Multi-User MIMO Downlink Channel Capacity for 4G Wireless Communication Systems

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Summary

This paper investigates the performance of multiple-input multiple-out (MIMO) wireless communication systems in a multi-user environment. The user information is defined by a data matrix generated by the observed input-output data of the system, from which subspace identification method is developed using LQ-decomposition of the matrix. The LQ-decomposition is basically used to solve for the user transmitted signals from a mobile station to the base station. Then the relation between the data matrix and the channel information based is derived using singular value decomposition (SVD) algorithm. The effect of additive white noise due to the wireless channel on the SVD of a rectangular matrix is considered to estimate channel capacity of a MIMO system in multi-user environment. Simulation results are then presented to evaluate the performance of the MIMO system in terms of channel capacity, which illustrate that multi-user MIMO systems have good channel capacity.

Index Terms:

Bit error rate (BER), channel capacity, multiple-input multiple-output (MIMO), multi-use, signal-to-noise ratio (SNR).

1. Introduction

The next generation wireless communication systems require high data rates and reliability as dictated by the ever increasing applications requirements. This calls for the design and implementation of communications systems to be highly adaptive and flexible enough to meet various quality-of-service (QoS) requirements. Recently, most communication systems deal with multiple users who share the same radio resources. Multiple-input multiple-output (MIMO) techniques promise to offer improved performance for future wireless communications systems [1]-[2]. They facilitate performance improvement by enhancement in the link quality (through spatial diversity) or throughput gain (through spatial multiplexing), which also leads to bandwidth efficiency and increased channel capacity in high signal-to-noise (SNR) environments [1], [3]. In particular, MIMO channels involve space-time coding which maps input symbol streams across space and time for diversity and coding gain at higher data rates [4]-[6], while spatial multiplexing involves transmitting independent data streams across multiple antennas [3]. In a nutshell, space-time coding provides diversity gains, while spatial multiplexing achieves high data rates. Generally, the performance of either of these mechanisms is highly dependent on the MIMO channel conditions, as it has been pointed out in [3] that if the MIMO channel is spatially uncorrelated, it is known to be well conditioned to achieve spatial multiplexing gain. On the other hand, if the MIMO channel is spatially correlated, it is much less able to support spatial multiplexing; in which case performance can be improved through spatial diversity [3], [5]-[6].

In a multi-user MIMO wireless communication system, a base-station with multiple antennas usually communicates with a group of users simultaneously, and the individual users are equipped with multiple antennas. Multiple antenna systems have been successfully deployed for emerging broadband wireless access networks such as Mobile WiMAX [1], [7]. With the advent of the 4th-Generation (4G) broadband wireless communications, the combination of MIMO wireless technology with orthogonal frequency division multiplexing (OFDM) has been recognized as one of the most promising techniques [1], [3], [5], [7]. In particular, coding over space, time and frequency domains provided by MIMO-OFDM communication systems enable a much more reliable detection and decoding of data from the transmitter to the receiver [5], [8]-[9].

The remainder of this paper is organized as follows: Section II. Section III presents the mathematical models used for MIMO channel techniques. Section IV and finally the concluding remarks in Section V.

2. Related work

Recent research works have culminated into a profound foundation for future developments in wireless communications theory and techniques. Some of the ground work was proposed by the authors in [10], whereby maximum likelihood (ML) technique was extended to build single-input, single-output (SISO) model based on input-out sequence data sequences. Then variety of statistical identification techniques emerged as prediction error methods, for which various identification algorithms have been established and tested for SISO. However, conducted research in the case of MIMO systems illustrate that the prediction error techniques do not satisfy the required

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optimization system parameters [3], [5]-[7]; hence have inherent difficulties for MIMO systems. As an alternative, stochastic realization theory evolved, which does not rely on optimization concepts to build models based on data; but apply deterministic realization theory with sample estimates of process covariances or apply canonical correlation analysis to the future and the past of the observed system processes. Such algorithms have been shown to be numerically stable in linear algebra, through the use of singular value decomposition (SVD) for example [11]. A great effort has been applied on SVD and QR decomposition in literature also through the works in [12], [13], [14]; which also lead to subspace identification methods as shown by the various works such as [15], [16], [17], [18] to mention a few. The key advantage of subspace identification methods is that they are not subject to inconveniences experienced when applying prediction error techniques in MIMO system identification as nonlinear optimization techniques are not required [17].

The SVD, together with LQ-decomposition have been extensively used in realization based stochastic subspace identification methods [14], [18]-[19]. In essence, the LQ-decomposition provides the preliminary orthogonal decomposition of an output process into deterministic and stochastic components to develop a stochastic realization theory for exogenous input [9]. Hence, the LQ-decomposition basically transforms a given data matrix into a product of lower triangular and an orthogonal matrix, in which case the triangular matrix carries the useful information for system identification, while the other provides orthogonal bases of the row space of data matrix. The LQ-decomposition has been mainly used for link adaptation in MIMO-OFDM systems, although the problem is still far from being completely solved. The key focus of this paper is basically to investigate the performance of MIMO communication systems in a multi-user environment. The next section presents the MIMO system model, as a derivative of a single-input, single-output (SISO) system.

3. MIMO System Model

It has been established in literature that the channel capacity of a single-user single-output (SISO) system with N_T transmit antennas by N_R receiver antennas is proportional to $N_{min} = \min(N_T, N_R)$ [3]-[4], [7], [17]. In general, however, MIMO channels change randomly. Therefore the variable **H**, which represents the channel matrix, is a random factor; which means that the channel capacity is also randomly time-varying. In other words, the MIMO channel capacity can be given by its time average. In practice, we can safely assume that the random channel is an ergodic process [17]. In general, the MIMO channel gains are not independent and identically distributed (*i.i.d.*). The channel correlation is closely related to the capacity of the MIMO channel.

3.1 MIMO Channel Capacity

In MIMO systems, a transmitter sends multiple streams using multiple transmit antennas. The transmit streams can then be represented by deterministic channel

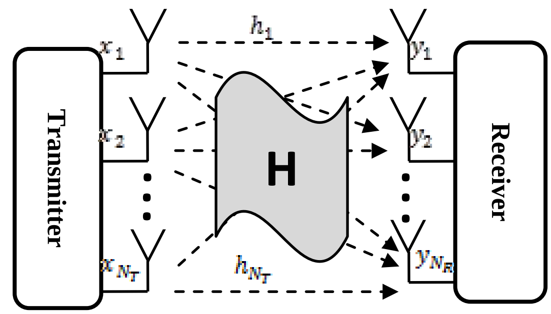

Fig. 1: MIMO System with $N_T \times N_R$ antennas.

matrix $\mathbf{H} \in \mathbb{C}^{N_R \times N_T}$ for a MIMO system with N_T transmit antennas and N_R receive antennas as shown in Fig 1. For any transmitted symbol vector $\mathbf{x} \in \mathbb{C}^{N_R \times 1}$ comprising N_T independent input symbols $x_1, x_1, \dots, x_{N_T}$; the receiver gets the received signal vectors by the multiple receive antennas, for which the received signal $\mathbf{y} \in \mathbb{C}^{N_R \times 1}$ can be expressed by the following:

$$\mathbf{y} = \sqrt{\frac{\mathrm{E}_x}{N_T}}\mathbf{Hx} + \mathbf{z} \qquad (1)$$

where represents E_x the energy of the transmitted signals, $\mathbf{z} = (z_1, z_2, \dots, z_{N_R})^T \in \mathbb{C}^{N_R \times N_T}$ represents a zero-mean *circular symmetric* complex Gaussian (ZMCSCG) noise vector [20]. Following the works in [7], [8], [14], [19], the channel model $\mathbf{H} \in \mathbb{C}^{N_R \times N_T}$ can be represented using singular value decomposition (SVD) as follows:

$$\mathbf{H} = \mathbf{U\Sigma V}^H \qquad (2)$$

where $\mathbf{U} \in \mathbb{C}^{N_R \times N_T}$ and $\mathbf{V} \in \mathbb{C}^{N_R \times N_T}$ are unary matrices, $\mathbf{\Sigma} \in \mathbb{C}^{N_R \times N_T}$ is a rectangular matrix comprising diagonal non-negative real numbers and off-diagonal elements with the values of zero. The singular values of the matrix **H** are the diagonal elements of $\mathbf{\Sigma}$ which can simply be denoted by σ_1, $\sigma_2, \dots, \sigma_{N_{\min}}$ where $N_{\min} \triangleq \min(N_T, N_R)$, with the assumption that the diagonal elements **H** are ordered singular values such that $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_{N_{\min}}$. The rank of **H** therefore corresponds to the number of singular values, ($\mathrm{rank}(\mathbf{H}) \leq N_{\min}$). From the SVD of matrix **H**, the following holds eigen-decomposition [14]:

$$\mathbf{HH}^H = \mathbf{U\Sigma\Sigma V}^H\mathbf{U}^H = \mathbf{Q\Lambda Q}^H \qquad (3)$$

where $\mathbf{Q} = \mathbf{U}$ such that $\mathbf{Q}^H\mathbf{Q} = \mathbf{I}_{N_R}$ (where $\mathbf{I}_{N_R}$ is an $N_R \times N_R$ identity matrix), and $\mathbf{\Lambda} \in \mathbb{C}^{N_R \times N_T}$ is a diagonal matrix for which the diagonal elements are given by the following expression [14]:

$$\lambda_i = \begin{cases} \sigma_i^2, & if\ i = 1,2,\dots,N_{min} \\ 0, & if\ i = N_{min}+1,\dots,N_R. \end{cases} \quad (4)$$

Since the diagonal elements of $\mathbf{\Lambda}$ in Eq. (2) above are eigenvalues $\{\lambda_i\}_{i=1}^{N_R}$, then Eq. (3) illustrates that the singular values $\{\sigma_i^2\}$ for the matrix $\mathbf{H}$ are the eigenvalues of the Hermitian symmetric matrix $\mathbf{HH}^H$ (*i.e.* $\mathbf{H}^H\mathbf{H}$). Based on the fundamental principle of information theory, the mutual information of two continuous random vectors $\mathbf{x}$ and $\mathbf{y}$ is given by the following:

$$I(\mathbf{x};\mathbf{y}) = H(\mathbf{y}) - H(\mathbf{y}|\mathbf{x}) \quad (5)$$

where $H(\mathbf{y})$ is the differential entropy of $\mathbf{y}$ and $H(\mathbf{y}|\mathbf{x})$ is the conditional differential entropy of $\mathbf{y}$ when $\mathbf{x}$ is given. As a result of the statistical independence of the two random variables $\mathbf{z}$ and $\mathbf{x}$ above, Eq. (1) can be used to establish the following relationship:

$$H(\mathbf{y}|\mathbf{x}) = H(\mathbf{z}) \quad (6)$$

Using Eq. (5) and Eq. (6) above, the following expression can be established:

$$I(\mathbf{x};\mathbf{y}) = H(\mathbf{y}) - H(\mathbf{z}) \quad (7)$$

From Eq. (7), the capacity (C) of a deterministic wireless channel can be defined as the maximum mutual information that can be achieved by varying $f(\mathbf{x})$, which is the probability density function (PDF) of a transmit signal vector $\mathbf{x}$ given by the following:

$$C = \max_{f(\mathbf{x})} I(\mathbf{x};\mathbf{y})\ bits/channel\ use \quad (8)$$

It has been established in [21] that the mutual information in Eq. (7) above can be expressed as

$$I(\mathbf{x};\mathbf{y}) = \log_2 \det\left(\mathbf{I}_{N_R} + \frac{\mathrm{E}_x}{N_T \mathrm{N}_0}\mathbf{HR}_{xx}\mathbf{H}^H\right) \mathrm{bps/Hz} \quad (9)$$

where $\mathbf{R}_{xx}$ is the autocorrelation of the transmitted signal vector defined as $\mathbf{R}_{xx} = E\{\mathbf{xx}^H\}$ for which the trace $\mathrm{Tr}(\mathbf{R}_{xx}) = N_T$ if the transmission power of each antenna is assumed to be 1. When the channel state information (CSI) is not known at the transmitter side, the transmit energy can be spread all equally among transmit antennas. The CSI simply refers to the known channel properties of a wireless communication link. When available at the transmitter side, the CSI makes it possible to adapt transmissions to current channel conditions of the wireless channel, which is crucial for achieving reliable communication with high data rates in multi-antenna systems [13], [21]. The CSI is usually estimated at the receiver side and fed back to the transmitter side. Because the channel conditions vary instantaneous, the CSI needs to be estimated on a short term basis for efficient adaptation.

Based on Eq. (8) and Eq. (9), the autocorrelation function of the transmit signal $\mathbf{x}$ therefore becomes $\mathbf{R}_{xx} = \mathbf{I}_{N_T}$; in which case the wireless channel capacity of a is given by the following expression:

$$C = \log_2 \det\left(\mathbf{I}_{N_R} + \frac{\mathrm{E}_x}{N_T \mathrm{N}_0}\mathbf{HH}^H\right) \mathrm{bps/Hz} \quad (10)$$

From Eq. (3), eigenvalues decomposition $\mathbf{HH}^H = \mathbf{Q\Lambda Q}^H$ and the identity $\det(\mathbf{I}_m + \mathbf{AB}) = \det(\mathbf{I}_n + \mathbf{BA})$ where $\mathbf{A} \in \mathbb{C}^{N_R \times N_T}$ and $\mathbf{B} \in \mathbb{C}^{N_R \times N_T}$ can be used to express the channel capacity as the following [21]:

$$\begin{aligned} C &= \log_2 \det\left(\mathbf{I}_{N_R} + \frac{\mathrm{E}_x}{N_T \mathrm{N}_0}\mathbf{Q\Lambda Q}^H\right) \\ &= \log_2 \det\left(\mathbf{I}_{N_R} + \frac{\mathrm{E}_x}{N_T \mathrm{N}_0}\mathbf{\Lambda}\right) \\ &= \sum_{i=1}^{r} \log_2\left(1 + \frac{\mathrm{E}_x}{N_T \mathrm{N}_0}\lambda_i\right) \end{aligned} \quad (11)$$

where r represents the rank of matrix $\mathbf{H}$, such that the following holds: $r = N_{min} \triangleq \min(N_T \times N_R)$.

As an illustration, Fig. 2 shows a cumulative distribution function (CDF) of capacity for a random MIMO channel when CSI is not available at the transmitter side. As the figure illustrates, 4×4 MIMO system achieves a higher capacity than a 2×2 MIMO system; hence, channel capacity improves with the increasing number of transmit antennas.

3.2 Multi user MIMO channels

In a multi user MIMO channel, uplink channel is called multi access channel (MAC) while downlink is referred to as broadcast channel (BC). Assume that the base transceiver station (BTS) and a mobile user node (MUS) have N_B and N_M respectively. We consider a downlink

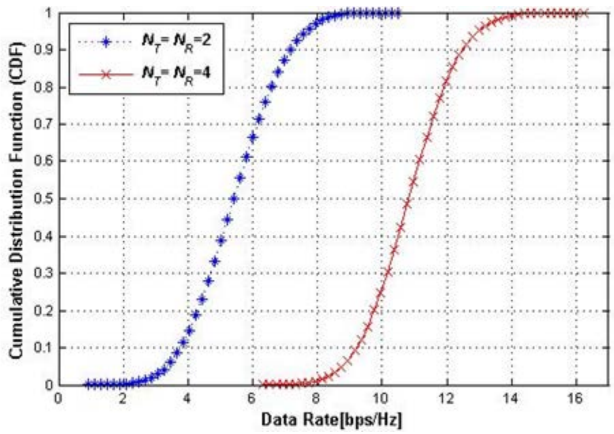

Fig. 2: MIMO systems channel capacity distribution.

BC where $\mathbf{x} \in \mathbb{C}^{N_B \times 1}$ is the transmit signal from the BTS, while $\mathbf{y}_i \in \mathbb{C}^{N_M \times 1}$ represents the received signal for the *i*th user, $i = 1,2,\dots,K$ for a total of K number of users. We let $\mathbf{H}_i^D \in \mathbb{C}^{N_M \times N_B}$ to represent the channel gain from the BTS to *i*th user. The received signal at the *i*th user can then be represented by the following

$$\mathbf{y}_i = \mathbf{H}_i^D \mathbf{x} + \mathbf{z}_i, \quad i = 1,2,\dots,K \tag{12}$$

where $\mathbf{z}_i \in \mathbb{C}^{N_M \times 1}$ is the additive ZMCSCG noise vector. From Eq. (12), all the user signals can be expressed as

$$\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \\ \mathbf{y}_K \end{bmatrix} = \begin{bmatrix} \mathbf{H}_1^D \\ \mathbf{H}_2^D \\ \vdots \\ \mathbf{H}_K^D \end{bmatrix} \mathbf{x} + \begin{bmatrix} \mathbf{z}_1 \\ \mathbf{z}_2 \\ \vdots \\ \mathbf{z}_K \end{bmatrix} \tag{13}$$

The capacity region of a Gaussian broadcast channel is problem yet to be solved [3]-[4], [6]-[7]. In this paper, we consider the situation whereby $N_B = 3$, $N_M = 1$ and $K = 3$, based on the works presented in [22]-[23] which used DPC algorithm and the duality of uplink and downlink channel capacities to derive the capacity of a BC. Based on Eq. (13), the received signal is given by

$$\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \mathbf{y}_3 \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{H}_1^D \\ \mathbf{H}_2^D \\ \mathbf{H}_3^D \end{bmatrix}}_{\mathbf{H}^D} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \tag{14}$$

where $\mathbf{H}_i^D \in \mathbb{C}^{1 \times 3}$ expresses the channel matrix between the BTS and the *i*th user for $i = 1,2,3$ and x_i is the signal transmitted by the *i*th transmit antenna. Assuming the channel state information is available at the BTS, the overall channel can be LQ-decomposed as follows:

$$\mathbf{H}^D = \begin{bmatrix} l_{11} & 0 \\ l_{21} & l_{22} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \tag{15}$$

where **L** represents block lower triangular matrices with a zero block at the upper right corner, with an orthogonal matrix **Q**. Each column of the *L*-matrix is an input-out. With reference to the works on [8]-[9], the components of the **L** and **Q** matrices can be expressed as follows:

$$\begin{aligned} l_{11} &= \|\mathbf{H}_1^D\|, \\ \boldsymbol{q}_1 &= \frac{1}{l_{11}} \mathrm{H}_1^D, \\ l_{21} &= \boldsymbol{q}_1 . (\mathbf{H}_2^D)^H, \\ l_{22} &= \|\mathbf{H}_2^D - l_{21}\, \boldsymbol{q}_1\|, \\ \boldsymbol{q}_2 &= \frac{1}{l_{22}} (\mathbf{H}_2^D - l_{21}\, \boldsymbol{q}_1). \end{aligned} \tag{16}$$

where $(\cdot)^H$ simply represents the conjugate transpose of **H** (transjugate). Based on the channel information given by Eq. (16) above, the transmitted signal $\mathbf{x}$ can therefore be precoded, such that the detected complex signal strength at the receiver side can be modelled by

$$\mathbf{y} = \mathbf{H}^D \mathbf{x} + \mathbf{z} \tag{17}$$

Assuming the transmission power P_T from the BTS is shared among the users by $\rho_1 P_T$ for the 1st user, $\rho_2 P_T$ for the 2nd user and $(1 - \rho_1 - \rho_1) P_T$ for the 3rd user, the channel capacities C_1, C_2 and C_3 for the three users is given by the following expressions [21]:

$$C_1 = \log\left(1 + \|\mathbf{H}_1^D\|^2 \frac{\rho_1 P_T}{\sigma_z^2}\right)$$

$$C_2 = \log\left(1 + \|\mathbf{H}_2^D\|^2 \frac{\rho_2 P_T}{\sigma_Z^2}\right) \qquad (18)$$

$$C_3 = \log\left(1 + \|\mathbf{H}_3^D\|^2 \frac{(1-\rho_1-\rho_2)P_T}{\sigma_Z^2}\right)$$

for the 1st, 2nd and 3rd users respectively, where σ_Z^2 represents the statistical information for the Gaussian noise. The key challenge for the downlink wireless channel data transmission is that the coordinated signal detection on the transmitter side is not straight forward, hence requires interference cancelations at the BTS node.

4. Simulation and Results

This section presents the simulation based evaluation of MIMO systems for channel capacity of a BC based on the models presented in the previous section. Fig. 3 estimates the ergodic capacity of a BC channel for varying SNR. The computation is based on the assumption when the CSI is not available at the transmitter side. The channel capacity is computed for different MIMO system configurations with varying number of transmit and receive antennas. It is worth noting from Fig. 3 that the channel capacity increases with the increase in number of antennas in a MIMO system. For further illustration, Fig. 4 computes ergodic channel capacities for two situations: when the CSI is known and when unknown at the transmitter side, using a 4×4 MIMO system. Evidently, the figure illustrates that the availability of CSI at the transmitter side improves channel capacity in comparison the when the CSI is unknown. However, the availability of CSI at the transmitter side in high SNR regime has little or no impact on the capacity of a BC as the figure illustrates.

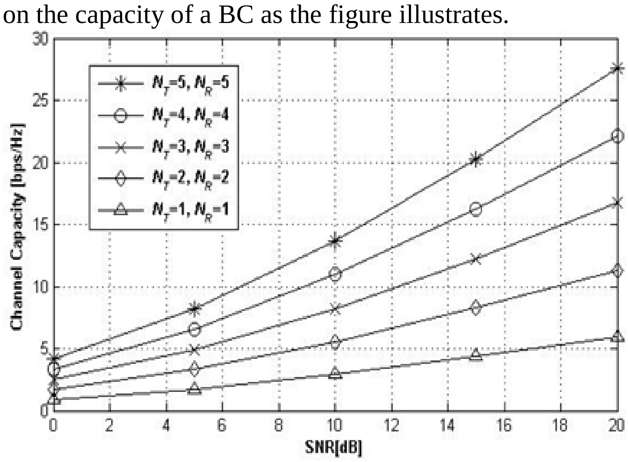

Fig. 3: MIMO channel capacity for varying number of antennas.

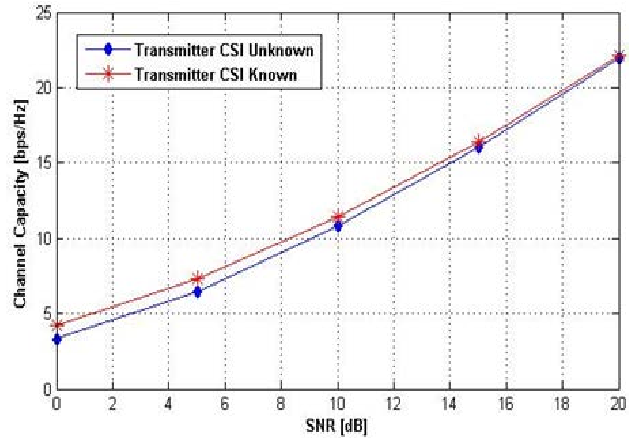

Fig. 4: MIMO channel capacity for unknown and known CSI.

5. Conclusions

This paper presented the capacity of multi-user MIMO systems, using singular value decomposition, from which eigenvalues are calculated, and LQ-decomposition to derive a downlink BC capacity. Ergodic channel capacity model has been presented, and evaluated for different configurations for a MIMO systems with varying number of transmit and receive antennas. Without loss of generality, the capacity of multi-user MIMO channels increase with the number of antennas. It has been pointed out also that the availability of CSI at the transmitter side achieves high channel capacity in low SNR conditions, in comparison to unknown CSI. In high SNR regime, the availability of CSI does not have significant impact on the achievable capacity of a BC.

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References

[1] W. Zhang, et.al. “Advances in space-time/frequency coding for next generation broadband wireless communications” IEEE Radio and Wireless Symposium, pp. 471-474, Jan. 2007.

[2] D. Raychaudhuri and N.B. Mandayam, “Frontiers of wireless and mobile communications,” in proceedings of the IEEE , vol. 100, no. 4, pp. 824-840, Apr. 2012.

[3] J.R. Lee and M.H. Ahmed, “Adaptive space-time coding and its implementation in MIMO antenna systems,” IEEE International Symposium on Signal Processing and Information Technology (ISSPIT’2007), Giza, pp. 425-430, Dec. 2007.

[4] M.A. Beach, D.P. McNamara, P.N. Fletcher and P. Karlson, “MIMO-a solution for advanced wireless access?” in proceedings of the IEEE International Conference on Antennas and Propagation, vol. 1, no. 480, pp. 231 – 235, Apr. 2011.
[5] S. Sanhdu, and A. Paulraj, “Space time block codes: a capacity perspective,” IEEE Communications Letters, vol. 4, no .12, pp. 384 386, Dec. 2000.1
[6] B. Hassibi and B.M. Hochwald, “High-rate codes that are linear in space and time,” IEEE Transactions on Information Theory, vol. 48, no.7, pp. 1804-1824, Jul. 2002.
[7] ZY. Huang and PY. Tsai, “High-throughput QR decomposition for MIMO detection in OFDM systems,” in proceedings of the IEEE International Symposium on Circuits and Systems (ISCAS’2010), pp. 1492 – 1495, Jun. 2010.
[8] N. Li and Y. Saad, “Crout versions of ILU factorization with pivoting for sparse symmetric matrices,” Electronic Transactions on Numerical Analysis, vol. 20, pp. 75-85, 2005.
[9] T. Katayama, “A note on LQ-decomposition in stochastic subspace identification,” Perspectives in Mathematical System Theory, Control, and Signal Processing Lecture Notes in Control and Information Sciences, vol. 398, pp. 355-364, 2010.
[10] K. J. Astrom and T. Bohlin, “Numerical identification of linear dynamic systems for normal operating records,” in proceedings of the 2nd IFAC Symposium on Theory of Self-Adaptive Systems, Teddington, pp. 96–111, 1965.
[11] D.J. Love and R.W. Heath Jr “Limited feedback unitary precoding for orthogonal space time block codes,” IEEE Transactions on Signal Processing, vol. 53, no. 1, pp. 64-73, Jan. 2005.
[12] W. E. Larimore, “System identification, reduced-order filtering and modeling via canonical variate analysis,” in proceedings of the 1983 American Control Conference, pp. 445–451, 1983.
[13] W. E. Larimore, “Canonical variate analysis in identification, filtering, and adaptive control,” in proceedings of the 29th IEEE Conference on Decision and Control, pp. 596–604, 1990.
[14] M. Moonen and J. Vandewalle, “QSVD approach to on- and off-line state-space identification,” International Journal of Control, vol. 51, no. 5, pp. 1133–1146, 1990.
[15] P. Van Overschee and B. De Moor, “Subspace algorithms for the stochastic identification problem,” Automatica, vol. 29, no. 3, pp. 649–660, 1993.
[16] M. Verhaegen, “Subspace model identification, Part 3: Analysis of the ordinary outputerror state-space model identification algorithm,” International Journal of Control, vol. 58, no. 3, pp. 555–586, 1993.
[17] M. Verhaegen, “Identification of the deterministic part of MIMO state space models given in innovations form from input-output data,” Automatica, vol. 30, no. 1, pp. 61–74, 1994.
[18] M. Verhaegen and P. Dewilde, “Subspace model identification, Part 2: Analysis of the elementary output-error state space model identification algorithm,” International Journal of Control, vol. 56, no. 5, pp. 1211–1241, 1992.
[19] P. Van Overschee and B. De Moor, “N4SID - Subspace algorithms for the identification of combined deterministic -stochastic systems,” Automatica, vol. 30, no. 1, pp. 75–93, 1994.
[20] N. Chiurtu, B. Rimoldi, and I.E. Telatar, “On the capacity of multi-antenna Gaussian channels,” in proceedings of IEEE International Symposium on Information Theory, Jun. 2001.
[21] E. Telatar, “Capacity of multi antenna Gaussian channels,” European Transactions on Telecommunications (ETT), vol. 10, no.6, pp. 585-595, 1999.
[22] G. Caire and S. Shamai, “On the achievable throughput of a multi antenna Gaussian broadcast channel,” IEEE Transactions on Information Theory, vol. 43, no. 7, pp. 1691-1706, 2003.
[23] S.S. Christensen and E. de Carvalho, “Achievable sum-rates in MIMO broadcast channels with vector precoding techniques using coded modulation,” in proceedings of the IEEE Vehicular Technology Conference, pp. 2261-2265, Apr. 2007.

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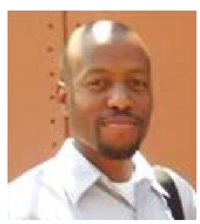

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