in 2007, and at the Royal Military College of Canada, in 2008-2009. In February 2009, He joined King Saud University, Riyadh, Saudi Arabia, as Assistant Professor.