Separation Axioms of Fuzzy Bitopological Spaces

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Abstract
In this paper, we give and study different types of separation axioms using the remoted neighbourhood of a fuzzy point and a fuzzy set in the fuzzy supratopological Spaces (X, τ_s) which is generated by the fuzzy bitopological space (X, τ_1, τ_2). Several properties on these separation axioms are researched.

Keywords:
Fuzzy supratopological, fuzzy bitopological, remoted neighbourhood, separation axioms.

1. Introduction

A.S.Mashhoue, F.H.Kehdr and M.H.Chenim [3] defined the fuzzy bitopological space (X, τ_1, τ_2). A.Kandil, A.D.Nouh and S.A. Sheikh [4] defined the fuzzy supratopological Spaces (X, τ_s) generated by the fuzzy bitopological space (X, τ_1, τ_2). C.K.wong [7] introduced the concepts of fuzzy point and their neighourhood. But there are some drawbacks in this study. For overcoming the problem that traditional neighbourhood method was no longer effective in fuzzy topology, Liu and Pu [6] introduced the concept of the so-called Q- neighbourhood. Nearly at the same time, Wang [5] introduced the concept of the so-called remoted neighbourhood to study fuzzy topology. The latter concept has more extensive application than the former one. Nishimura [8] defined the strong neighbourhood of the fuzzy point and he defined the fuzzy filter generated by all the open neighbourhood and opened strong neighbourhood of the fuzzy point. In sections 3-6, we define two types of FPT-spaces and FPR-spaces and study some properties on them.

2. Notations and Preliminaries

All fuzzy sets on universe X will be denoted by I^X . The class of all fuzzy points in universe X will be denoted by $FP(X)$. Use Greek letters as $\mu, \eta, \delta \cdots$ etc. to denote fuzzy sets on X. Also P stands for pairwise.

A fuzzy point [5] p_σ be defined as the ordered pair $(p,\sigma) \in X \times (I - \{0\})$, where $I = [0,1]$. If $\sigma \le \mu(p)$, then $(p,\sigma) \in \mu$ and we call (p,σ) belongs to μ . Also if $\sigma < \mu(p)$, then $(p,\sigma) \in^* \mu$ and we call (p,σ) belongs strongly to μ .

Definition2.1[3]. Let X be any set and τ_1, τ_2 be two fuzzy topologies on X. The triple (X, τ_1, τ_2) is said to be a fuzzy bitopological space.

Definition2.2[2,8]. Let (X, τ)be an fuzzy topological space and $(p,\sigma) \in FP(X)$. Then (i) An fuzzy set μ s.t. $(p,\sigma) \in \mu$ is said to be an open neighbourhood of (p,σ). The fuzzy filter generated by all the open neighbourhood of (p,σ) is denoted and defined as : $V_{(p,\sigma)} = \{\mu \in I^X : \exists \rho \in \tau, (p,\sigma) \in \rho \subseteq \mu\}$. Each fuzzy set belonging to $V_{(p,\sigma)}$ is said to be an neighbourhood of (p,σ). (ii) An open fuzzy set μ s.t. $(p,\sigma) \in^* \mu$ is said to be an open *-neighbourhood of (p,σ) . The fuzzy filter generated by all the open *-neighbourhood of (p,σ) is denoted and defined as :

$$V^*_{(p,\sigma)} = \{\mu \in I^X : \exists \rho \in \tau, (p,\sigma) \in^* \rho \subseteq \mu\}.$$

Each fuzzy set belonging to $V^*_{(p,\sigma)}$ is said to be an *-neighbourhood of (p,σ).

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Definition2.3[6]. The fuzzy filter generated by all the open Q-neighbourhood of (p,σ) is denoted and defined as : $V^Q_{(p,\sigma)} = \{\mu \in I^X : \exists \rho \in \tau, (p,\sigma) q \rho \subseteq \mu\}$.
Each fuzzy set belonging to $V^Q_{(p,\sigma)}$ is said to be an Q-neighbourhood of (p,σ).

Definition2.4[6]. Let Y be a crisp subset of an fuzzy topological space (X, τ). Then (X, τ_Y)is said to be a subspace of (X, τ), where τ_Y is a fuzzy topology on Y given by $\tau_Y = \{Y \cap \mu, \mu \in \tau\}$. A subspace (X, τ_Y) is open (closed) if the crisp fuzzy set Y is open (closed) in τ .

Definition2.5[2,9]. Let (X, τ)be an fuzzy topological space and $\mu, \rho \in I^X$. A fuzzy set ρ is said to be: (i) R-neighbourhood of a fuzzy point (p,σ) if for some closed fuzzy set λ have $(p,\sigma) \notin \lambda \supseteq \rho$. (ii) R*- neighbourhood of a fuzzy point (p,σ) if for some closed fuzzy set λ have $(p,\sigma) \notin^* \lambda \supseteq \rho$.

The collection of all the R- neighbourhoods of (p,σ) (resp. R*- neighbourhoods of (p,σ)) is denoted by $R_{(p,\sigma)}$ (resp. $R^*_{(p,\sigma)}$) i.e.
$R_{(p,\sigma)} = \{\rho \in I^X : \exists \lambda \in \tau, (p,\sigma) \notin \lambda, \lambda \supseteq \rho\}$.
($R^*_{(p,\sigma)} = \{\rho \in I^X : \exists \lambda \in \tau, (p,\sigma) \notin^* \lambda, \lambda \supseteq \rho\}$).

Definition2.6[4]. Let (X, τ_1, τ_2) be fuzzy bitopological space and $\mu \in I^X$. Then:Associated with the fuzzy closure operators $\tau_1 - cl \; and \; \tau_2 - cl$ define the mapping
$C_{12} : I^X \to I^X$ as: $C_{12}(\mu) = \tau_1 - cl(\mu) \cap \tau_2 - cl(\mu)$.

Definition2.7[4]. Let (X, τ_1, τ_2) be fuzzy bitopological space. Then : the pair (X, τ_s) is said to be the associated fuzzy supratopological space of (X, τ_1, τ_2), where
$\tau_s = \{\mu \in I^X : \mu = \mu_1 \cup \mu_2, \mu_1 \in \tau_1, \mu_2 \in \tau_2\}$. $\mu \in \tau_s$ is said to be fuzzy τ_s -open or fuzzy supraopen in (X, τ_1, τ_2) and its complement is said to be fuzzy supreclosed in (X, τ_1, τ_2).

Theorem2.1[9]. Let (X, τ)be an fuzzy topological space and $\lambda \in I^X$. Then : (i) $V^*_{(p,\sigma)} = V^Q_{(p,\sigma)}$. (ii) $\lambda \in V_{(p,\sigma)}$ iff $\lambda' \in R^*_{(p,1-\sigma)}$. (iii) $\lambda \in R_{(p,\sigma)}$ iff $\lambda' \in V^*_{(p,1-\sigma)}$. (iv) $\lambda \in V^Q_{(p,\sigma)}$ iff $\lambda' \in R_{(p,\sigma)}$.

Theorem2.2[9]. If (X, τ)be an fuzzy topological space and (Y, τ_Y) is a subspace of (X, τ), $\mu \in I^Y$. We define:
(i) $R^Y_{(p,\sigma)} = \{Y \cap \rho, \rho \in R_{(p,\sigma)}\}$.
(ii) $R^{*Y}_{(p,\sigma)} = \{Y \cap \rho, \rho \in R^*_{(p,\sigma)}\}$.
Then $R^Y_{(p,\sigma)}$ (resp. $R^{*Y}_{(p,\sigma)}$) is the collection of R-neighbourhoods (resp. R*- neighbourhoods) of μ in the space (Y, τ_Y).

3. Separation axioms FP^0T_0 and FPT_0

Definition3.1. An fuzzy bitopological space (X, τ_1, τ_2) is said to be) (i) $\in FP^0T_0$ (resp. $\in FPT_0$) iff (p,σ) , (q,δ) $\in FP(X)$, $p \neq q$ implies that there exists a fuzzy τ_s -closed set μ (resp. $\mu \in \tau'_1 \cap \tau'_2$) s.t. ($\mu \in R_{(p,\sigma)}, \mu \notin R_{(q,\delta)}$) or ($\mu \in R_{(q,\delta)}, \mu \notin R_{(p,\sigma)}$).
(ii) $*FP^0T_0$ (resp. $*FPT_0$) iff (p,σ) , (q,δ) $\in FP(X)$, $p \neq q$ implies that there exists a fuzzy τ_s -closed set μ (resp. $\mu \in \tau'_1 \cap \tau'_2$) s.t. ($\mu \in R^*_{(p,\sigma)}, \mu \notin R^*_{(q,\delta)}$) or ($\mu \in R^*_{(q,\delta)}, \mu \notin R^*_{(p,\sigma)}$).

Theorem3.1. Let (X, τ_1, τ_2) be fuzzy bitopological space. Then : (i) (X, τ_1, τ_2) is $\in FP^0T_0$ (resp. $\in FPT_0$) iff (p,σ), $(q,\delta) \in FP(X)$, $p \neq q$ implies that there exists a fuzzy τ_s -open set λ (resp. $\lambda \in \tau_1 \cup \tau_2$) s.t. ($\lambda \in V^Q_{(p,\sigma)}, \lambda \notin V^Q_{(q,\delta)}$) or ($\lambda \in V^Q_{(q,\delta)}, \lambda \notin V^Q_{(p,\sigma)}$). Iff $(p,\sigma),(q,\delta) \in FP(X)$, $p \neq q$ implies that there exists a fuzzy τ_s -open set λ (resp. $\lambda \in \tau_1 \cup \tau_2$) s.t. ($\lambda \in V^*_{(p,1-\sigma)}, \lambda \notin V^*_{(q,1-\delta)}$) or ($\lambda \in V^*_{(q,1-\delta)}, \lambda \notin V^*_{(p,1-\sigma)}$).
(ii)(X, τ_1, τ_2)is $*FP^0T_0$ (resp. $*FPT_0$) iff (p,σ) , $(q,\delta) \in FP(X)$, $p \neq q$ implies that there exists a fuzzy τ_s -open set λ (resp. $\lambda \in \tau_1 \cup \tau_2$) s.t.
($\lambda \in V_{(p,1-\sigma)}, \lambda \notin V_{(q,1-\delta)}$) or ($\lambda \in V_{(q,1-\delta)}, \lambda \notin V_{(p,1-\sigma)}$).
Proof. (i) Follows from Theorem 2.1 (iii)(iv) and by putting $\mu' = \lambda$ in Definition 3.1(i).
(ii) Follows from Theorem 2.1 (ii) and by putting $\mu' = \lambda$ in Definition 3.1(ii).

Theorem3.2. Let (X, τ_1, τ_2) be fuzzy bitopological space. Then : (i) $\in FP^0T_0 \Rightarrow \in FPT_0$ (ii) $*FP^0T_0 \Rightarrow *FPT_0$.

Proof. We prove part (i)and proof of the other part is similar. Suppose (X, τ_1, τ_2) is $\in FP^0T_0$. Let (p,σ), $(q,\delta) \in FP(X)$, $p \neq q$. Since (X, τ_1, τ_2) is $\in FP^0T_0$, then $\exists \mu \in \tau_s'$ s.t. ($\mu \in R_{(p,\sigma)}, \mu \notin R_{(q,\delta)}$) or ($\mu \in R_{(q,\delta)}, \mu \notin R_{(p,\sigma)}$). But $\tau_s' \subseteq \tau_1' \cap \tau_2'$, then $\exists \mu \in \tau_1' \cap \tau_2'$ s.t. ($\mu \in R_{(p,\sigma)}, \mu \notin R_{(q,\delta)}$) or ($\mu \in R_{(q,\delta)}, \mu \notin R_{(p,\sigma)}$). Hence (X, τ_1, τ_2) is $\in FPT_0$.

Theorem3.3. A subspace of a $\in FPT_0$ (resp. $*FPT_0$) is $\in FPT_0$ (resp. $*FPT_0$).

Proof. We prove the case $\in FPT_0$, and the proof of the other case is similar. Suppose (Y, τ_1^0, τ_2^0) is a subspace of a $\in FP^0T_0$. Let $(p,\sigma),(q,\delta) \in FP(Y)$, $p \neq q$. Then $(p,\sigma),(q,\delta) \in FP(X)$. Since (X, τ_1, τ_2) is $\in FPT_0$, then $\exists \mu \in \tau_1' \cap \tau_2'$ s.t. ($\mu \in R_{(p,\sigma)}, \mu \notin R_{(q,\delta)}$) or ($\mu \in R_{(q,\delta)}, \mu \notin R_{(p,\sigma)}$). So , $\exists \mu^0 = \mu \cap Y \in \tau_1^{0'} \cap \tau_2^{0'}$ s.t. ($\mu^0 \in R^Y_{(p,\sigma)}, \mu^0 \notin R^Y_{(q,\delta)}$) or ($\mu^0 \in R^Y_{(q,\delta)}, \mu^0 \notin R^Y_{(p,\sigma)}$), where $R^Y_{(p,\sigma)} = \{Y \cap \rho : \rho \in R_{(p,\sigma)}\}$. Hence (Y, τ_1^0, τ_2^0) is $\in FPT_0$.

Theorem3.4. A subspace of a $\in FP^0T_0$ (resp. $*FP^0T_0$) is $\in FP^0T_0$ (resp. $*FP^0T_0$).

Proof. It is similar to that of Theorem3.3.

4. Separation axioms FP^0T_1 and FPT_1

Definition4.1. An fuzzy bitopological space (X, τ_1, τ_2) is said to be) (i) $\in FP^0T_1$ (resp. $\in FPT_1$) iff (p,σ), $(q,\delta) \in FP(X)$, $p \neq q$ implies that there exists a fuzzy τ_s -closed set μ, λ (resp. $\mu, \lambda \in \tau_1' \cap \tau_2'$) s.t. ($\mu \in R_{(p,\sigma)}, \mu \notin R_{(q,\delta)}$) and ($\lambda \in R_{(q,\delta)}, \lambda \notin R_{(p,\sigma)}$).

(ii) $*FP^0T_1$ (resp. $*FPT_1$) iff $(p,\sigma),(q,\delta) \in FP(X)$, $p \neq q$ implies that there exists a fuzzy τ_s -closed set μ, λ (resp. $\mu, \lambda \in \tau_1' \cap \tau_2'$) s.t.($\mu \in R^*_{(p,\sigma)}, \mu \notin R^*_{(q,\delta)}$) and ($\lambda \in R^*_{(q,\delta)}, \lambda \notin R^*_{(p,\sigma)}$).

Theorem4.1. Let (X, τ_1, τ_2) be fuzzy bitopological space. Then : (i) (X, τ_1, τ_2) is $\in FP^0T_1$ (resp. $\in FPT_1$) iff (p,σ), $(q,\delta) \in FP(X)$, $p \neq q$ implies that there exists a fuzzy τ_s -open set η, ρ (resp. $\eta, \rho \in \tau_1 \cup \tau_2$) s.t. ($\eta \in V^Q_{(p,\sigma)}, \eta \notin V^Q_{(q,\delta)}$) and ($\rho \in V^Q_{(q,\delta)}, \rho \notin V^Q_{(p,\sigma)}$). Iff $(p,\sigma),(q,\delta) \in FP(X)$, $p \neq q$ implies that there exists a fuzzy τ_s -open set η, ρ (resp. $\eta, \rho \in \tau_1 \cup \tau_2$) s.t. ($\eta \in V^*_{(p,1-\sigma)}, \eta \notin V^*_{(q,1-\delta)}$)and($\rho \in V^*_{(q,1-\delta)}, \rho \notin V^*_{(p,1-\sigma)}$).

(ii)(X, τ_1, τ_2)is $*FP^0T_1$ (resp. $*FPT_1$) iff $(p,\sigma),(q,\delta) \in FP(X)$, $p \neq q$ implies that there exists a fuzzy τ_s -open set η, ρ (resp. $\eta, \rho \in \tau_1 \cup \tau_2$) s.t. ($\eta \in V_{(p,1-\sigma)}, \eta \notin V_{(q,1-\delta)}$) and ($\rho \in V_{(q,1-\delta)}, \rho \notin V_{(p,1-\sigma)}$).

Proof. (i) Follows from Theorem 2.1 (iii)(iv) and by putting $\mu' = \eta,\ \lambda' = \rho$ in Definition4.1(i).

(ii) Follows from Theorem 2.1 (ii) and by putting $\mu' = \eta,\ \lambda' = \rho$ in Definition4.1(ii).

Theorem4.2. Let (X, τ_1, τ_2) be fuzzy bitopological space. Then : (i) $\in FP^0T_1 \Rightarrow \in FPT_1$ (ii) $*FP^0T_1 \Rightarrow *FPT_1$.

Proof. It follows from the face that $\tau_s' \subseteq \tau_1' \cap \tau_2'$.

Theorem4.3. Let (X, τ_1, τ_2) be fuzzy bitopological space. Then :(i) $\in FP^0T_1$ (resp. $\in FPT_1$) $\Rightarrow \in FP^0T_0$ (resp. $\in FPT_0$).

(ii) $*FP^0T_1$ (resp. $*FPT_1$) $\Rightarrow *FP^0T_0$ (resp. $*FPT_0$).

Proof. Obvious.

Theorem4.4. A subspace of a $\in FPT_1$ (resp. $*FPT_1$) is $\in FPT_1$ (resp. $*FPT_1$).

Proof. We prove the case $\in FPT_1$, and the proof of the other case is similar. Suppose (Y, τ_1^0, τ_2^0) is a subspace of a $\in FPT_1$. Let $(p,\sigma),(q,\delta) \in FP(Y)$, $p \neq q$. Then (p,σ), $(q,\delta) \in FP(X)$. Since (X, τ_1, τ_2) is $\in FPT_1$, then: $\exists \mu, \lambda \in \tau_1' \cap \tau_2'$ s.t. ($\mu \in R_{(p,\sigma)}, \mu \notin R_{(q,\delta)}$) and ($\lambda \in R_{(q,\delta)}, \lambda \notin R_{(p,\sigma)}$). So , $\exists \mu^0 = \mu \cap Y \in \tau_1^{0'} \cap \tau_2^{0'}$ and $\exists \lambda^0 = \lambda \cap Y \in \tau_1^{0'} \cap \tau_2^{0'}$ s.t.($\mu^0 \in R^Y_{(p,\sigma)}, \mu^0 \notin R^Y_{(q,\delta)}$) and ($\lambda^0 \in R^Y_{(q,\delta)}, \lambda^0 \notin R^Y_{(p,\sigma)}$). Hence (Y, τ_1^0, τ_2^0) is $\in FPT_1$.

Theorem4.5. A subspace of a $\in FP^0T_1$ (resp. $*FP^0T_1$) is $\in FP^0T_1$ (resp. $*FP^0T_1$).

Proof. It is similar to that of Theorem4.4.

5. Separation axioms FP^0T_2 and FPT_2

Definition5.1. An fuzzy bitopological space (X, τ_1, τ_2) is said to be) (i) $\in FP^0T_2$ (resp. $\in FPT_2$) iff (p,σ) ,

$(q,\delta) \in FP(X)$, $p \neq q$ implies that there exists a fuzzy τ_s -closed set μ,λ (resp. $\mu,\lambda \in \tau_1' \cap \tau_2'$) where $\mu \in R_{(p,\sigma)}, \lambda \in R_{(q,\delta)}$ s.t. $\lambda \cup \mu = 1_X$. (ii) $*FP^0T_2$ (resp. $*FPT_2$) iff (p,σ) , $(q,\delta) \in FP(X)$, $p \neq q$ implies that there exists a fuzzy τ_s -closed set μ,λ (resp. $\mu,\lambda \in \tau_1' \cap \tau_2'$) where $\mu \in R^*_{(p,\sigma)}, \lambda \in R^*_{(q,\delta)}$ s.t. $\lambda \cup \mu = 1_X$.

Theorem5.1. Let (X, τ_1,τ_2) be fuzzy bitopological space. Then : (i) (X, τ_1,τ_2) is $\in FP^0T_2$ (resp. $\in FPT_2$) iff (p,σ), $(q,\delta) \in FP(X)$, $p \neq q$ implies that there exists a fuzzy τ_s -open set η,ρ (resp. $\eta,\rho \in \tau_1 \cup \tau_2$) where $\eta \in V^Q_{(p,\sigma)}, \rho \in V^Q_{(q,\delta)}$ and $\eta \cap \rho = 0_{X'}$. Iff $(p,\sigma),(q,\delta) \in FP(X)$, $p \neq q$ implies that there exists a fuzzy τ_s -open set η,ρ (resp. $\eta,\rho \in \tau_1 \cup \tau_2$) where $\eta \in V^*_{(p,1-\sigma)}, \rho \in V^*_{(q,1-\delta)}$ and $\delta \cap \rho = 0_{X'}$.
(ii) (X, τ_1,τ_2)is $*FP^0T_2$ (resp. $*FPT_2$) iff $(p,\sigma),(q,\delta) \in FP(X)$, $p \neq q$ implies that there exists a fuzzy τ_s -open set η,ρ (resp. $\eta,\rho \in \tau_1 \cup \tau_2$) where $\eta \in V_{(p,1-\sigma)}, \rho \in V_{(q,1-\delta)}$ and $\eta \cap \rho = 0_{X'}$.
Proof. It is similar to that of Theorem4.1

Theorem5.2. Let (X, τ_1,τ_2) be fuzzy bitopological space. Then : (i) $\in FP^0T_2 \Rightarrow \in FPT_2$ (ii) $*FP^0T_2 \Rightarrow *FPT_2$.
Proof. It follows from the face that $\tau_s' \subseteq \tau_1' \cap \tau_2'$.

Theorem5.3. Let (X, τ_1,τ_2) be fuzzy bitopological space. Then :(i) $\in FP^0T_2$ (resp. $\in FPT_2$) $\Rightarrow \in FP^0T_1$ (resp. $\in FPT_1$). (ii) $*FP^0T_2$ (resp. $*FPT_2$) $\Rightarrow$ $*FP^0T_1$ (resp. $*FPT_1$).
Proof. We prove the case (i) and the proof of the other case is similar. Suppose (X, τ_1,τ_2) is $\in FPT_2$, Let (p,σ) , $(q,\delta) \in FP(X)$, $p \neq q$. then: $\exists \mu,\lambda \in \tau_1' \cap \tau_2'$ where $\mu \in R_{(p,\sigma)}, \lambda \in R_{(q,\delta)}$ s.t. $\lambda \cup \mu = 1_X$. Since $\mu \in R_{(p,\sigma)}$, then $(p,\sigma) \notin \mu$ and since $\lambda \cup \mu = 1_X$, then $(p,\sigma) \in \lambda$. So $\lambda \notin R_{(p,\sigma)}$. Similar we have $\mu \notin R_{(q,\delta)}$. Hence (X, τ_1,τ_2) is $\in FPT_1$.

Theorem5.4. A subspace of a $\in FPT_2$ (resp. $*FPT_2$) is $\in FPT_2$ (resp. $*FPT_2$).
Proof. We prove the case $\in FPT_2$, and the proof of the other case is similar. Suppose (Y, τ_1^0,τ_2^0) is a subspace of a $\in FPT_2$. Let (p,σ) , $(q,\delta) \in FP(Y)$, $p \neq q$. Then $(p,\sigma),(q,\delta) \in FP(X)$. Since (X, τ_1,τ_2) is $\in FPT_2$, then: $\exists \mu,\lambda \in \tau_1' \cap \tau_2'$ where $\mu \in R_{(p,\sigma)}, \lambda \in R_{(q,\delta)}$ s.t. $\lambda \cup \mu = 1_X$. So , $\exists \mu^0 = \mu \cap Y \in \tau_1^{0'} \cap \tau_2^{0'}$ and $\exists \lambda^0 = \lambda \cap Y \in \tau_1^{0'} \cap \tau_2^{0'}$ s.t. $\lambda^0 \cup \mu^0 = 1_Y$ Hence (Y, τ_1^0,τ_2^0) is $\in FPT_2$.

Theorem5.5. A subspace of a $\in FP^0T_2$ (resp. $*FP^0T_2$) is $\in FP^0T_2$ (resp. $*FP^0T_2$).
Proof. It is similar to that of Theorem5.4.

6. Separation axioms FP^0R_0 and FPR_0

Definition6.1. An fuzzy bitopological space (X, τ_1,τ_2) is said to be (i) $\in FP^0R_0$ (resp. $\in FPR_0$)iff τ_i -cl((p,σ)) $\bar{q}\mu$, $i = 1,2$ (resp. $C_{12}((p,\sigma))\ \bar{q}\mu$) $\forall \mu \in R_{(p,1-\sigma)}$.
(ii) $*FP^0R_0$ (resp. $*FPR_0$) iff τ_i -cl((p,σ)) $\bar{q}\mu$, $i = 1,2$ (resp. $C_{12}((p,\sigma))\ \bar{q}\mu$) $\forall \mu \in R^*_{(p,1-\sigma)}$

Theorem6.1. Let (X, τ_1,τ_2) be fuzzy bitopological space. Then : (i) (X, τ_1,τ_2) is $\in FP^0R_0$ (resp. $\in FPR_0$) iff τ_i -cl((p,σ)) $\subseteq \lambda$, $i = 1,2$ (resp. $C_{12}((p,\sigma)) \subseteq \lambda$) $\forall \lambda \in V^Q_{(p,1-\sigma)}$. Iff τ_i -cl((p,σ)) $\subseteq \lambda$, $i = 1,2$ (resp. $C_{12}((p,\sigma)) \subseteq \lambda$) $\forall \lambda \in V^*_{(p,\sigma)}$. (ii) (X, τ_1,τ_2)is $*FP^0R_0$ (resp. $*FPR_0$) iff τ_i -cl((p,σ)) $\subseteq \lambda$, $i = 1,2$ (resp. $C_{12}((p,\sigma)) \subseteq \lambda$) $\forall \lambda \in V_{(p,\sigma)}$.
Proof. It is similar to that of Theorem3.1.

Theorem6.2. Let (X, τ_1,τ_2) be fuzzy bitopological space. Then : (i) $\in FP^0R_0 \Rightarrow \in FPR_0$ (ii) $*FP^0R_0 \Rightarrow *FPR_0$.
Proof. We prove part (i)and proof of the other part is similar. Suppose (X, τ_1,τ_2) is $\in FP^0R_0$ .Let $\mu \in R_{(p,1-\sigma)}$, then τ_i -cl((p,σ)) $\bar{q}\mu$, $i = 1,2$ .Thus $C_{12}((p,\sigma))\ \bar{q}\mu$. Hence (X, τ_1,τ_2) is $\in FPR_0$.

Theorem6.3. A subspace of a $\in FPR_0$ (resp. $*FPR_0$) is $\in FPR_0$ (resp. $*FPR_0$).
Proof. We prove the case $\in FPR_0$, and the proof of the other case is similar. Suppose (Y, τ_1^0,τ_2^0) is a subspace of

an $\in FPR_0$. Let $\mu \in R^{Y}_{(p,1-\sigma)}$, then $\mu \in R_{(p,1-\sigma)}$. Since (X, τ_1, τ_2) is $\in FPR_0$, then $C_{12}((p,\sigma))\ \bar{q}\mu$. So $C^0_{12}((p,\sigma))$ $\bar{q}\mu$, where $C^0_{12}((p,\sigma)) = C_{12}((p,\sigma)) \bigcap Y$ and $R^{Y}_{(p,1-\sigma)} = \{Y \bigcap \rho : \rho \in R_{(p,1-\sigma)}\}$. Hence (Y, τ^0_1, τ^0_2) is $\in FPR_0$.

Theorem6.4. A subspace of a $\in FP^0R_0$ (resp. $*FP^0R_0$) is $\in FP^0R_0$ (resp. $*FP^0R_0$).
Proof. It is similar to that of Theorem6.3.

Similar with 4 and 5, we can further studySeparation axioms FP^0R_1 and FPR_1 (resp. FP^0R_2 and FPR_2). Omitted here.

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