Development and Performance of a New SAC Optical CDMA code with in-phase Cross Correlation

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Summary
A new code structure for Spectral Amplitude coding (SAC) in Optical Code Division Multiple Access (OCDMA) is developed and presented. This new code is called Multi Weight (MW) code. Besides the ease of implementation, this code has the advantage due to the flexibility of choosing any number of users and any weight with minimum code length. Results also indicate that this code perform well from other codes in its class in term of cardinality and system performance. The performance of this code in the existence of noise is also evaluated using Optisys version 7.0. It has been shown that the performance of OCDMA system can be improved by utilizing the new code compared to other codes in the SAC code family.

Key words:
Spectral Amplitude Coding, Optical Code Division Multiple Access, Optisys 7.0, code length.

1. Introduction
Optical code division multiple access (OCDMA) is a multiple access technology introduced over than two decades ago gaining much attraction due to its interesting features as multiple users can access the network simultaneously with high level of security [1],[2],[3]. However, one main degrading factor of OCDMA is the Multiuser Interference (MUI) especially when large numbers of simultaneous users are involved. With Spectral Amplitude Coding (SAC) OCDMA systems, the MUI can be eliminate with the use of codes with fixed in phase cross correlation [4]. The cross correlation, $\lambda_c$ can be defined as the code words overlapping between two code sequences in the SAC OCDMA code families. To eliminate MUI and further suppress other noise limiting factor such as Phase Induced Intensity Noise (PIIN), it is desirable to design codes that have $\lambda_c$ being kept as small as possible [5]. Following this, many codes have been proposed to meet the requirements such as the Optical Orthogonal codes, OOC [6], Modified Quadratic Congruence Code, MQC[7], Modified Frequency Hoppoing code (MFH) [8], Modified Double Weight, MDW [9], Random Diagonal code[10], and recently the Zero Cross Correlation code[11] and Multi Diagonal code[12]. These codes, however, still suffers from some bounds and limitations. OOC, for example exhibits long code lengths constructions, MQC and MFH are bound to exist for prime numbers, $p$ and $q$ only. MDW is a code with maximum of one cross correlation yet is only limited to even number of weight. RD basically has zero cross correlation between its data segment in the code, but had multiple cross correlations between the code segments. The fact that only one chip i.e. one weight will be carrying the data while other weights acts as dummies for security purposes, does not help in improving the system performance. Realizing that one main key element to an effective OCDMA system is to develop an efficient address codes that have low or zero cross correlation properties, this new code, called Multi Weight (MW) is proposed and developed. This code structure only have one overlapping bits with one other code within its mapping sector, while zero cross correlation between any other code sequences. This means that the MW code possesses an ideal in phase cross correlation, where $\lambda_c$$\leq$1. This property ensures that MW can be eliminated while PIIN noise to be largely suppressed.

The rest of this paper is organized as follows: Section II describes briefly on the MW code construction. Section III is devoted to code comparison in term of code lengths and performance comparison in term of cardinality. Network simulations and its results are discussed in Section IV, while conclusions are drawn in Section V.

2. Multi Weight (MW) Code Construction
In SAC OCDMA, a code with length $N$, weight $W$ and cross-correlation $\lambda_c$ can be denoted by $(N, W, \lambda_c)$. The MW code can be constructed based on the following steps:

**Step 1:**
The MW code can be represented by using a $K x N$ matrix. In a MW code structure, the matrix $K$ rows and $N$ columns represent the number of users and the minimum code length for the particular user. A basic MW code is given by a $2 x 3$ matrix, as shown below:

$$MW_{basic} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

(1)
Note that MW_basic has a chip combination sequence of 1-2-1 for the three columns. The chip combination sequence is defined by the summation of the values of the corresponding elements in every two rows. For example, at the first column, the summation is 1+0=1, the summation of the values in the second column is 1+1=2, while in the third column the summation is 0+1=1. The purpose of the 1-2-1 combination is to maintain the cross correlation value, $\lambda_c=1$; meaning that only one overlapping between two codes are allowed.

**Step 2:**

To increase the number of code weight, from the initial weight 2 (W=2), a $(W-2)$ identity matrix with size $n \times n$ (where $n=K$); i.e. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is added at the end of the basic code matrix columns, as illustrated below:

\[ \text{[MW new weight]} = \text{consist of 2 parts, i.e. } \langle Y_1 | Y_2 \rangle \]

\[ [Y_1] = \text{basic matrix of MW code } = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \]

\[ [Y_2] = (W-2) \times \text{identity matrix } \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \]

For example, to increase from weight 2 to weight 3, it is necessary to add $(3-2)$ identity matrix at the end of the basic MW matrix as shown:

\[ MW_{w=3} = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \end{bmatrix} \]

Here, the $K \times N$ matrix becomes $2 \times 5$. One increase in code weight will mean that the column matrix will be increased by two columns. In the case of $K=2$ and $W=4$, the $K \times N$ matrix will become $2 \times 7$. In general, for basic number of users, $K$ equal to two, the minimum required code length for a specific weight is:

\[ N=2W-1 \]  

(2)

To increase the number of users, $K$ for the same weight, a mapping technique is applied where the basic code $Y_{[M=0]}$ is mapped diagonally to achieve $Y_{[M=1]}$ as shown below:

\[ Y_{[M=1]} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \]

Where $M$ is the mapping sector denoted as

\[ M = \begin{cases} 0 & \text{when } 1 < K < 2 \\ Q & \text{when } K \text{ is odd} \\ Q-1 & \text{when } K \text{ is even} \end{cases} \]  

(3)

$Q$ represents the quotient of $K/2$.

In every mapping, the number of rows will be increased by 2; while the number of columns will also increase by $N_{\text{initial}}=N_{[M=0]}$. For example, $Y_{[M=0]}=2 \times 3$ and $Y_{[M=1]}=4 \times 6$ and $Y_{[M=2]}=6 \times 9$ matrices. The relation between the mapping process ($M$) and the number of rows, $K$ and number of columns, $N$ is given by:

\[ K=2 \times 2^M \]  

(4)

\[ N=N_{[M=0]} + M \times N_{[M=0]} \]  

(5)

It is necessary to emphasize that as the number of users, $K$ increases, the minimum required code lengths, $N$ also increases. The relationship between the two parameters, $K$ and $N$ is given by:

\[ N = \begin{cases} \frac{3K}{2} & \text{when } K \text{ is even} \\ \frac{3K}{2} + \frac{1}{2} & \text{when } K \text{ is odd} \end{cases} \]  

(6)

In general, the equation can be re-written as in Equation (6) for both odd and even numbers

\[ N = \frac{3K}{2} + \frac{1}{2} \left[ \sin \left( \frac{K\pi}{2} \right) \right]^2 \]  

(7)

Equation (7) above is only valid when code weight is equal to two ($W=2$). It is also very important to notice that every mapping sector($M$) only consists of two users ($K$), where the cross correlation, $\lambda_c=1$ only exist in the code sequence in the same mapping sector, while for code sequences from different mapping sector, the cross correlation is always zero. This is one important features of the MW code advantage compared to other SAC OCDMA codes family, where the cross correlation is a major limitation. Using the mapping technique, code weights are being remained unchanged while the code length increases with the increase of users.

In general, for any given number of users, $K$ and code weight, $W$, with cross correlation, $\lambda_c=1$, the relation between the parameters is given as below:

\[ N = \frac{3K}{2} + \frac{1}{2} \left[ \sin \left( \frac{K\pi}{2} \right) \right]^2 + (W-2) \left[ K + \left( \sin \left( \frac{K\pi}{2} \right) \right)^2 \right] \]  

(8)

To illustrate the code construction process explained above, an example of MW code generation with weight, $W=3$ and users, $K=6$ is demonstrated as follow:
Step a. Add number of weights by adding the 2 x 2 identity matrix to the next available columns.

\[ Y_{[w=3; K=2]} = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \end{bmatrix} \]

Step b. Perform the mapping technique to increase the number of users in the system. To get \( K=6 \) the mapping technique (\( M=2 \)) must be done twice as show below:

\[ Y_{[w=3; K=6]} = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix} \]

From this matrix representation, the unique codewords for different users can be represented as:

User 1 = \( \{\lambda_1, \lambda_2, \lambda_4\} \)
User 2 = \( \{\lambda_2, \lambda_3, \lambda_5\} \)
User 3 = \( \{\lambda_6, \lambda_7, \lambda_9\} \)
User 4 = \( \{\lambda_7, \lambda_8, \lambda_{10}\} \)
User 5 = \( \{\lambda_{11}, \lambda_{12}, \lambda_{14}\} \)
User 6 = \( \{\lambda_{12}, \lambda_{13}, \lambda_{15}\} \)

### 3. Code Comparison and Performance Evaluations

In Spectral Amplitude Coding OCDMA, one of the most important parameter that needs to be taken into account while designing a code is the code length. For a given number of users and weight, a code with the most minimum code length is preferable. This is due to the fact that longer code length will lead to larger bandwidth consumptions of each code. Apart from minimizing the capacity that can be dealt by each system, larger bandwidth would contribute to higher noise level. Furthermore, Long code length is a disadvantage since either very wide band sources or very narrow filter bandwidths are required.

Table 1 summarized the code lengths needed for different code for a fixed number of subscribers, \( K=30 \). Assuming that each filter is of 0.8nm bandwidth (chip width=0.8nm), the OOC code, with weight 4 will need a spectrum width of 291.2nm while Prime code with weight 6 will need up to 1036.8nm. In addition, ZCC and MD code will consume 96nm, respectively. Consequently, the new purpose MW code with weight 4 will only require 84nm spectrum width, which represents the most minimum code lengths compared to other existing code. Figure 1 shows the performance of the MW code in term of cardinality compared to other SAC OCDMA codes such as the MQC and RD code I the existence of noise.

<table>
<thead>
<tr>
<th>Codes</th>
<th>No. of User (K)</th>
<th>Weight (W)</th>
<th>Code Length (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>OOC</td>
<td>30</td>
<td>4</td>
<td>364</td>
</tr>
<tr>
<td>Prime Code</td>
<td>30</td>
<td>6</td>
<td>1296</td>
</tr>
<tr>
<td>ZCC</td>
<td>30</td>
<td>4</td>
<td>120</td>
</tr>
<tr>
<td>MW</td>
<td>30</td>
<td>4</td>
<td>105</td>
</tr>
<tr>
<td>MD</td>
<td>30</td>
<td>4</td>
<td>120</td>
</tr>
</tbody>
</table>

It is clearly shown that at BER of \( 10^{-9} \), the MQC code with weight 8 can accommodate users only up to 30 users, while about 55 simultaneous users can be accommodated using RD code with weight 5 for the same BER. Accordingly, using this newly proposed MW code at weight 4, an OCDMA network can increase its cardinality up to about 75 simultaneous subscribers for the same BER. This is equivalent to 150% and 37% of cardinality increase from the MQC code and the RD code. This result thus proved that although using a lower weight compared to MQC and RD codes, MW code can still outperform these two codes.

### 4. Network Simulation and Results

The performance of MW code was simulated using OptiSys version 7.0. Illustrated in Figure 2 is the simulation setup used in the simulation.
The code sequences are generated from different wavelengths that are sliced from the broadband light source with the chip width of 0.8 nm each. The tests were carried out at the rates of 622 Mbps and 1 Gbps. The distances are varied from 0 - 100 km length of the ITU-T G.652 standard single mode optical fiber. Attenuation are set to of 0.25 dB/km, dispersion of 18 ps/nm-km and non linear effects such as four wave mixing and self phase modulation were activated with industrial specification to simulate as close as industry real environment. The dark current value is 10 nA and the thermal noise coefficient is $1.0 \times 10^{22}$ W/Hz for each of the photo-detectors. The performance of the system was characterized by referring to the BER. In the transmitter a pseudo random bit generator is used to generate the data signal with Non Return to Zero data format. WDM Multiplexers are used to encode the codewords from the different wavelengths. At the receiver, FBGs are used as decoders while PIN photo detectors are used to convert back the optical signals to electrical data signals. Results are taken at the eye diagram analyzer placed at the receiving end of the simulated network. The performances of the system were characterised by referring to the bit error rate, BER and the eye patterns.

Figure 3 shows the BER curves over distances for MDW code weight 4 and MW code weight 3 involving three simultaneous users. The bit rate is fixed to 622 Mbps. As observed, the BER increased as the distances increased. This is mainly due to the fiber attenuations and nonlinear effects that increase as the fiber length increases. It is clearly seen that the for BER of $10^{-9}$, the system using MW code can go up to 65 km compared to the MDW code only up to 50 km although its weight is smaller than the MDW code. In SAC OCDMA larger weight indicate higher quality of signals because each weight carries the power spectrum of information bearing signal. Thus, systems using MW code are proved to perform better for longer distances compared to MDW code.

Figure 4 reveals the eye diagram taken at the receiving end of the simulation setup for bit rate of 1 Gbps for (a) MW

![Figure 2 Simulation Setup](image)

![Figure 3 BER curves over distance for MW and MDW code (K=3)](image)
code weight 3, (b) MDW code weight 4 and (c) RD code weight 3 at 30km distance. The eye opening patterns shown indicate that the signal received using MW code is better compared to the RD and MDW code with the widest eye opening and height. The BER taken at this point is 1.359 x 10^{-17} for MW code, 7.8 x 10^{-13} for MDW code, and 5.25 x 10^{-9} for RD code.

5. Conclusions

In this paper, the development of a new code in the family of Spectral Amplitude Coding OCDMA has been successfully presented. This code has been shown to have minimum code length in its construction compared to other codes. The MW code has been proven to be providing better performance compared to systems encoded with MQC, MDW and RD code. In addition, this new proposed code possesses numerous advantages such as cross correlation ≤ 1, existence is for any integer value, \( n \), easy code construction with simple encoder decoder design, and could occupy larger number of simultaneous subscribers asynchronously.

Figure 4. (a) MW code weight 3, (b) MDW code weight 4 and (c) RD code weight 3 at 30km distance.
References


J.M.Nordin received B.Sc. in Electrical and Electronics from Universiti Tenaga Nasional, Malaysia in 2002 and MSc in RF Communication System from University of Southampton, United Kingdom in 2005. She joined Universiti Tenaga Nasional as a tutor and currently is working towards her PhD at School of Computer and Communication Engineering, University Malaysia Perlis (UniMAP), Malaysia. Her research interests include OCDMA technologies and wavelength division multiplexing and radio frequency communication system.