Numerical Method for Modeling Transient Flow in Distribution Systems

Mehdi Salmanzadeh

Department of Mechanical Engineering Shoushtar Branch, Islamic Azad University, Shoushtar

Abstract: - A transient is a temporary flow and pressure condition that occurs in a hydraulic system between an initial steady-state condition and a final steady-state condition. When velocity changes rapidly in response to the operation of a flow-control device(for instance, a valve closure or pump start), the compressibility of the liquid and the elasticity of the pipeline cause a transient pressure wave to propagate throughout the system. If the magnitude of this transient pressure wave and the resulting transient flow variation is great enough and adequate transient-control measures are not in place, a transient can cause system hydraulic components to fail (for instance, a pipe burst). In general, transients resulting from relatively slow changes in flow rate are referred to as surges, and those resulting from more rapid changes in flow rate are referred to as water hammer events. Surges in pressurized systems are different than tidal or storm surges, flood waves, or dam breaks, which can occur in open-water bodies. A water hammer wave travels much faster in a pressurized system and it can burst even the strongest pipes. In general engineering practice, the terms surge, transient, hammer, and water hammer are synonymous.
Key-Words: - Surge Analysis, Transient flow, Characteristis Method, Velocity and Pressure Equations.

1. Introduction

The study of hydraulic transients is generally considered to have begun with the works of Joukowsky (1898) and Allievi (1902). The historical development of this subject makes for good reading (Wood F., 1970). A number of pioneers made breakthrough contributions to the field, including R. Angus and John Parmakian (1963), who popularized and refined the graphical calculation method. Benjamin Wylie and Victor Streeter (1993) combined the method of characteristics with computer modeling. The field of fluid transients is still rapidly evolving worldwide (Brunone et al., 2000 Koelle and Luvizotto, 1996; Filion and Karney, 2002; Hamam and McCorquodale1982; Savic and Walters, 1995; Walski and Lutes, 1994; Wu and Simpson, 2000).

Various methods have been developed to solve transient flow in pipes. These range from approximate equations to numerical solutions of the nonlinear Navier-Stokes equations:

Arithmetic method—Assumes that flow stops instantaneously (in less than the characteristic time, 2 L/a), cannot handle water column separation directly, and neglects friction (Joukowski, 1898; Allievi, 1902).

Graphical method—Neglects friction in its theoretical development but includes a means of accounting for it through a correction (Parmakian, 1963). It is timeconsuming and not suited to solving networks or pipelines with complex profiles.

Method of Characteristics (MOC)—Most widely used and tested approach, with support for complex boundary conditions and friction and vaporous cavitation models. It converts the partial differential equations(PDEs) of continuity and momentum (e.g., Navier-Stokes) into ordinary differential equations that are solved algebraicially along lines called **characteristics**. An MOC solution is exact along characteristics, but friction, vaporous cavitation, and some boundary representations introduce errors in the results (Gray, 1953; Streeter and Lai, 1962; Elansary, Silva, and Chaudhry, 1994).

Field Tests—Field tests can provide key modeling parameters such as the pressure- wave speed or pump inertia. Advanced flow and pressure sensors equipped with high-speed data loggers make it possible to capture fast transients, down to 5 milliseconds. Methods such as inverse transient calibration and leak detection use such data. Like all tests, however, data are obtained at a finite number of locations and generalizing the findings requires assumptions, with uncertainties spread across the system. At best, tests provide local data and a feel for the systemwide response. At worst, tests can lead to physically doubtful conclusions limited by the scope of the test program.

The three most common causes of transient initiation, or source devices, are all moving system boundaries.

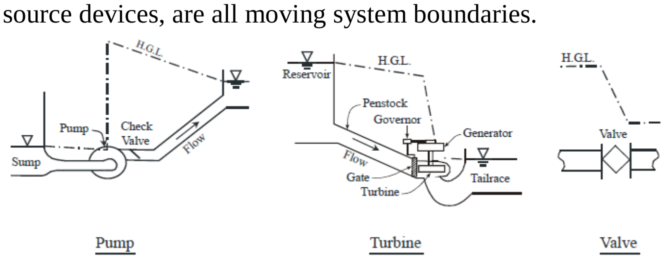

Fig 1: Common Causes of Hydraulic Transients

Manuscript received January 5, 2013
Manuscript revised January 20, 2013

Pumps—A pump's motor exerts a torque on a shaft that delivers energy to the pump's impeller, forcing it to rotate and add energy to the fluid as it passes from the suction to the discharge side of the pump volute. Pumps convey fluid to the downstream end of a system whose profile can be either uphill or downhill, with irregularities such as local high or low points. When the pump starts, pressure can increase rapidly. Whenever power sags or fails, the pump slows or stops and a sudden drop in pressure propagates downstream (a rise in pressure also propagates upstream in the suction system).

Turbines—Hydropower turbines are located at the downstream end of a conduit, or penstock, to absorb the moving water's energy and convert it to electrical current Conceptually, a turbine is the inverse of a pump, but very few pumps or turbines can operate in both directions without damage. If the electrical load generated by a turbine is rejected, a gate must rapidly stop flow, resulting in a large increase in pressure, which propagates upstream (in the penstock).

Valves—A valve can start, change, or stop flow very suddenly. Energy conversions increase or decrease in proportion to a valve's closing or opening rate and position, or stroke. Orifices can be used to throttle flow instead of a partially open valve. Valves can also allow air into a pipeline and/or expel it, typically at local high points.

2. Problem Formulation

2.1 Hydraulic Transient Theory

In pressurized networks, a steady-state condition or transient event at one point in the system can affect all other parts of the system. Consequently, computer models must consider every pipe that is directly connected to a pressurized system, regardless of administrative or political boundaries.

While a systemwide approach increases the information an engineer must consider, the physical principles that govern the behavior of the network provide a unified conceptual basis for tackling the problem. Two fundamental laws apply to steadystate, EPS or transient models:

• **Conservation of mass**—also expressed as the continuity equation, which states that matter cannot be created or destroyed.

• **Conservation of energy**—also expressed as the momentum equation, which states that energy cannot be created or destroyed. The best way to arrive at sound, physically meaningful conclusions and recommendations is to keep these principles in mind whenever you interpret the results of a hydraulic model. In this paper makes this easy by tracking the mass inflow or outflow of air or water at any location and by plotting or animating the resulting total energy at any point and time in the system.

2.2 Governing Equations for Steady-State Flow

Steady-state models, such as WaterCAD or WaterGEMS, are capable of two modes of analysis: steady state and extended period simulation (EPS). EPS solves a series of consecutive steady states using a gradient algorithm and accounting for mass in reservoirs and tanks (e.g., net inflows and storage). Both methods assume the system contains an incompressible fluid, so the total volumetric or mass inflows at any node must equal the outflows, less the change in storage.

In addition to pressure head, elevation head, and velocity head, there may also be head friction. These changes in head are referred to as head gains and head losses, respectively. Balancing the energy across two points in the system yields the energy or Bernoulli equation for steady-state flow:

$$\frac{P_1}{\gamma} + \frac{V_1}{2g} + Z_1 + h_p = \frac{P_2}{\gamma} + \frac{V_2}{2g} + Z_2 + h_L$$

The components of the energy equation can be combined to express two useful quantities, the hydraulic grade and the energy grade:

• **Hydraulic grade**—The hydraulic grade is the sum of the pressure head (p/γ) and elevation head (z). The hydraulic head represents the height to which a water column would rise in a piezometer. The plot of the hydraulic grade in a profile is often referred to as the hydraulic grade line or HGL.

• **Energy grade—**The energy grade is the sum of the hydraulic grade and the velocity head ($V2/2g$). This is the height to which a column of water would rise in a pitot tube. The plot of the hydraulic grade in a profile is often referred to as the energy grade line or EGL. At a lake or reservoir, where the velocity is essentially zero, the EGL is equal to the HGL, as can be seen in the following figure.

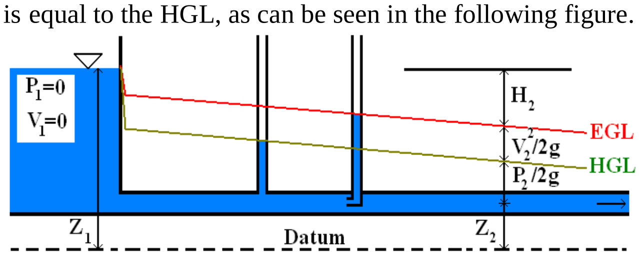

Fig 2: Hydraulic grade Line and Energy grade line

2.3 Governing Equations for Unsteady (or Transient) Flow

Hydraulic transient flow is also known as unsteady fluid flow. During a transient analysis, the fluid and system boundaries can be either elastic or inelastic:

• **Elastic theory** describes unsteady flow of a compressible liquid in an elastic system (e.g., where pipes can expand and contract). In this paper uses the Method of Characteristics (MOC) to solve virtually any hydraulic transient problems.

• **Rigid-column theory** describes unsteady flow of an incompressible liquid in a rigid system. It is only applicable to slower transient phenomena. Both branches of transient theory stem from the same governing equations.

The continuity equation and the momentum equation are needed to determine *V* and *p* in a one-dimensional flow system. Solving these two equations produces a theoretical result that usually corresponds quite closely to actual system measurements if the data and assumptions used to build the numerical model are valid. Transient analysis results that are not comparable with actual system measurements are generally caused by inappropriate system data (especially boundary conditions) and inappropriate assumptions.

3. Problem Solution

3.1 Continuity Equation for Unsteady Flow

The continuity equation for a fluid is based on the principle of conservation of mass. The general form of the continuity equation for unsteady fluid flow is as follows:

$$\frac{\partial P}{\partial t} + V\frac{\partial P}{\partial x} + \frac{a^2}{g}\frac{\partial V}{\partial x} = 0 \qquad (1)$$

The second term on the left-hand side of the preceding equation is small relative to other terms and is typically neglected, yielding the following simplified continuity equation, as used in the majority of unsteady models:

$$\frac{\partial P}{\partial t} + \frac{a^2}{g}\frac{\partial V}{\partial x} = 0 \qquad (2)$$

3.2 Momentum Equation for Unsteady Flow

The equations of motion for a fluid can be derived from the consideration of the forces acting on a small element, or control volume, including the shear stresses generated by the fluid motion and viscosity. The three-dimensional momentum equations of a real fluid system are known as the Navier-Stokes equations. Since flow perpendicular to pipe walls is approximately zero, flow in a pipe can be considered one-dimensional, for which the continuity equation reduces to:

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} + g\frac{\partial P}{\partial x} + \frac{f}{2D}V\left|V\right| = 0 \qquad (3)$$

The last term on the left-hand side(f*V|V|/2D) represents friction losses in the direction of flow.

The first term on the left-hand side is the local acceleration term, while the second term represents the convective acceleration, proportional to the spatial change of velocity at a point in the fluid, which is often neglected to yield the following simplified equation:

$$\frac{\partial V}{\partial t} + g\frac{\partial P}{\partial x} + \frac{f}{2D}V\left|V\right| = 0 \qquad (4)$$

3.3 Method of Characteristics (MOC)

In this paper uses the most widely used and tested method, known as the Method of Characteristic (MOC), to solve governing equations 2 and 4 for unsteady pipe flow. Using the MOC, the two partial differential equations can be transformed to the following two forward differences of equations:

$$\frac{\partial V}{\partial x} = \frac{V_{i+1}^{n} - V_{i}^{n}}{\Delta x}$$

$$\frac{\partial P}{\partial t} = \frac{P_{i}^{n+1} - P_{i}^{n}}{\Delta t}$$

$$\frac{\partial V}{\partial t} = \frac{V_{i}^{n+1} - V_{i}^{n}}{\Delta t}$$

$$\frac{\partial P}{\partial x} = \frac{P_{i+1}^{n} - P_{i}^{n}}{\Delta x}$$

$$\frac{P_i^{n+1} - P_i^{n}}{\Delta t} + \frac{a^2}{g}\frac{V_i^{n+1} - V_i^{n}}{\Delta x} = 0 \qquad (5)$$

$$\frac{V_i^{n+1} - V_i^{n}}{\Delta t} + g\frac{P_i^{n+1} - P_i^{n}}{\Delta x} + \frac{f}{2D}V_i^{n}\left|V_i^{n}\right| = 0 \qquad (6)$$

With unknown amount V and P of time n. Their value at the time n+1 using Equations 5 and 6 is calculated.

Also put : $P = \rho g H$ *and* $\frac{dZ}{dx} = 0$

$$H_i^{n+1} = H_i^{n} - \frac{a^2\Delta t}{g\Delta x^2}\left(V_{i+1}^{n} - V_i^{n}\right) \qquad (7)$$

$$V_i^{n+1} = V_i^{n} - \frac{g\Delta t}{\Delta x}\left(H_{i+1}^{n} - H_i^{n}\right) - \frac{f\Delta t}{2D}V_{i+1}^{n}\left|V_{i+1}^{n}\right| \qquad (8)$$

The friction factor f in the above equations 3 to 8 is replaced by the following Churchill explicit approximations which covers full range of flow conditions, from laminar to turbulent.

$$f = 8\left[\left(\frac{8}{\mathrm{Re}}\right)^{12} + \frac{1}{(A+B)^{1.5}}\right]^{\frac{1}{12}}$$

$$A = \left[2.457\ln\frac{1}{(\frac{7}{\mathrm{Re}})^{0.9} + \frac{0.27e}{D}}\right]^{16} \quad , \quad B = \left(\frac{37530}{\mathrm{Re}}\right)^{16}$$

3.4 Boundary conditions

At the principle of orifice is taken to evaluate the values up to complete closure and Finite difference equation along positive characteristics is evaluated respectively as:

$$V_m^n = V_0\left(1 - \frac{t}{T_c}\right) \quad \text{if} \quad 0 \le t \le T_c$$

$$H_m^{n+1} = H_m^n - \frac{a^2\Delta t}{g\Delta x}\left(V_m^n - V_{m-1}^n\right)$$

When the valve is completely closed :

$$V_m^{n+1} = 0 \qquad and \qquad A = 0 \quad \text{if} \quad t > T_c$$

At upstream i.e., reservoir is assumed to be infinite hence pressure remains constant i.e., and discharge evaluated along the negative characteristics respectively as:

$$H_0^{n+1} = H_0$$

$$V_0^{n+1} = V_1^n - \frac{g\Delta t}{\Delta x}\left(H_1^n - H_0^n\right) - \frac{f\Delta t}{2D}V_1^n\left|V_1^n\right|$$

3.5 One-way surge tank

One-way surge tanks is used for inhibit low pressure and for water column separation and it has no effect for inhibiting of high pressure. An one-way valve is installed on tank input for inhibit water flowing in to the tank. therefore, water in one-way surge tanks just flow from tank to pipe line and do not return to tank because of one way valve.

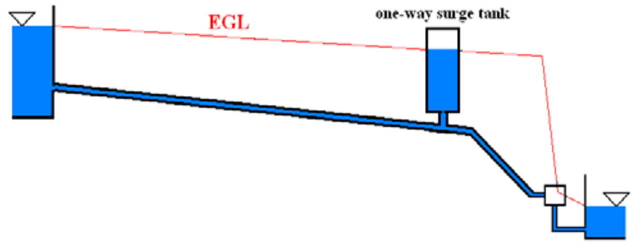

Fig 3: One-way surge tank in hydraulic installations

3.5.1 Analyze of One-way surge tank

For analyzing, we should consider one-way surge tanks as a special boundary conditions. If position, geometry and hydraulic features of tanks are known, we can develop dominant equations for these boundary conditions. For doing so, we assume that tank is located at intersection of two series lines. following equations are for internal boundary conditions that exist in low pressure location of tank.

$$C_1 = V_{i+1}^n - \frac{g}{a}H_{i+1}^n - \frac{f\Delta t}{2D}V_{i+1}^n\left|V_{i+1}^n\right|$$

$$C_2 = V_{i-1}^n + \frac{g}{a}H_{i-1}^n - \frac{f\Delta t}{2D}V_{i-1}^n\left|V_{i-1}^n\right|$$

$$V_A^n = C_2 - (\frac{g}{a})H_A^n$$

$$V_B^n = C_1 + (\frac{g}{a})H_A^n$$

$$H_A^n = H_B^n \quad \text{and} \quad V_B^n A_B = V_A^n A_A + Q_s$$

$$Q_s = C_0 A_n \sqrt{2g(H_s^n - H_A^n)}$$

And we should look for another equation. for writing these equation, we should consider variation of H_s with time. Let H_s be initial head of water in tank, then by writing continuity rule for tank volume control we have:

$$H_s^{n+1} = H_s^n - \frac{\Delta t}{A_s}Q_s$$

Where, A_s is cross section of pipe between surge tank and pipe.

$$Q_s = 0.5C_3\left(-1 + \sqrt{1 + \frac{4C_4}{C_3^2}}\right)$$

$$C_3 = \frac{2gC_0^2 A_n^2}{(g/a)(A_B - A_A)}$$

$$C_4 = C_3\left[C_1 A_B - C_2 A_A + (g/a)(A_A + A_B)H_s^n\right]$$

Q_s magnitude should be controlled because if Q_s is negative, then it equal zero because in this situation valve closed and it is no counter flow in line.
In next steps, we use one-way surge tank in pipeline route to examine its effect in different points of line and find best location for installing tank.

3. Problem Solution

3.1 Case studies

The case study used the water pipeline system shown in figure 4. This system consisted of a 182.87m head reservoir feeding a network of five pipe sections and five junctions. The Hazen-Williams roughness and weve speed for each pipe were 0.029 and 918m/s. The elevation of each junction was essumed to 0 m. A raped demand decrease over a 1-s time period at the terminal junction 5 was initiated at 4 s to introduce a transient condition.

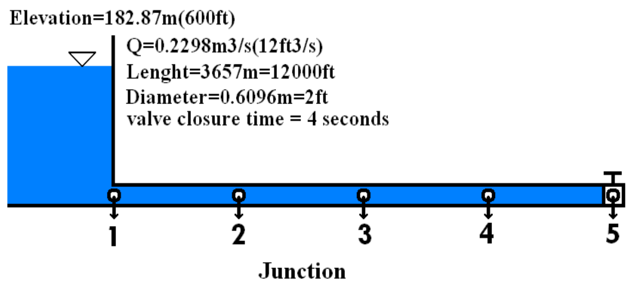

Fig 4: Schematic of hydraulic system considered for the case study

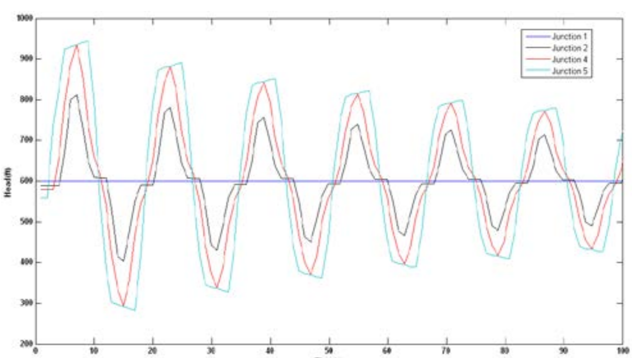

Fig 5: Pressure Head Vs time up to 100 sec by MoC

Figures 5 show the transient head profiles at junction 1, 2, 4, 5 using head-insensitive and head-sensitive demands, respectively.

3.2 Case study 1:

In first step, one-way surge tank installed near valve in joint 4 (figure 6). By closing valve in 4 second, produced head in system for joints 1,2,4,5 are showed in figure 7.

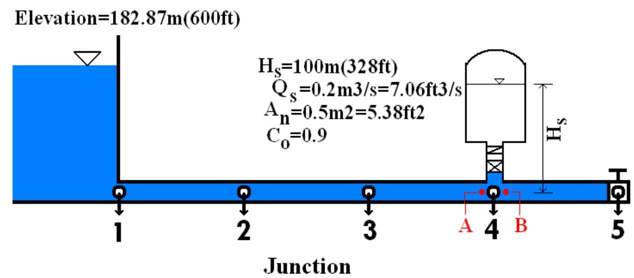

Fig 6: Pipeline system by one-way surge tank near valve

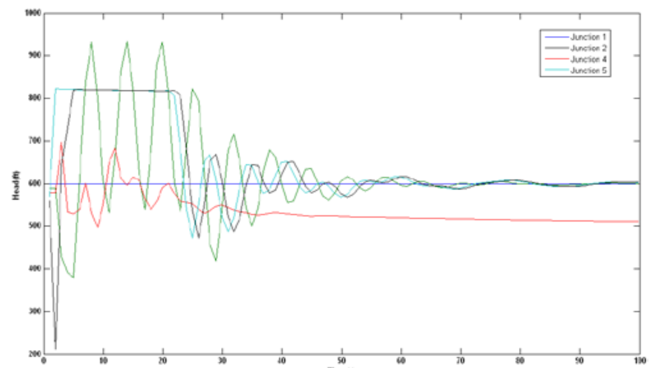

Fig 7: Pressure Head Vs time for case 1

3.3 Case study 2:

At second step, one-way surge tank installed in middle of pipe at joint 3 (figure 8). By closing valve at 4 second, produced head in system for joints 1,2,3,5 showed in figure 9.

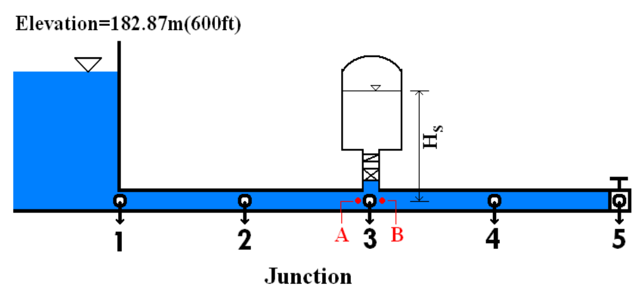

Fig 8: Pipeline system by one-way surge tank in meddle of pipe

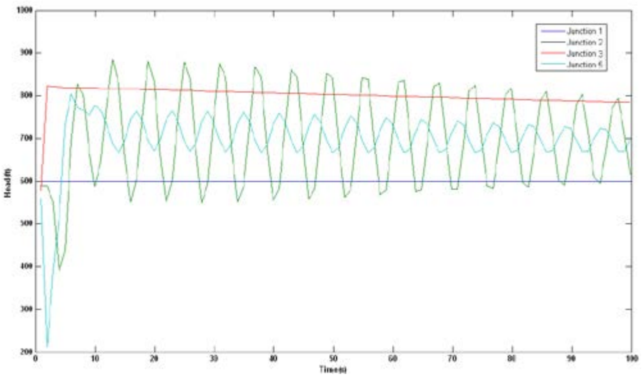

Fig 9: Pressure Head Vs time for case 2

3.4 Case study 3:

At third step, one-way surge tank installed in end of pipe at joint 2 (figure 10). By closing valve at 4 second, produced head in system for joints 1,2,4,5 showed in figure 11.

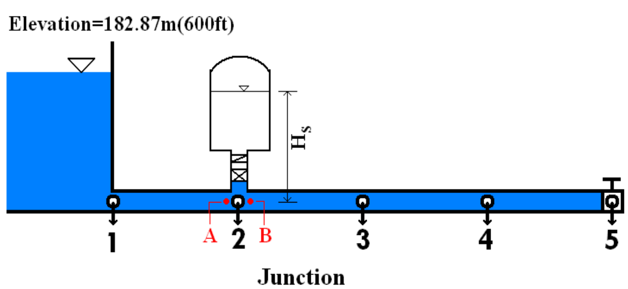

Fig 10: Pipeline system by one-way surge tank in end of pipe

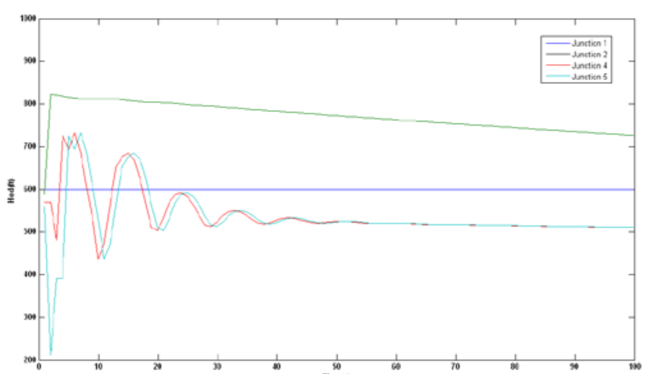

Fig 11: Pressure Head Vs time for case 3

Now, compare produced head near valve for pipe without tank tank and 1,2,3 steps (figure 12). Maximum and minimum produced head in pipe without surge tank and 1,2,3 steps (figure 13,14) compared. In result of this comparing, we can easily determine best location for installing one-way surge tank.

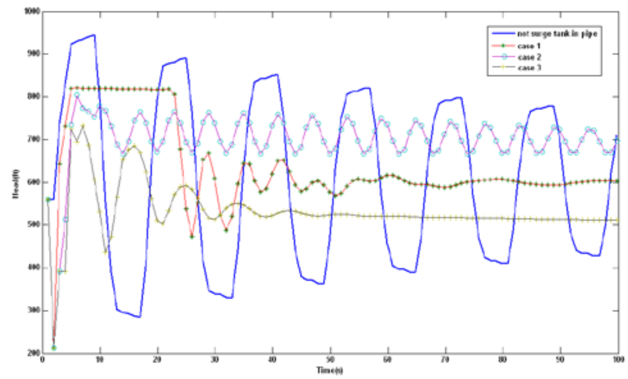

Fig 12: Pressure Head Vs time at valve for case 1,2,3 and no surge tank case

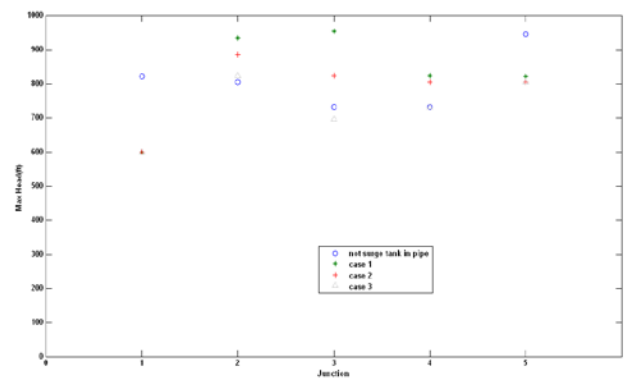

Fig 13: Maximum pressure head created during the pipeline for case 1,2,3 and no surge tank

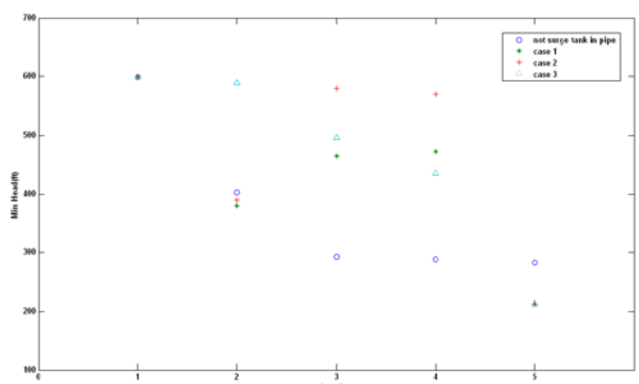

Fig 14: Minimum pressure head created during the pipeline for case 1,2,3 and no surge tank

4. Conclusion

Oone-way surge tank only used for low pressure and inhibit flow separation. These type of tank used when

EGL has insignificant distance with pipe axis. Tests shows that if tank installed near to end of pipe then water pressure reduced at steady state flow and maximum pressure reduce significantly.

List of symbols

a	Wave velocity, m / s
A_n	Pipe area between surge tank and pipeline, m^2
C_0	Cofficient of orifice
e	Roughness, mm
f	Coefficient Darcy-Weisbac
g = 9.81	Acceleration of gravity, m / s
H	Head or height of fluid, m
H_s	Water level in surge tank, m
P	Pressure, Pa
Q_s	Flow rate frome surge tank to pipeline, m^3/s
Re	Reynolds number
T_c	Valve closure time, sec
V	Speed, m / s
ρ	Density, kg / m ^{3}

References:

[1] Benard, CJ 1991, "*Handbook of fluid flowmetering*", 1 st Ed., Mcgraw-Hill, New york, pp.241.

[2] Box, PO 1996 "*Valves, Pipeline Handbook*", 2 nd Ed., Kidlingto, Oxford, pp. 50-56 and 156-161 and 288-291.

[3] Newton, I. 1729 "*Mathematical Principles*", Vol.One, Berkeley, Losangeles, London, pp.44-46.

[4] Stephenson, D. 1984, "*Pipeflow Analysis*", Elsevier, New york.

[5] Streetr, V. 1950 "Valves *for control of fluid*", SW London, pp.98- 114.

[6] Penitent, or.And Grass, 1379, *Description municipal water networks*, Isfahan University Press, pp 411-478.

[7] Excited, Vol.Medical And Race, 1369, *Applied Hydraulic currents Mira*, the first volume, type Dehkhoda Institute Publicati.

[8] ARPN Journal of Engineering and Applied Sciences I. VOLODKO, V. KOLISKINAS, "*Transient Flows in Pipes and Channels: Analytical Solutions*" 4th IASME/WSEAS International Conference on ENERGY, ENVIRONMENT, ECOSYSTEMS and SUSTAINABLE DEVELOPMENT(EEESD'08) in 2008 Proc. , pp. 265-268.

[9] M. TÖPPEL, PROF. DR. E. PASCHE, DR. F. PUNTIGLIANO, M. RADA, DR. U. TESCHKE., "*Hydrodynamic Analysis of Aircraft Water Supply*" in 2008 Proc. WSEAS Conf.

[10] VOLODKO I., V. KOLISKINAS, "*A Java-based Program for Numerical computation of Hydraulic Shock*"Mathematical methods, computational techniques, intelligent systems., pp. 201-204.

[11] M. Salmanzadeh, S.Torfi, "*Numerical Simulations of Transient Flow Occurrence in Pipelines by Appropriate Division Of The Pipeline Length Method*" in 2010 Proc. WSEAS Conf. Athens, Greece, pp. 141-147.

[12] M. Saikia and A.K Sarma, "*SIMULATION OF WATER HAMMER FLOWS WITH UNSTEADY FRICTION FACTOR*" in ARPN Journal of Engineering and Applied Sciences, VOL.1, NO.4, DECEMBER 2006.

[13] Wood, D.J, "*Waterhammer Analysis—Essential and Easy .and Efficient*" in JOURNAL OF ENVIRONMENTAL ENGINEERING © ASCE / AUGUST 2005 / 1131.

[14] Wood, D.J, "*Numerical method for modeling transient flow in distribution systems*" in American Water Works Association , Julay 2005.

[15] Wood, D. J., Lingireddy, S., and Boulos, P. F. ~2004! *Pressure wave analysis of transient flow in pipe distribution systems*, MWHSoft Inc., Pasadena, Calif.

[16] W.F Silva-Arya, and M.H.Chaudhury. 1997. *Computaion of enegu dissipation in transient flow. Journal Hydraulic Engineering*, ASCE. Vol. l123(2): 108-115.

[17] Watters, G. Z. ~1984!. *Analysis and control of unsteady flow in pipelines*, 2nd Ed., Ann Arbor Science, Boston.

[18] M.H Chudhury. 1994. *Open Channel Flow*. Prentice-Hall of India Pvt. Ltd., New Delhi, India.

[19] K. Hariri Asli, F.B. Nagiyev and A.K. Haghi. *Water hammer analysis; some computational aspects and practical hints, Computational Methods in Applied Science and Engineering*, Nova Science Publications, New York, USA, 2010, chapter 16, pp 263-282.