Digital Beam Forming using RLS–QRD Algorithm

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Abstract
Digital beam formers are a means for separating a desired signal from interfering signals. This paper describes the GSC technique using the QRD Algorithm and RLS QRD Algorithm for digital Beamforming.

1. INTRODUCTION

Today the demand of high data rate services is increasing very highly in wireless communication. At the same time the need to support more users per base station is also increasing. The result is that the higher data rates and higher capacities become the pressing need. To increase the capacity the attempts are made to increase the traffic within the fixed bandwidth. But increasing the traffic within the fixed bandwidth creates more interference in the system and the result is degradation of signal quality. The interference can be reduced by using the sectored antenna in place of omnidirectional antenna.

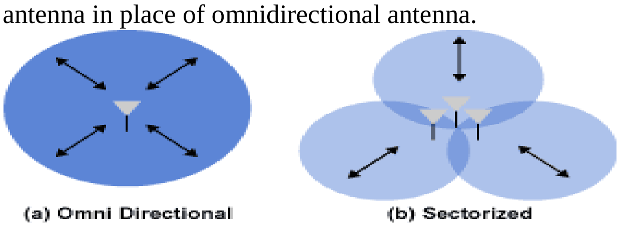

Figure-1.

Smart antenna technology can also be used to reduce the interference level. With this technology each user's signal is transmitted or received by the base station only in the direction of that particular user. This results in the reduction of interference. A smart antenna system consists of an array of antennas that together direct different transmission/reception beams toward each user in the system. This method of transmission and reception is called Beamforming.

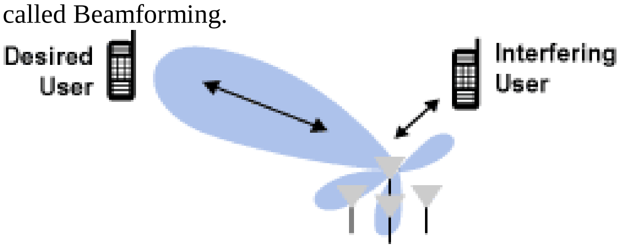

Figure-2: Smart Antenna

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The magnitude and phase of the signal to and from each antenna is adjusted by multiplying each user signal by complex weights. This results in a transmit/receive beam in the desired direction and minimizes the output in other directions.
If the complex weights are selected from a table of weights, the beam is formed in specific, predetermined direction; this type of Beamforming is called switched Beamforming. In this case the base station basically switches between the different beams based on the received signal strength measurements. On the other hand, if the weights are computed and adaptively updated in real time, the process is called adaptive Beamforming. Through adaptive Beamforming, the base station can form narrower beams towards the desired user and nulls towards interfering users, considerably improving the signal-to-interference-plus-noise ratio.

2. BEAMFORMING

Beamforming is one type of processing used to form beams to simultaneously receive a signal radiating from a specific location and attenuate signals from other locations. Systems designed to receive spatially propagating signals often encounter the presence of interference signals. If the desired signal and interference occupy the same frequency band, unless the signals are uncorrelated, e. g., CDMA signals, the temporal filtering often cannot be used to separate signal from interference. However, the desired and interfering signals usually originate from different spatial locations. This spatial Separation can be exploited to separate signal from interference using a spatial filter at the receiver. Implementing a temporal filter requires processing of data collected over a temporal aperture. Similarly, implementing a spatial filter requires processing of data collected over a spatial aperture. A beamformer is a processor used in conjunction with an array of antennas to provide a versatile form of spatial filtering. The antenna array collects spatial samples of propagating wave fields, which are processed by the Beamformer. Typically a beamformer linearly combines the spatially sampled time series from each antenna to obtain a scalar output time series in the same manner that an FIR filter linearly combines temporally sampled data. There are two types of beamformer, narrowband beamformer, and wideband beamformer. A narrowband beamformer is as shown in

Figure 3, the output at time M, y (M), is given by a linear combination of the data at the K.

$$y(M) = \sum_{i=1}^{K} W_i^* x_i(M) \quad \text{--- Eq(1)}$$

Where * denotes complex conjugate. Since we are now using the complex envelope representation of the received signal, both W_i and $x_i(M)$ are complex.

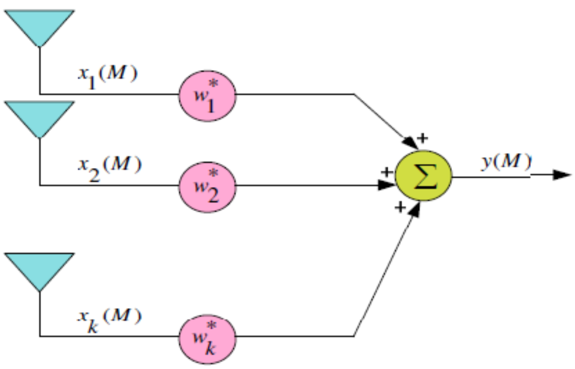

Figure 3: Narrowband Beamformer

2.1 ADAPTIVE BEAMFORMING

An adaptive beamformer can separate signals collocated in the frequency band but separated in the spatial domain. To optimize the array the elemental control weights are adjusted until a prescribed objective function is satisfied. For calculating the adaptive weights the choice of adaptive algorithm is very important as the adaptive algorithm determines the speed of convergence and hardware complexity required. To calculate the adaptive weights the various algorithms used are LMS (Least Mean Squares) algorithm, the SMI (Sample Matrix Inversion) technique and RLS (Recursive Least Squares) algorithm.

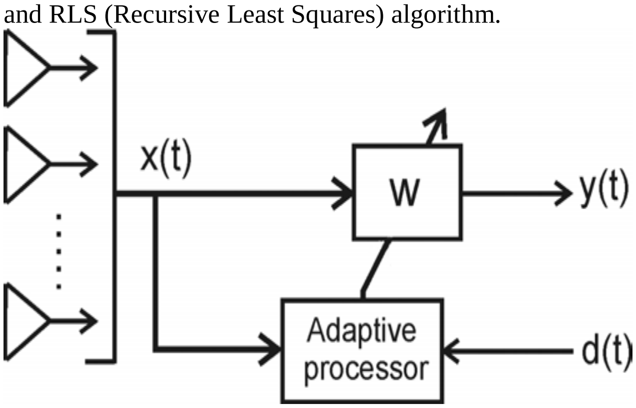

Figure 4: Adaptive Beamforming system

3. SMI (Sample matrix inversion) Technique

There are various ways for SMI like QRD decomposition, SVD, LU decomposition, RLS QRD Decomposition.

3.1 QR Decomposition (QRD)

QR matrix decomposition (QRD), sometimes referred to as orthogonal matrix triangularization, is the decomposition of a matrix (A) into an orthogonal matrix (Q) and an upper triangular matrix (R). QRD is useful for solving least squares' problems and simultaneous equations.

Consider the following equation:
AX= b -------eq(2)

Where:
A, X and b are matrices
A is of order N × N
X and b are column vectors of order N × 1
A and b are known; X is unknown. The objective is to determine the N different unknowns in the X matrix.

Performing QRD (substituting QR for A) results in:
(QR)X = b ------- eq(3)
Moving Q to the right hand side of the equation gives:
RX = Q-1 b ------- eq(4)
Q is an orthogonal (unitary) matrix, thus Q-1 is equal to the complex Conjugate transpose of Q. This operation requires minimal resources to Perform in hardware.

So:
RY = b' ------- eq (5)

Where:
b' = Q-1 b. ------- eq (6)

To find the Q and R the method used is given Rotation method.

3.2 Given Rotation

Given rotation are orthogonal plane rotation used to eliminate the elements within a matrix for [aij] =o when i>j.This method is known as QR decomposition method, by using this the matrix A can be reduced to upper triangular matrix R(n) and Orthogonal matrix Q(n).
A (n) =R (n) Q (n) ------- eq (7)

The A (n) matrix is pre-multiplied by rotation matrices one element at a time. The rotation parameters are calculated so that the sub-diagonal elements of the first column are zeroed. Then the next column's sub-diagonal elements are zeroed and so forth, until an equivalent upper triangular matrix is formed. The following example illustrate the given rotation method, by the following matrix.

$$\begin{matrix} a11 & a12 & a13 \\ a21 & a22 & a23 \end{matrix}$$

This matrix is transformed into pseudo-triangular matrix by eliminating the element; a21.This is achieved by multiplying the matrix by the rotation of matrix.

$$\begin{matrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{matrix}$$

Thus:

$$\begin{matrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{matrix} * \begin{matrix} a11 & a12 & a13 \\ a21 & a22 & a23 \end{matrix} =$$

$$\begin{matrix} a11\cos\alpha + a21\sin\alpha & a12\cos\alpha + a22\sin\alpha & a13\cos\alpha + a23\sin\alpha \\ -a11\sin\alpha + a21\cos\alpha & -a12\sin\alpha + a21\cos\alpha & -a13\sin\alpha + a23\cos\alpha \end{matrix}$$

To eliminate a21
-a11sinα + a21cosα =0

Therefore from trigonometry
Sinα = a21/(a112+a212)1/2

Cosα = a11/ (a112+a212)1/2

The same procedure is repeated till matrix get converted into upper triangular matrix .The orthogonal matrix is got by multiplying transpose of all rotation matrices used to convert the given matrix into the upper triangular matrix. Hence any matrix can be expressed as the product of upper triangular matrix and the orthogonal matrix by using the given rotation method.

4. RLS Solved by QRD

The P*N dimensional data matrix, X(n) is decomposed into an N*N dimensional upper triangular matrix R(n) ,through the application of unitary matrix ,Q(n),such that:

$$Q(n) * X(n) = \begin{bmatrix} R(n) \\ 0 \end{bmatrix}$$

---- eq(8)

Where 0 is the zero matrix resulting if N<p. Since Q (n) is a unitary matrix, then:
φ(n)=

$$X^T(n)X(n) = X^T(n)Q^T(n)Q(n)X(n) = R^T(n)R(n)$$

--- eq(9)

The triangular matrix, R (n) is the cholesky factor of the data correlation matrix. Since Q(n) is unitary then the original system equation may be expressed as:

---- eq(10)

Where QT(n) X(n) =R(n) and QT(n)y(n)=u(n)

The least square vector square weight vector $W_{LS}(n)$ must satisfy the equation

R (n) $W_{LS}(n)$ + u (n) = 0 ----eq (11)

As R(n) is a upper triangular matrix, the weights can be solved by using Back substitution . QR decomposition is an extension of this QR factorization, which enables the matrix to be triangularized again when new data enter the data matrix, without having to compute the triangularization from the original square matrix format. In other words, it updates the old triangular matrix when new data are entered. The data matrix X(n) and the measurement vector y(n) at time n can be represented in a recursive manner by the previous resulting matrix and vector and the new data, such that:

$$X(n) = \begin{bmatrix} \lambda(n)X(n-1) \\ X^T(n) \end{bmatrix} \text{ and } X(n) = \begin{bmatrix} \lambda(n)y(n-1) \\ y(n) \end{bmatrix}$$

Where XT(n) and y(n) form the append Row at time n. A square root of the Algorithm is achieved as follows.

$$Q^T \begin{bmatrix} \lambda^{.5}R(n-1) \\ X^T(n) \end{bmatrix} W_{LS}(n) = Q^T(n) \begin{bmatrix} \lambda^{.5}U(n-1) \\ y(n) \end{bmatrix} + Q^T(n)e(n)$$

--eq(12)

Where β= $\lambda^{.5}$

This is computed to give

$$\begin{bmatrix} R(n) \\ 0 \end{bmatrix} W_{LS}(n) = \begin{bmatrix} U(n) \\ \alpha(n) \end{bmatrix}$$

Where e(n)= $\alpha(n)\ \gamma(n)$

Where $\gamma(n)$ is the product of cosines generated in course of eliminating $X^T(n)$

$$\|J(n)\| = \|Q(n)e(n)\| = \left\| Q^T(n)X(n)W_{LS}(n) + Q^T(n)y(n) \right\|$$

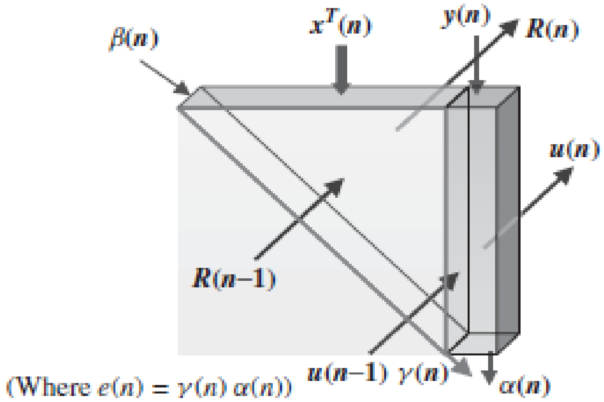

Figure 5. High-level dependence graph for the QR-RLS solution

5. GSC BEAMFORMER

A uniform linear array (ULA) with M sensors has been considered between each element λ /2 spacing is given, where λ is the smallest signal wavelength of the signal with specified gain/null arrangements. If the spacing between the elements is increased beyond λ /2 than it will result in large side lobes in radiation pattern .Assume that K narrowband and far field signals are impinging on the array from direction angles θ_i =1, 2, 3………..K .At the mth array sensor the signal received can be expressed as :

$$\sum_{i=1}^{K} s_i(t) a_m(\theta_i) + n_m(t)$$

$$m = 1, 2, 3 \ldots\ldots M$$

---eq(13)

Where $a_m(\theta_i)$ =exp ($j2\pi d_m \sin(\theta_i)/\lambda$) and d_m is the distance between the first and mth array sensor. Si (t) is the ith signal complex waveform and nm(t) is the spatially white noise .The data received by the array is given as x(t)=As(t)+n(t).

Where A= $[a(\theta_1)a(\theta_2)........a(\theta_K)]$ .The signal source vector is given as S(t)= $[s_1(t)s_2(t)..........s_K(t)]^T$ and the noise vector n(t)= $[n_1(t)n_2(t)..........n_M(t)]^T$.

In GSC structure the Blocking Matrix (B) function is to remove the desired signal from the received array data. d(n) is given as wqHx(t).

The quiescent weight vector wq is utilized to realize the constrained weight subspace and is chosen such that the output signal power $E[d(t)^2]$ is minimized subject to a set of L linear constraints.

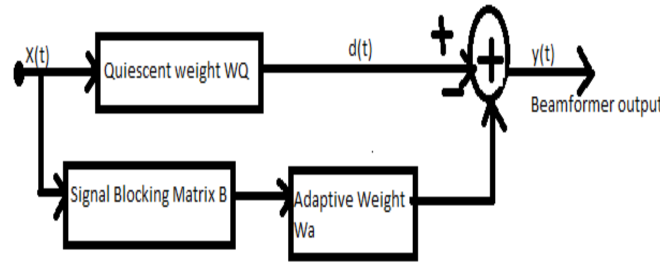

Figure 6. GSC Beamformer

To find the optimum weights Wa using LS criteria the following deterministic equation must be solved.
RX Wa=b.Where RX is the correlation matrix of the input x(t) to the unconstrained section of GSC and the vector b is the cross-correlation of input x(t) and the ideal response.
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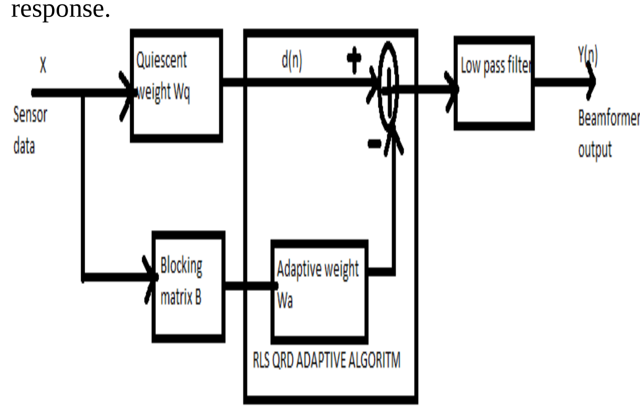

Figure 7. Adaptive GSC Beamformer

The above equation can be solved without any need of matrix inversion by using the RLS QRD ALGORITHM.

6. SIMULATED SYSTEM

The GSC beamformer model has been designed for performing the simulations .The feature of the design includes.
1 A uniform linear array of four sensors.
2 An input signal impinging at an angle of 0 degree.

3 A narrow band interfering signal at an angle of 10 degree.
4 Uncorrelated white noise at a level of - 20 db.

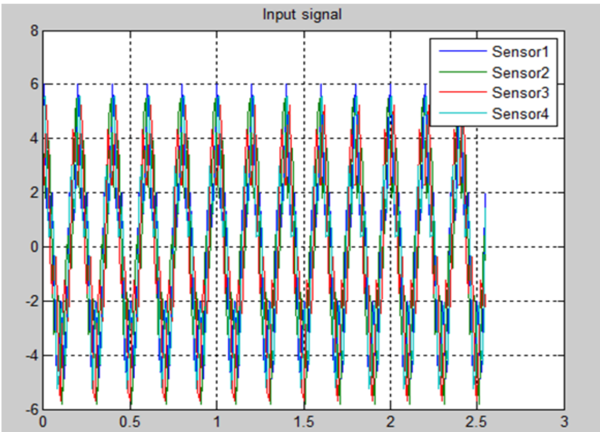

Figure 8: Input Signal

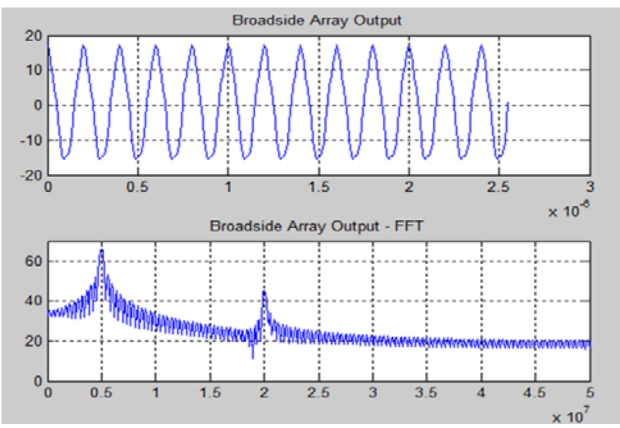

Figure 9: Broad-side array output & its FFT using QRD

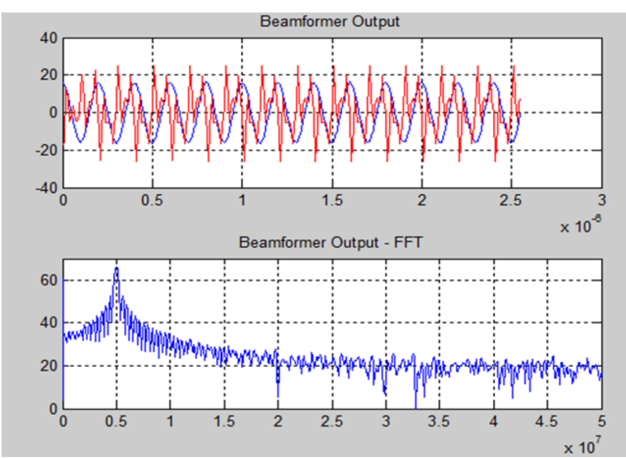

Figure 10: Beamformer output & its FFT using QRD

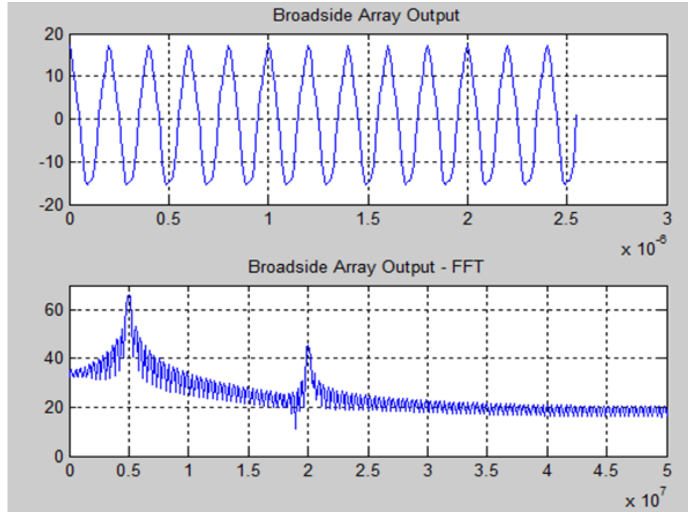

Figure 11: Broadside array output & its FFT using RLS QRD

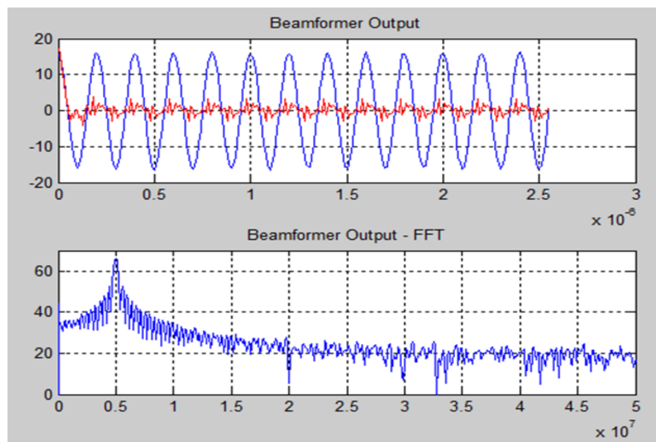

Figure 12: Beamformer output & its FFT using RLS QRD

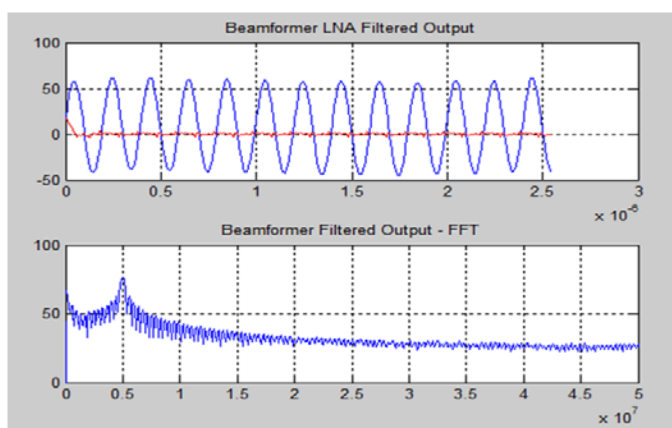

Figure 13: Beamformer output & its FFT after passing output of RLS QRD from Low pass filter.

CONCLUSION

An efficient beamforming technique has been proposed and the system level simulation is performed. The overall system was simulated for four sensors. The results are calculated by using QRD, RLS-QRD and than after applying the low pass filter on RLS - QRD .The result shows that the error has been reduced in the beamformer output when we used the RLS QRD ALGORITHM and it has been further reduced by applying the low pass filter to the results of the RLS-QRD.

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