Formalization and Petri net-Based Extension to Model Factors of State-Varying Failures for Reliability Modeling

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Summary
Failures are very often affected by many other factors besides the time. In reality, very often, these factors are neglected, leading to misleading conclusions about reliability of stochastic systems. In this paper we identify the key factors that contribute to failure rates of various stochastic systems. We further formalize them and provide solutions for their modeling using stochastic Petri nets. The latter one is a challenging task for certain classes of factors, thus we have extended the classic Petri net formalism by new elements to facilitate the accurate modeling of these types of failures, which we term as state-varying failures. We illustrate how each of the factors is modeled by Petri nets.

Key words:
reliability modeling, Petri nets, stochastic systems, state-varying failures.

1. Introduction

There has been intensive research on failure rates, including their significant impact on reliability [1, 2, 3, 4], which have been defined as such almost two decades ago [5]. Recently, Xie developed an analytical model of unavailability due to aging failures too [6]. Long time ago, it has been shown that constant failure rates are inadequate for describing systems' failures [7]. Nevertheless, they are still widely used due to the fact that the methodology for their analysis is less complex and more accurate. The popular MTTF (meantime to failure) measure is still a widely used one [8, 9], even though it has been deemed many times as inadequate [10]. We advance one step further as to claim that even time-varying failure rates are insufficient, as in many systems the rates completely change their functions based on the occurrence of some relevant events or based on the complete state of the system. For instance, if a part has been replaced by a new one that is based on a new technology, or if a mechanical part has been physically broken, then it is logical that the failure rate would increase with each time it breaks. This is what we term as a state-varying failure. To support the occurrence of such failure rates and justify the need for their formalization and modeling approach, we further summarize relevant studies and research.

According to a study of medical equipment [11], it was shown that there was a decreasing hazard of (first) failure after repair for some types of equipment. The explanation was that it is a consequence of imperfect or hazardous repair, and also, because of differing failure rates among a population of machines.

Likewise, in [12] a pizza production line is studied and it was found that most of the failures have a decreasing failure rate because proactive maintenance improves the operating conditions at different parts in the line, and a few failures have an almost constant failure rate. It was also concluded that the longer the time between two failures, the more problems accumulate, and therefore, it takes longer time to fix the latter failure. It also suggests that the more time the technicians spend fixing a failure, the more careful job they do, and therefore, the time period until the next failure is longer. This is a very interesting observation that calls for state-varying failure rates and it can be addressed using our approach.

More recently, in [13] an algorithm to evaluate substation reliability is proposed that considers operation, internal aging and external weather conditions. The authors further show that the operating conditions and failure types have a great impact on system reliability.

These are some examples that show that failures need to be described more realistically to obtain accurate and useful simulation results. Unfortunately, this has very rarely been the case.

Our goal is to provide an approach to model systems that exhibit not only time-, but also, more importantly, state-varying failure rates. For this we use the Petri nets formalism, which we extend with new elements to accommodate the state-varying failure rates. In [14] we have analyzed and described state-dependent transitions and used proxel-based simulation for their analysis. These are the types of transitions that correspond and can be used to describe state-varying failure rates. Thus, in addition to the simulation approach, this paper provides a concept of how to model this type of failure rates and what changes need to be undertaken in the standard stochastic Petri net (SPN) models to introduce them.

The paper is organized as follows. In the subsequent section we describe the state-varying failure rates, along with a formal description of the formalism of stochastic Petri nets. Further, we provide a concept for modeling state-varying failures using SPN. Next, we present an

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example model that we use to demonstrate our modeling approach. Finally, we present the conclusions of our proposed extension to the SPN modeling formalism.2. Tables, Figures and Equations

2. Preliminaries

In the following we provide the background and definition on the definition of state-varying failures and stochastic Petri nets.

2.1 State-varying Failures

It is a common observation that a failure rate cannot be simply described by one function during the entire simulation time. Even more, failure rates in reality can change not solely based on time [1], but also based on the occurrence of certain events in the system (e.g. replacing the service person by another one which fixes them in a different manner, i.e. more thoroughly, which would influence the failure rate function for failure's next occurrence). We refer to these types of failures as state-varying failure rates.
Description of failure rate functions of state-varying failure rates is a complex process and would require an algorithmic description to supplement the graphical model. To illustrate it, one such description may be:

if machine is repaired by repairman A
    then failure rate function *follows* Probability Distribution $f(a, b)$
else if machine is repaired by repairman B
    then failure rate function *follows* Probability Distribution $f(c, d)$

If we add another factor to this, i.e. the age of the machine, and then the description would change to:

if machine is repaired by repairman A
    then failure rate function *follows* Probability Distribution $f(g(t), b)$
else if machine is repaired by repairman B
    then failure rate function *follows* Probability Distribution $f(h(t), d)$

where t is the age of the machine (which can easily be exchanged to represent the number of failures or any other relevant quantity) and $g(t)$ and $f(t)$ are functions of the age of the machine. This observation is more general than the one that uses fixed failure rate functions, and as such, more realistically models the phenomenon of a machine that exhibits failures.

Obviously, these models would need a more advanced (or extended) modeling formalisms to be described. Thus, we extend stochastic Petri nets to account for the state-varying rates.

2.2 Stochastic Petri Nets

Stochastic Petri Nets are widely popular modeling formalism, which is very powerful in its expression potential [15]. In the following we provide the formal description and a basic example to describe its basic features. A stochastic Petri Net SPN is defined as:

$$\text{SPN} = (\text{P, T, A, G, m}_0)$$

where:

- $P = \{P_1, P_2, \ldots, P_n\}$, the set of places, drawn as circles
- $T = \{T_1, T_2, \ldots, T_m\}$, the set of transitions along with their distribution functions or probability values, drawn as bars
- $A = A^I \cup A^O \cup A^H$, the set of arcs, where A^O is the set of output arcs, A^I is the set of input arcs, and A^H is the set of inhibitor arcs; each arc has a multiplicity assigned to it,
- $G = \{g_1, g_2, \ldots, g_r\}$, the set of guard functions which are associated with different transitions,
- m_0 – the initial marking of the Petri net.

Each transition is defined as $T_i = (F, type)$, where $type \in \{enabling, age, immediate\}$ is the type of memory policy if it is a timed transition or "*immediate*" if the corresponding transition is an immediate one. F is a cumulative distribution function if the corresponding transition is a timed one. Immediate transitions have a constant value instead of a distribution function assigned to them, which is used for computing the probability of firing of an immediate transition if more than one are enabled at once. The sets of arcs are defined such that

$$A^O = \{a^o{}_1, a^o{}_2, \ldots, a^o{}_k\},\ A^I = \{a^i{}_1, a^i{}_2, \ldots, a^i{}_j\}, \text{ and } A^H = \{a^h{}_1, a^h{}_2, \ldots, a^h{}_i\},$$

where

$$A^H, A^I \subseteq P \times T \times \mathbb{N}, A^O \subseteq T \times P \times \mathbb{N}.$$

The multiplicity of the tracking arcs can be a real number, unlike the others, where it is a non-negative integer number. We denote by $M = \{m_0, m_1, m_2, \ldots\}$ the set of all reachable markings of the Petri net. Each marking is a vector made up of the number of tokens in each place in the Petri net. The set of all reachable markings is the discrete state space of the Petri net. The changes from one marking to another are consequences of firing of enabled transitions which move (destroy and create) tokens; thus, creating the dynamics in the Petri net. This makes the firing of a transition analogous to an event in a discrete-event system. The markings of a Petri net, viewed as nodes, and the possibilities of movement from one to another, viewed as arcs, form the reachability graph of the Petri net.

3. Modeling State-Varying Failures

According to observations, studies and research, we identify several classes of factors that state-varying failure rates are dependent on, listed as follows:

a) number of failure occurrences up to the observed point in time,

b) age of a machine up to the observed point in time,

c) duration of the last repair,

d) time between the last two failures,

e) properties of the repair facilities, introduced as additional parameters, and

f) types of failures that have occurred.

We allow a combination of a number of these factors to occur in our sample model to illustrate their effects through Petri net models. In the following, we will provide the details of the formal classification of the state-varying failures and our modeling approach. This will be further demonstrated using example models

3.1 Formal Model

The underlying discrete stochastic model that exhibits state-varying failure rates is described using a stochastic Petri net (SPN) [16]. We propose an extension to the basic description of SPN to support tracking of, what is termed as, *relevant rewards*. Relevant rewards are variables that additionally affect the distribution functions of timed transitions, besides age intensities of relevant transitions. To model the less complex types/factors of the listed state-varying failure rates' factors, we extend the basic SPN with additional places and transitions that facilitate the tracking, as shown in Figure 1 (the details of this figure are shown in Figure 4, here we only show the high-level description of the proposed extensions). However, to model the more complex relevant rewards, such as *duration of the last repair* (type (c)), we introduce a novel element that we term as *tracking variable* (TV), and it is represented by a hexagon in the SPN graphical model. TVs are connected by diamond-shape-ended arrows (on the TV's side) to the transitions for which they record the last firing time, termed as *tracking arcs*. We selected a different type of arrowhead for the tracking arcs to distinguish them from the standard input/output arcs, as they have different function, namely to assign a value to a tracking variable. E.g. in the example of "*duration of the last repair*" it would record the last random firing time that the *repair* transition was assigned to, and store it in one of the tracking variables. Tracking arc can also have algebraic function associated with it, i.e.

- type "+" would mean to add the enabling time to the current value stored in the tracking variable,
- type "reset" or "0" would mean to reset the value of the TV to zero.

To summarize, the extension that we propose to account for the state-varying failures is at both:

1) the level of SPN formalism, and
2) the actual Petri net model.

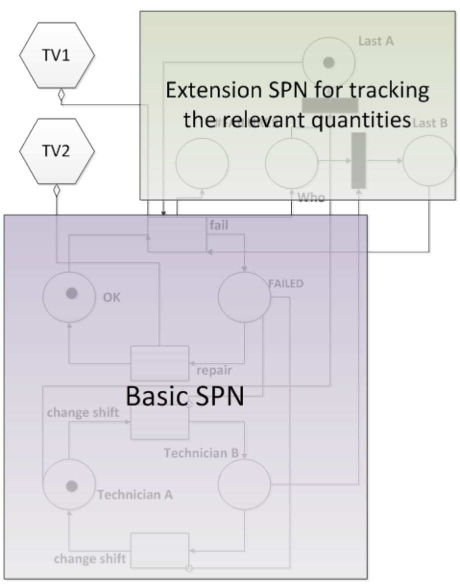

Fig. 1 Illustration of the extended SPN model

The latter (2) implies enriching the SPN by a number of extra places and transitions to support tracking of relevant rewards. As for (1) - SPN formalism extension: we add new elements: tracking variables and tracking arcs. Furthermore, in order to model the various state-varying transitions, we allow discrete states, i.e. markings, to be parameters of distribution functions that determine firings of transitions. In the following, we show by example how a SPN can be extended to allow the tracking of the various relevant quantities.
Table 1 illustrates at what level extensions need to be performed, with respect to the factors that influence a state-varying failure rate. For instance, failure rate of type

(a), i.e. one that depends on the number of failures that have occurred, can be recorded using an additional place in the SPN. For failure rate of type (b), no extension at either level is needed, as this is implicitly recorded as "simulation time". However, if the system allows for renewal types of repairs/maintenance, i.e. repairs/maintenance which renew the lifetime of a machine by either replacing it by a new one or repairing it with the same effect, an additional tracking variable would be required that would record each renewal repair/maintenance time. However, for failure rate of type (c), as previously explained, the existing formalism of SPN is inadequate and need the support of tracking variables.

Table 1. Extension level for the six state-varying failure classes

State-varying failure class	Extension Level
a) number of failure occurrences up to the observed point in time,	SPN Model
b) age of a machine up to the observed point in time,	None (implicitly recorded) or SPN Formalism (TV)
c) duration of the last repair,	SPN Formalism (TV)
d) time between the last two failures,	SPN Formalism (TV)
e) properties of the repair facilities, introduced as additional parameters,	SPN Model
f) types of failures that have occurred.	SPN Model

In the following subsection, we provide the definition of the extended SPN formalism.

3.2 Petri Net Specifications to Accommodate State-Varying Failure Rates

In the following we provide the formal definition of the extension of the SPN to account for the state-varying failure rates. With that respect, each extended stochastic Petri net *SPN* is defined as:

$$\mathrm{SPN} = (\mathrm{P, T, A, G, TV, m_0})$$

where:

- $P = \{P_1, P_2, \ldots, P_n\}$, the set of places, drawn as circles
- $T = \{T_1, T_2, \ldots, T_m\}$, the set of transitions along with their distribution functions or probability values, drawn as bars
- $A = A^I \cup A^O \cup A^H \cup A^T$, the set of arcs, where A^O is the set of output arcs, A^I is the set of input arcs, A^H is the set of inhibitor arcs, and A^T is the set of tracking arcs (connect transition to a tracking variable and are ended by a diamond-shape at the tracking variable end); each arc has a multiplicity assigned to it,
- $G = \{g_1, g_2, \ldots, g_r\}$, the set of guard functions which are associated with different transitions,
- $TV = \{TV_1, TV_2, \ldots, TV_m\}$, the set of tracking variables that store the duration of the last enabling time of a transition (drawn as hexagons),
- m_0 – the initial marking of the Petri net.

Each transition is defined as $T_i = (F, type)$, where $type \in \{enabling, age, immediate\}$ is the type of memory policy if it is a timed transition or "*immediate*" if the corresponding transition is an immediate one. F is a cumulative distribution function if the corresponding transition is a timed one. Immediate transitions have a constant value instead of a distribution function assigned to them, which is used for computing the probability of firing of an immediate transition if more than one are enabled at once. The sets of arcs are defined such that

$$A^O = \{a^o{}_1, a^o{}_2, \ldots, a^o{}_k\},\ A^I = \{a^i{}_1, a^i{}_2, \ldots, a^i{}_j\},$$

$$A^H = \{a^h{}_1, a^h{}_2, \ldots, a^h{}_i\}, \text{ and } A^T = \{a^t{}_1, a^t{}_2, \ldots, a^t{}_l\},$$

where

$$A^H, A^I \subseteq P \times T \times \mathbb{N}, A^O \subseteq T \times P \times \mathbb{N}, A^T \subseteq T \times P \times \mathbb{R}.$$

The multiplicity of the tracking arcs can be a real number, unlike the others, where it is a non-negative integer number. We denote by $M = \{m_0, m_1, m_2, \ldots\}$ the set of all reachable markings of the Petri net. Note that, different to the standard SPN description, in this case each marking is a vector made up of the number of tokens in each place in the Petri net *along with the values of the tracking variables*, $m_i = (\#P_1, \#P_2, \ldots, \#P_n, val(TV_1), val(TV_2), \ldots, val(TV_m))$. The set of all reachable markings is the discrete state space of the Petri net.
With this formal model, state-varying failure rates can be easily described and further analyzed. We anticipate building a tool for design and analysis of the extended Petri nets.

4. Examples for the Classes of Factors that Affect State-Varying Failure Rates

In this section we present simple example models to illustrate all factors that can affect state-varying failure rates. In the first subsection (4.1) we present a simple Petri net model of the example system to show the shortcomings of the standard SPN formalism in the presence of state-varying failure rates. In the following subsection (4.2) we extend the model, one-by-one with the newly defined elements, tracking variables and tracking arcs.

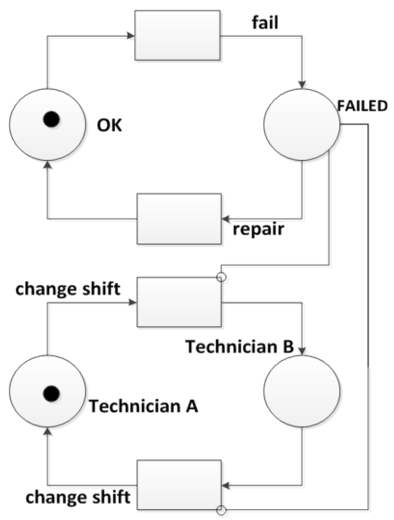

Fig. 2. Basic Petri net model of the example

4.1 Basic SPN Model

The model that we use to demonstrate our approach is a straightforward model that describes a machine that incorporates both time- and state- varying failure rates, similar to the scenarios described in Section 2.1.

Using Petri nets, the example model can be described as shown in Figure 2. It represents a machine that exhibits one of two possible states: OK and FAILED. When the machine has failed, one of the two repairmen arrives and repairs it, after what the machine's state changes to OK. We assume that changing shifts of repairmen during repair is not allowed. The reachability graph, with the following mapping format (OK, FAILED, Technician A, Technician B), is shown in Figure 3.

In the following subsection, we further complicate the model, step-by-step, to illustrate the real-world scenarios that lead to state-varying failure rates.

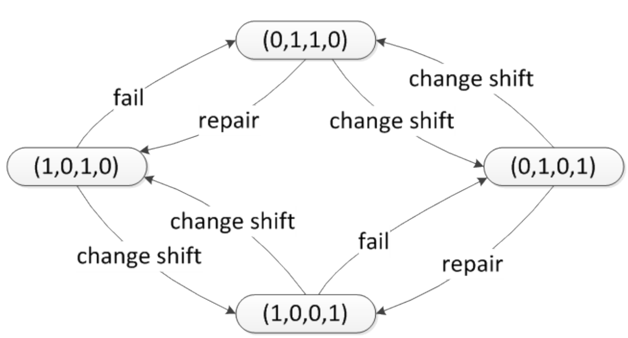

Fig. 3. Reachability graph of the simple Petri net

4.2 State-varying Failures Examples

To improve the reflection of reality in our model, we include various different scenarios that illustrate the six different classes of factors that influence failure rates. In a step-by-step fashion, we include all classes and extend our model consequently.

a) Number of failure occurrences

To illustrate this factor, we assume that the distribution function of the time to failure takes the number of failures as parameter. In the Petri net, this is achieved by extending the Petri net model itself, as shown in Figure 4.

b) Age of machine

The age of the machine in our example is implicitly modeled as the simulation time. However, if repair could be of the type of replacement of a machine, then this would need a tracking variable connected with tracking arcs of type "+" to track the age of the new machine, and a "reset" tracking arc to reset the TV when a machine has been replaced.

c) Duration of Last Repair

If we assume that the time to failure distribution function depends on the duration of the last repair, this would imply adding a tracking variable that would keep track of the duration of the last repair (TV1). This is shown in Figure 5.

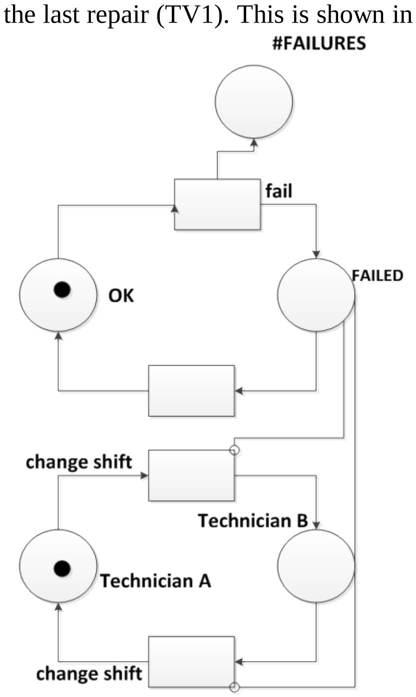

Fig. 4. Basic Petri net + factor (a) model of the example

d) Time between Last Two Failures

If we assume that the time between last two failures affects the time to failure distribution function, this would imply adding another tracking variable that would keep track of the this (TV2). This is shown in Figure 5.

e) Properties of Repair Facilities

To illustrate this factor we assume that the distribution function of the time to failure depends on the repairman that completed the last repair, as we assume that both repairmen have varying expertise. The two repairmen have different lengths of working experience, which is reflected in their repairing skills. Thus, when the machine is fixed by the Repairman A, the time to the next failure is on average longer, than when it is repaired by Repairman B. For this reason we need the extra places: Who, LastA, and LastB. Thus, this factor is easily modeled using existing Petri net elements.

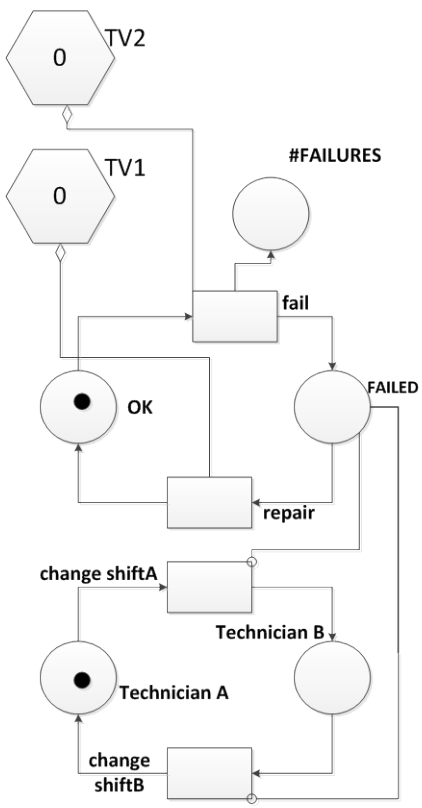

Fig. 5. Basic Petri net + factors (c,d) model of the example

e) Types of Failures

To illustrate this factor we can assume that in our model there are two different types of failures that can occur and each of them occurs with a certain probability, i.e. *p* and 1-*p*. Additionally, we need to keep track of the number of each type of failure. Thus, this can be modeled using existing Petri net elements.

4.2 Extended Petri Net Reachability Graph

In Figure 7, a small fragment of the reachability graph of the Petri net model from Figure 4, is shown. We use the following order of places/tracking variables to format the marking:

(OK, FAILED, TechnicianA, TechnicianB, #Failures, Who, LastA, LastB, TV1, TV2)

Thus, the initial marking would be (as shown in Figure 4):

$$(1, 0, 1, 0, 0, 0, 1, 0, 0, 0).$$

There are two enabled transitions in this marking: (1) - “change shiftA” and (2) - “fail”, which can correspondingly transit the SPN in one of the two markings:

1) $(1, 0, 0, 1, 0, 0, 1, 0, 0.0, 0.0)$, and
2) $(0, 1, 1, 0, 1, 0, 1, 0, 0\ .0, x)$.

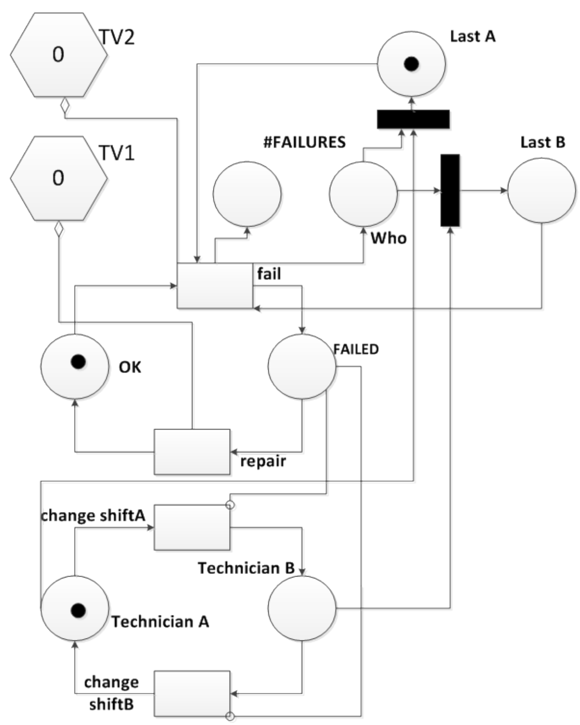

Fig. 6. Basic Petri net + factor (a) model of the example

In case (2) the TV1 value gets updated with the enabling time duration of the transition “fail”, denoted as *x*. Note that the model is an unbounded Petri net, i.e. it is practically impossible to accurately analyze it using numerical approaches. Thus, it can be effectively analyzed using either discrete-event simulation or the proxel-based simulation method [17]. To simplify the representation each state in the Petri net contains the marking of the Petri net along with the values of a selection of the relevant rewards. E.g. ((OK, A) (0, A)) would mean that the machine is OK, it is Technician A's shift, the time since the machine has spent in this state is 0, and the last repair was performed by Technician A.

Besides the repair duration probability distribution function, as shown by the Equation (1), the remaining distribution functions of the time to failure can be described as $f_{fail}(age, n_f, lastR)$, where age is the age of the machine, n_f is the number of failures and $lastR$ is the repairman that completed the last repair.

5. Summary and Outlook

We emphasized the importance of modeling failures in a more realistic manner, as this reflects their true nature more accurately. We presented a Petri net based approach to model failures that exhibit a wide range of dependencies, which are typically neglected. Their neglecting, however, can provide highly misleading results, and thus, it is imperative to avoid their oversimplification. We, furthermore, illustrated our approach and all of the elements in an example model. The proposed extension of stochastic Petri nets, by introducing tracking variables, is highly flexible in describing the complex types of dependencies that typically occur in stochastic models. We anticipate extending of the presented work to provide a tool that would facilitate reliability modeling and simulation considering state-varying failure rate functions.

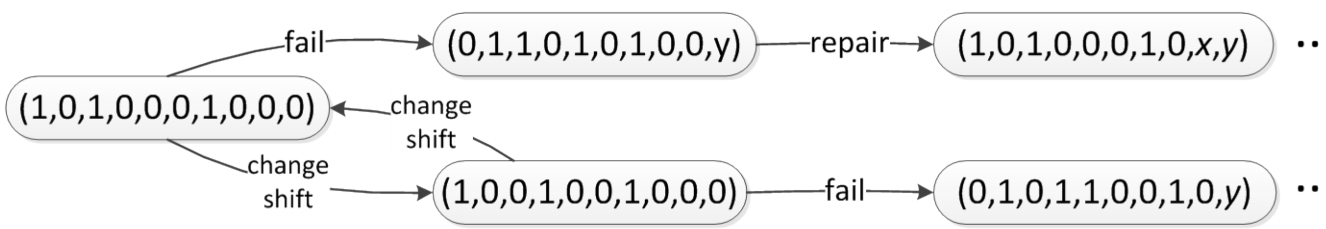

Fig. 7. State-transition diagram of the unbounded Petri net model from Figure 3

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