Simulation of Ultra Wideband Miscrostrip Antenna Based on Laguerre-Finite-Difference Time-Domain (LFDTD) Algorithm

NGENDAKUMNA Gaspard[1] and Nabih EL OUAZZANI[2]

Sidi Mohamed Ben Abdellah University – Faculty of Sciences and Technology – Fès
Signals, Systems and Components Laboratory –EMC Team
FST–Fès BP 2202–Route d'Imouzzer–Fès–Maroc

Summary
The use of Weighted Laguerre Polynomials to overcome Courant-Friedrich-Levy (CFL) stability condition leads to unconditionally Finite-Difference Time-Domain ($FDTD$) method. This formulation results in a huge sparse matrix equation which needs important memory storage when modelling Three Dimensional ($3D$) electromagnetic problems with fine meshes. Recently, a new efficient algorithm which consists in using factorization splitting scheme based on two sub-steps to decompose the previous huge sparse matrix equation has been proposed. In this paper, we develop a New Node Numbering Scheme in order to optimize LU Decomposition and to overcome the memory storage limitation. Therefore, the programming code is partially modified and finally used to model rectangular patch antenna which is fed by uniform or non uniform microstrip Lines, in order to achieve the Ultra Wideband Matching of the rectangular patch antenna. The results obtained from the Implementation of Laguerre Finite-Difference Time-Domain ($LFDTD$) method are compared to those of the conventional $FDTD$ method.

Key words:
$FDTD$ method, Laguerre Polynomials, $LFDTD$ formulation, microstrip antenna and microstrip lines, node numbering scheme.

1. Introduction

Numerical formulations to analyse electromagnetic problems have been chosen since they have many advantages. Among these formulations, the Finite-Difference Time-Domain ($FDTD$) method (known as conventional $FDTD$ method) still taking the main place and it is easy to implement [1]-[2]. This Marching-On in Degree formulation is conditionally stable and the Courant-Friedrich-Levy (CFL) stability condition, which is its bottleneck, depends on the smallest cell size. Therefore, for applications requiring fine meshes, the time step is small and the CPU Computation Time increases too. Over the past decades, many modifications have been proposed in order to overcome the CFL stability condition [3]-[8].

The Alternating-Direction Implicit (ADI) formulation has been proposed first [3]. This formulation is unconditionally stable but suffers from the numerical dispersion error when the time step is greater than the maximum time step in conventional $FDTD$ method. Few years later, Crank-Nicolson (CN) and Crank-Nicolson-Douglas-Gunn ($CNDG$) formulations [9]-[11] were proposed and found more efficient and accurate than the ADI formulation. These formulations involve inverse matrix operations which need more memory storage and the problem of memory limitation is considered as the main bottleneck in unconditionally stable formulations.

Recently, significant progresses have been made to optimize the resolution of the huge sparse matrix equation [14]-[15][18]-[19]. Simulation using weighted Laguerre polynomials was proofed to be much faster than conventional $FDTD$ formulation [7], [15]. Laguerre formulation is suitable for one and two dimension problems but presents long time limitation [13]-[14] [19]. Many improvements such as the Balanced Laguerre Polynomials [19], the equivalent circuit in the $FDTD$ grid [14] and an efficient algorithm to implement the huge sparse matrix equation [18] have been proposed. The use of Factorization splitting scheme to resolve the huge matrix equation into two sub-steps [9]-[11] reduces the memory storage limitation. In this paper, we propose a new node numbering scheme leading to optimum tri-diagonal matrices, suitable for LU Decomposition. Therefore, the incorporation into the code of the Perfectly Matched Layers (PML) Absorbing Boundary Conditions (ABCs)[20] for truncating $FDTD$ grids to model Rectangular Patch Antenna, fed by either Uniform or Non Uniform microstrip Lines, is carried out.

2. Laguerre Formulation

Numerical methods based upon Laguerre Polynomials combined with $FDTD$ algorithm have been proposed first in

Manuscript received November 5, 2013
Manuscript revised November 20, 2013

[7]. Laguerre polynomials of order p exist for (t ≥ 0) and satisfy a recursive relation (Equation 1-3):

$$L_0(t) = 1 \quad (1)$$

$$L_1(t) = 1 - t \quad (2)$$

$$pL_p(t) = (2p - 1 - t)L_{p-1}(t) - (p-1)\, L_{p-2}(t) \;\; for\; p \geq 2 \quad (3)$$

Laguerre Polynomials have been chosen among others [19] because they are causal and defined from t = 0 to ∞ (Equation 4). They present orthogonal property with respect to the weighted or exponential function $e^{-\frac{t}{2}}$ (Equation 6):

$$\int_0^{+\infty} e^{-\frac{\bar{t}}{2}}\, L_p(\bar{t})\, e^{-\frac{\bar{t}}{2}}\, L_q(\bar{t})\, d\bar{t} = \delta_{pq} \quad (4)$$

where the scaled time $\bar{t} = S * t \geq 0$ represents the real time multiply by the time scale factor s. In fact, the weighted Laguerre Polynomials or Laguerre Basis functions result in the product of Laguerre polynomials and the weighted functions and they present also orthogonal property according to weighted function.

$$\varphi_p(\bar{t}) = e^{-\frac{\bar{t}}{2}}\, L_p(\bar{t}) \quad (5)$$

$$\int_0^{+\infty} \varphi_p(\bar{t})\, \varphi_q(\bar{t})\; d\bar{t} = \delta_{pq} \quad (6)$$

where:

$$\delta_{pq} = \begin{cases} 1 & if\;\; p = q \\ 0 & if\;\; p \neq q \end{cases}$$

The main objective is to form an orthonormal set Laguerre basis functions $(\varphi_0, \varphi_1, \varphi_2, \ldots, \varphi_{Nl})$ where N_l represents the number of Laguerre basis functions. In the Laguerre Domain, each time-domain quantity F (t) is expressed in terms of basis functions [7], as follow:

$$F(t) = \sum_{p=0}^{Nl} F(p)\;\; \varphi_p(\bar{t}) \quad (7)$$

$$\frac{\partial F(t)}{\partial t} = S\left(0.5F(p) + \sum_{k=0}^{p-1} F(k)\right) \quad (8)$$

On the other hand, Laguerre polynomials are involved in order to eliminate the time dependence in the Maxwell's equations. So, Laguerre coefficients F (p) which are needed to update different equations can be calculated by using (Equation 9):

$$F(p) = \int_0^{+\infty} F(\bar{t})\; \varphi_p(\bar{t})\; d\bar{t} \quad (9)$$

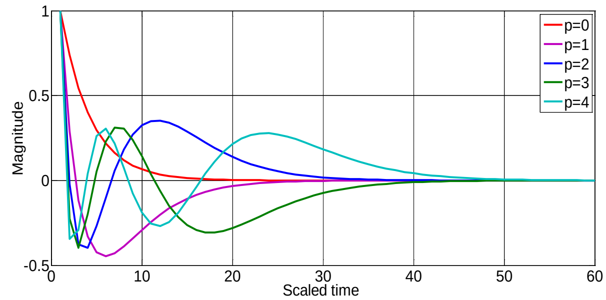

Figure 1: Laguerre basis functions for p = 0 – 4

As example, we plot (Figure 1) basis functions for p ranging from 0 to 4. We notice that basis functions decay to zero as the time increases. Therefore, each temporal quantity that is spanned by these functions decay also to zero.

3. Laguerre-FDTD Formulation

3.1. Maxwell's equations in Laguerre Domain

Consider Maxwell's equations with PML ABCs according to the general expressions:

$$\left(\varepsilon\frac{\partial}{\partial t} + \sigma_x\right) E_x(r,t) = \frac{\partial H_z(r,t)}{\partial y} - \frac{\partial H_y(r,t)}{\partial z} - J_x(r,t) \quad (10)$$

$$\left(\varepsilon\frac{\partial}{\partial t} + \sigma_y\right) E_y(r,t) = \frac{\partial H_x(r,t)}{\partial z} - \frac{\partial H_z(r,t)}{\partial x} - J_y(r,t) \quad (11)$$

$$\left(\varepsilon\frac{\partial}{\partial t} + \sigma_z\right) E_z(r,t) = \frac{\partial H_y(r,t)}{\partial x} - \frac{\partial H_x(r,t)}{\partial y} - J_z(r,t) \quad (12)$$

$$\left(\mu\frac{\partial}{\partial t} + \sigma_x^*\right) H_x(r,t) = \frac{\partial E_y(r,t)}{\partial z} - \frac{\partial E_z(r,t)}{\partial y} \quad (13)$$

$$\left(\mu\frac{\partial}{\partial t} + \sigma_y^*\right) H_y(r,t) = \frac{\partial E_z(r,t)}{\partial x} - \frac{\partial E_x(r,t)}{\partial z} \quad (14)$$

$$\left(\mu\frac{\partial}{\partial t} + \sigma_z^*\right) H_z(r,t) = \frac{\partial E_x(r,t)}{\partial y} - \frac{\partial E_y(r,t)}{\partial x} \quad (15)$$

where:

$$\begin{cases} \sigma_x = \sigma_{xy} + \sigma_{xz} \;\; and \;\; \rho_x = \rho_{xy} + \rho_{xz} \\ \sigma_y = \sigma_{yx} + \sigma_{yz} \;\; and \;\; \rho_y = \rho_{yx} + \rho_{yz} \\ \sigma_z = \sigma_{zx} + \sigma_{zy} \;\; and \;\; \rho_z = \rho_{zx} + \rho_{zy} \end{cases}$$

and $\varepsilon, \mu, \sigma_i\; and\; \rho_i$ represent permittivity, permeability, electric and magnetic losses in direction i (with $i = x, y, z$), respectively. Using Laguerre basis functions and Laguerre Coefficients expressions, Maxwell's equations with PML ABCs can be expressed in terms of Laguerre coefficients (Equation 17–21), as follow:

$$E_x(x,y,z,t) = \sum_{p=0}^{N_l} E_x^p(x,y,z)\varphi_p(\bar{t}) \quad (16)$$

$$E_y(x,y,z,t) = \sum_{p=0}^{N_l} E_y^p(x,y,z)\varphi_p(\bar{t}) \quad (17)$$

$$E_z(x,y,z,t) = \sum_{p=0}^{N_l} E_z^p(x,y,z)\varphi_p(\bar{t}) \quad (18)$$

$$H_x(x,y,z,t) = \sum_{p=0}^{N_l} H_x^p(x,y,z)\varphi_p(\bar{t}) \tag{19}$$

$$H_y(x,y,z,t) = \sum_{p=0}^{N_l} H_y^p(x,y,z)\varphi_p(\bar{t}) \tag{20}$$

$$H_z(x,y,z,t) = \sum_{p=0}^{N_l} H_z^p(x,y,z)\varphi_p(\bar{t}) \tag{21}$$

where N_l represents the basis function number and directly depends on time duration and frequency of the excitation. In fact, by using (Equations 17-21) and Galerkin's testing procedure, Maxwell's equations in Laguerre Domain can be expended as follow:

$$E_x^p = a_x\left[D_y H_z^p - D_z H_y^p - J_x^p\right] - a_{1x}\sum_{k=0}^{p-1} E_x^k \tag{22}$$

$$E_y^p = a_x\left[D_z H_x^p - D_x H_z^p - J_y^p\right] - a_{1y}\sum_{k=0}^{p-1} E_y^k \tag{23}$$

$$E_z^p = a_x\left[D_x H_y^p - D_y H_x^p - J_z^p\right] - a_{1z}\sum_{k=0}^{p-1} E_z^k \tag{24}$$

$$H_x^p = b_x\left[D_z E_y^p - D_y E_z^p\right] - b_{1x}\sum_{k=0}^{p-1} H_x^k \tag{25}$$

$$H_y^p = b_x\left[D_x E_z^p - D_z E_x^p\right] - b_{1y}\sum_{k=0}^{p-1} H_y^k \tag{26}$$

$$H_z^p = b_x\left[D_y E_x^p - D_x E_y^p\right] - b_{1z}\sum_{k=0}^{p-1} H_z^k \tag{27}$$

where:

$$\begin{cases} a_x = \dfrac{S\,\varepsilon}{2} + \sigma_x \text{ and } b_x = \dfrac{S\,\mu}{2} + \rho_x \\ a_y = \dfrac{S\,\varepsilon}{2} + \sigma_y \text{ and } b_y = \dfrac{S\,\mu}{2} + \rho_y \\ a_z = \dfrac{S\,\varepsilon}{2} + \sigma_z \text{ and } b_z = \dfrac{S\,\mu}{2} + \rho_z \end{cases}$$

$$\begin{cases} a_{1x} = S\,\varepsilon\,a_x \text{ and } b_{1x} = S\,\mu\,b_x \\ a_{1y} = S\,\varepsilon\,a_y \text{ and } b_1 = S\,\mu\,b_y \\ a_{1z} = S\,\varepsilon\,a_z \text{ and } b_1 = S\,\mu\,b_z \end{cases}$$

Unlike Marching-In-on Degree formulation, electric field components depend on the magnetic components at the same order p. By substituting magnetic field expressions into electric field expressions, we ensure the conventional Laguerre-FDTD formulation where the electric field in Laguerre domain is function of the magnetic field with order lower than p (Equation 28).

$$[I - D_H D_E]E^p = D_H V_H^{p-1} + V_E^{p-1} + J_E^p \tag{28}$$

where:

$$E^p = \begin{bmatrix} E_x^p \\ E_y^p \\ E_z^p \end{bmatrix};\ D_H = \begin{bmatrix} 0 & -a_x D_z & a_x D_y \\ a_y D_z & 0 & -a_y D_x \\ -a_z D_y & a_z D_x & 0 \end{bmatrix}$$

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix};\ H^p = \begin{bmatrix} H_x^p \\ H_y^p \\ H_z^p \end{bmatrix}$$

$$D_E = \begin{bmatrix} 0 & a_x D_z & -a_x D_y \\ -a_y D_z & 0 & a_y D_x \\ a_z D_y & -a_z D_x & 0 \end{bmatrix};\ J_E^p = \begin{bmatrix} -a_x J_x^p \\ -a_y J_y^p \\ -a_z J_z^p \end{bmatrix}$$

$$V_H^{p-1} = \begin{bmatrix} -b_{1x}\sum_{k=0}^{p-1} H_x^k & -b_{1y}\sum_{k=0}^{p-1} H_y^k & -b_{1z}\sum_{k=0}^{p-1} H_z^k \end{bmatrix}^T$$

$$V_E^{p-1} = \begin{bmatrix} -a_{1x}\sum_{k=0}^{p-1} E_x^k & -a_{1y}\sum_{k=0}^{p-1} E_y^k & -a_{1z}\sum_{k=0}^{p-1} E_z^k \end{bmatrix}^T$$

3.2. Finite-Difference scheme

The decomposition of the sparse matrix equation [18] leads to three general matrix equations, as given bellow:

$$\left[1 - \left(a_x\, b_y D_{2z} + a_x b_z D_{2y}\right)\right]E_x^p + a_x b_z D_x D_y E_y^p + a_x b_y D_x D_z E_z^p = a_x b_{1y}\sum_{k=0}^{p-1} H_y^k - a_x b_{1z}\sum_{k=0}^{p-1} H_z^k - a_{1x}\sum_{k=0}^{p-1} E_x^k - J_x^p \tag{29}$$

$$\left[1 - \left(a_y\, b_x D_{2z} + a_y b_z D_{2x}\right)\right]E_y^p + a_y b_z D_x D_y E_x^p + a_y b_x D_x D_z E_z^p = a_y b_{1x}\sum_{k=0}^{p-1} H_z^k - a_y b_{1x}\sum_{k=0}^{p-1} H_x^k - a_{1y}\sum_{k=0}^{p-1} E_y^k - J_y^p \tag{30}$$

$$\left[1 - \left(a_z\, b_y D_{2x} + a_z b_x D_{2y}\right)\right]E_z^p + a_z b_y D_x D_z E_x^p + a_z b_x D_z D_y E_y^p = a_z b_{1y}\sum_{k=0}^{p-1} H_x^k - a_z b_{1z}\sum_{k=0}^{p-1} H_y^k - a_{1z}\sum_{k=0}^{p-1} E_z^k - J_z^p \tag{31}$$

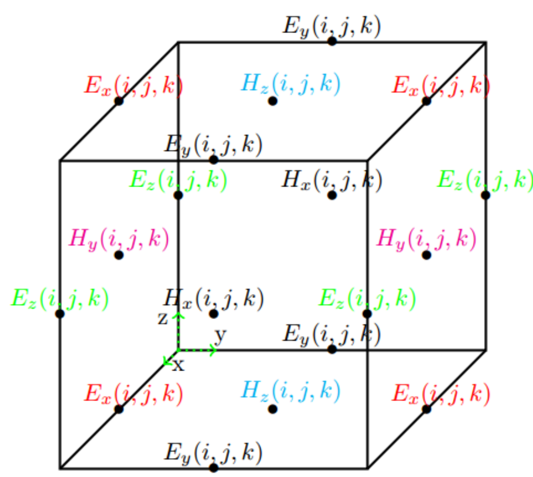

Figure 2: Position of the electric and magnetic field components about a unit cell in the Yee space Lattice (proposed in 1966).

By applying the Finite-Difference Scheme [1] and the Yee Cell (Figure 2) to one of the three equations, we obtain:

$$-\frac{a_x(i,j,k)}{\Delta z^2} b_y(i,j,k-1)E_x^p(i,j,k-1) - \frac{a_x(i,j,k)}{\Delta y^2} * b_z(i,j-1,k)E_x^p(i,j-1,k) + \left[1 + \frac{a_x(i,j,k)}{\Delta z^2} * \left(b_y(i,j,k) + b_y(i,j,k-1)\right) + \frac{a_x(i,j,k)}{\Delta y^2}\left(b_z(i,j,k) + b_z(i,j-1,k)\right)\right] E_x^p(i,j,k) - \frac{a_x(i,j,k)}{\Delta z^2} b_y(i,j,k)E_x^p(i,j,k+1) - \frac{a_x(i,j,k)}{\Delta y^2} b_z(i,j,k)E_x^p(i,j+1,k) + \frac{a_x(i,j,k)}{\Delta x \Delta y}\left[b_z(i,j,k)\left(E_y^p(i+1,j,k) - E_y^p(i,j,k)\right) - b_z(i,j-1,k)\left(E_y^p(i+1,j-1,k) - E_y^p(i,j-1,k)\right)\right] + \frac{a_x(i,j,k)}{\Delta x \Delta z}\left[b_y(i,j,k)\left(E_z^p(i+1,j,k) - E_z^p(i,j,k)\right) - b_y(i,j,k-1)\left(E_z^p(i+1,j,k-1) - E_z^p(i,j,k-1)\right)\right] = \frac{a_x(i,j,k)}{\Delta z}\sum_{k=0}^{p-1}\left[b_{1y}(i,j,k)H_y^k(i,j,k) - b_{1y}(i,j,k-1)H_y^k(i,j,k-1)\right] - \frac{a_x(i,j,k)}{\Delta y}\sum_{k=0}^{p-1}\left[b_1(i,j,k)H_z^k(i,j,k) - b_1(i,j-1,k)H_z^k(i,j-1,k)\right] - a_{1x}(i,j,k)\sum_{k=0}^{p-1}E_x^k(i,j,k) - a_x(i,j,k)J_x^p \quad (32)$$

From the previous equation, we notice that each electric field variable is in relationship with adjacent 10 electric components. In addition, the magnetic field variables are known as the order is still lower than those of the electric field. Therefore, the three general equations should be performed in order to obtain matrix equation, as follow:

$$AE^p = J^p + \beta^{p-1} \quad (33)$$

where β^{p-1} and J^p represent summation on the second member for $k = 0\ to\ p-1$ and excitation at order p, respectively. Even if the inverse matrix A^{-1} is performed once at the beginning of the simulation, important resources remain needed when using conventional Laguerre--FDTD formulation. Moreover, this formulation has been turned out to be not suitable for long time duration [14].

4. Efficient implementation of 3-D LFDTD Method

This part is devoted to overcoming time limitation by adopting Balanced Laguerre Polynomials [19] and optimising the memory storage limitation. By this way, the memory storage optimization can be achieved by employing factorizing splitting technique [18] and solving the matrix system in two sub-steps. As proposed in [19], the technique consists on performing the huge sparse matrix equation ($Equation$ 28) by adding perturbation term and finally by proposing an efficient node numbering scheme in order to obtain tri-diagonal matrices which are suitable for LU Decomposition.

4.1. Factorizing-Splitting Scheme

As mentioned previously, the main step consists in decomposing the matrix $D_H D_E$ into two triangular matrices$A\ and\ B$, as shown:

$$D_H D_E = A + B \quad (34)$$

where:

$$A = \begin{bmatrix} a_x b_z D_{2y} & 0 & 0 \\ a_y b_z D_x D_y & a_y b_x D_{2z} & 0 \\ a_z b_y D_x D_z & a_z b_x D_y D_z & a_z b_y D_{2x} \end{bmatrix}$$

$$B = \begin{bmatrix} a_x b_y D_{2z} & a_x b_z D_x D_y & a_x b_y D_x D_z \\ 0 & a_y b_z D_{2x} & a_y b_x D_y D_z \\ 0 & 0 & a_z b_x D_{2y} \end{bmatrix}$$

Therefore, we use $A\ and\ A$ instead of $D_H D_E$ and the huge sparse matrix becomes:

$$[I - A - B]E^p = D_H V_H^{p-1} + V_E^{p-1} + J_E^p \quad (35)$$

By adding a perturbation term $AB\left(E^p + V_E^{p-1}\right)$, the huge matrix equation becomes:

$$[I - A - B + AB]E^p = [AB + I]V_E^{p-1} + D_H V_H^{p-1} + J_E^p \quad (36)$$

As explained in [10], one can solve the previous equation according to sub-steps, as given:

$$[I - A]E^{*p} = [A + I]V_E^{p-1} + D_H V_H^{p-1} + J_E^p \quad (37)$$

$$[I - B]E^p = E^{*p} B V_E^{p-1} \quad (38)$$

where:

$$\begin{cases} E^{*p} = \left[E_x^{*p}\ E_y^{*p}\ E_z^{*p}\right]^T & are\ nonphysical\ values \\ E^p = \left[E_x^p\ E_y^p\ E_z^p\right]^T & are\ physical\ values \end{cases}$$

This new formulation leads to six matrix equation system:

$$[1 - a_x b_z D_{2y}] E_x^{*p} = -a_{1x}[1 + a_x b_y D_{2z}] \sum_{k=0}^{p-1} E_x^k + a_x a_{1y} b_z D_x D_y \sum_{k=0}^{p-1} E_y^k + a_x a_{1z} b_y D_x D_z \sum_{k=0}^{p-1} E_z^k + a_x b_{1y} \sum_{k=0}^{p-1} H_y^k - a_x b_{1z} \sum_{k=0}^{p-1} H_z^k - a_{1x} \sum_{k=0}^{p-1} E_x^k - J_x^p \quad (39)$$

$$a_{1y}[1 + a_y b_z D_{2x}] \sum_{k=0}^{p-1} E_y^k - a_y b_z D_x D_y E_x^{*p} + a_{1z} a_y b_x D_y D_z \sum_{k=0}^{p-1} E_z^k - a_y b_{1x} D_z \sum_{k=0}^{p-1} H_x^k + a_y b_{1z} D_x \sum_{k=0}^{p-1} H_z^k - a_{1y} \sum_{k=0}^{p-1} E_y^k - J_y^p \quad (40)$$

$$[1 - a_z b_y D_{2x}] E_z^{*p} = -a_{1z}[1 + a_z b_x D_{2y}] \sum_{k=0}^{p-1} E_z^k - a_z b_y D_x D_z E_x^{*p} - a_z b_x D_y D_z E_y^{*p} a_z b_{1z} \sum_{k=0}^{p-1} H_y^k - a_{1z} \sum_{k=0}^{p-1} E_z^k - J_z^p \quad (41)$$

$$[1 - a_z b_x D_{2y}] E_z^p = E_z^{*p} + a_z a_{1z} b_x D_{2y} \sum_{k=0}^{p-1} E_z^k \quad (42)$$

$$[1 - a_y b_z D_{2x}] E_y^p = E_y^{*p} + a_y a_{1y} b_z D_{2x} \sum_{k=0}^{p-1} E_y^k - a_y b_x D_y D_z E_z^p - a_{1z} a_y b_x D_y D_z \sum_{k=0}^{p-1} E_z^k \quad (43)$$

$$[1 - a_x b_y D_{2z}] E_x^p = E_y^{*p} + a_x a_{1x} b_y D_{2z} \sum_{k=0}^{p-1} E_x^k - a_x b_z D_x D_y E_y^p - a_x b_y D_x D_z E_z^p - a_x b_z D_x D_y \sum_{k=0}^{p-1} E_y^k - a_x b_y D_x D_z \sum_{k=0}^{p-1} E_z^k \quad (44)$$

By applying the Finite-Difference Scheme to the previous six matrix equations, six tri-diagonal matrices which are suitable for LU Decomposition when using an optimum node numbering scheme, are obtained. Therefore, one can propose an efficient node numbering technique [19] to optimize the memory storage limitation. As mentioned in the introduction, we propose an optimum node numbering scheme to obtain tri-diagonal matrices where the non null coefficients are three diagonals (Figure 3).

Therefore, the lower and upper triangular matrices which result in LU Decomposition of each matrix are two-diagonal matrices. Furthermore, the two matrices remain sparse and are very useful. This new node numbering scheme is applied to six tri-diagonal matrices.

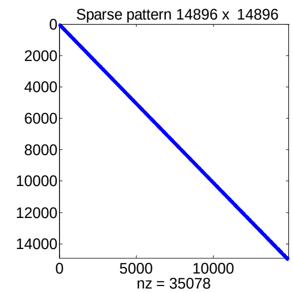

Figure 3: One of the six tri-Diagonal Matrices obtained after the new node numbering Technique.

5. Rectangular Patch Antenna

As mentioned above, the second purpose in this paper is to model Rectangular Patch Antenna with numerical methods (conventional $FDTD$ and Laguerre $FDTD$ Method). When modelling radiating structures, one should model ground plane, radiating patch and the strip line. The proposed rectangular patch antenna is fed either by uniform or non uniform lines. Therefore, non uniform microstrip lines are dealt with in order to achieve Ultra Wideband matching.

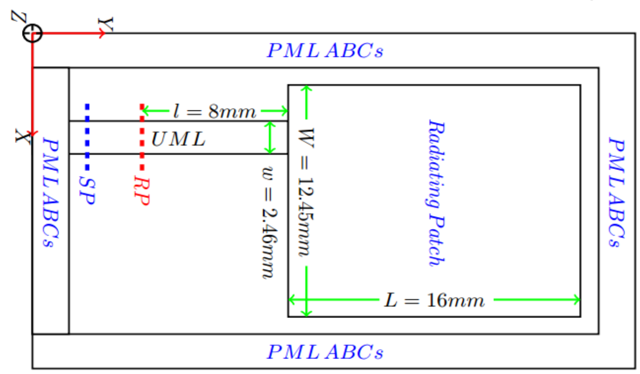

Figure 4. To view of the rectangular patch antenna fed by uniform microstrip line (UML).

On the contrary of the conventional $FDTD$ where PEC (Perfect Electric Conductor) or PMC (Perfect Magnetic Conductor) are modelled by putting to zero all electric or magnetic components, in the Laguerre $FDTD$ formulation, PEC, ground plane (taking as a PEC) and radiating patch are modelled by replacing the correspondent coefficients by zeros except those of the main diagonal of the matrix. The patch antenna of interest (Figures 4-5-6) has $f_r = 7.5\ GHz$, $\varepsilon_r = 2.2$ and $h = 0.8mm$ as resonant frequency, relative permittivity of the substrate and the substrate thickness, respectively [2].

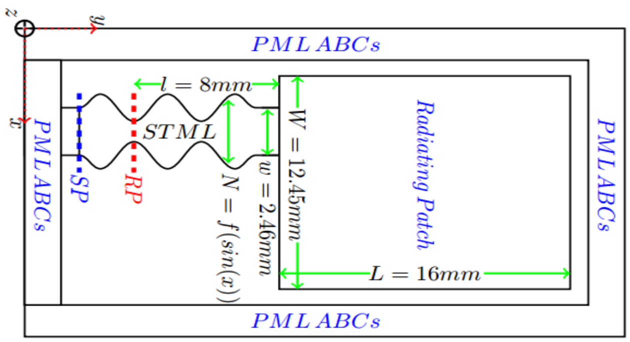

Figure 6: To view of the rectangular patch antenna fed by sinus tapered microstrip line (STML).

Although, knowing these parameters, its dimensions can be obtained through formulas given below [21]:

$$W = \frac{c}{2f_r\sqrt{\frac{\varepsilon_r+1}{2}}} \quad (45)$$

$$L_{eff} = \frac{c}{2f_r\sqrt{\varepsilon_{eff}}} \quad (46)$$

$$\Delta L = 0.412h\frac{(\varepsilon_{eff}+0.3)}{(\varepsilon_{eff}-0.258)}\frac{\left(\frac{W}{h}+0.264\right)}{\left(\frac{W}{h}+0.8\right)} \quad (47)$$

$$L = L_{eff} - 2\Delta L \quad (48)$$

where c is velocity in the vacuum.

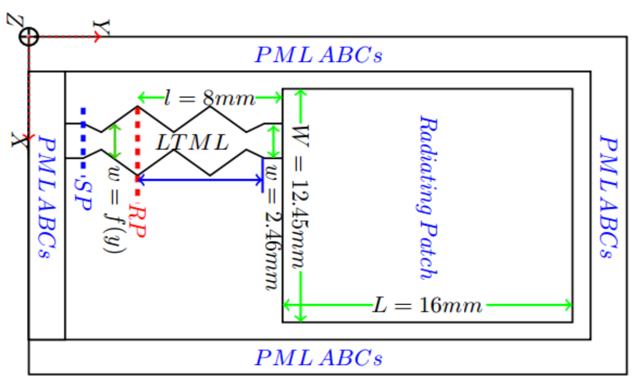

Figure 5: To view of the rectangular patch antenna fed by linear tapered microstrip line (LMTL).

When modelling patch antennas, one can either pick up the pattern, gain, directivity, return loss, input impedance, etc. In this paper, we take into account two configurations of microstrip to determine the return loss S_{11} and the input impedance Z_{in} of the antenna (Figures 5-6). In fact, Return loss S_{11} can be obtained from the FFT (Fast Fourier Transformer) based on a relationship between the reflected and incident waves [2].

6. Results and Discussion

First, we simulate the microstrip line which is seen as an infinity microstrip line in order to generate the incident wave at the reference plane. An infinity microstrip line is modelled by extending its ends in the PML area. The total wave is calculated at the reference plane with respect to both the antenna and the line parameters. So, the difference between the total and incident waves gives us the reflected wave in time-domain (Figures 4-5-6). Other parameters are summarized in the tables 1-2.

Table 1: N_l (Basis functions number), Time scale factor S, Δx, Δy, Δz cell size.

Δx(mm)	Δy(mm)	Δz(mm)	Factor S	N_l
0.389	0.4	0.265	$2.4* 10^{12}$	400

Table 2: Time step number T_{max}, N_{pml} : thickness of the PML area and Δt the time step

N_x	N_y	N_z	T_{max}	N_{pml}	$\Delta t(ps)$
50	88	9	1320	7	0.441

Table 3: Comparison of Simulation results of the Rectangular Patch Antenna

Method	Δt	Iterations	Memory (MB)
FDTD	0.4417ps	11448	9
LFDTD	0.4417ps	400	73

The results obtained (Incident and Total Waves (Figure 8)) by means of the $Laguerre-FDTD$ formulation are similar to those of the $FDTD$ formulation. One can increase the number of Laguerre Basis functions in order to improve $LFDTD$ accuracy.

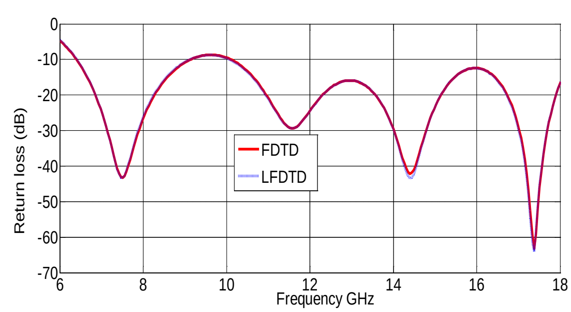

Figure 7: Return Loss S_{11} of the Rectangular Patch Antenna which is fed by UML.

We notice that from 5 GHz to 19 GHz that the antenna presents 4 resonances with narrow bandwidth (Figure 7) and the results of both methods are also identical in the frequency domain. As previously mentioned, the main goal of the use of the non uniform microstrip lines is to achieve the ultra wideband matching of the input impedance. So, two types of tapered microstrip line (Figure 5-6) are dealt with in this paper.

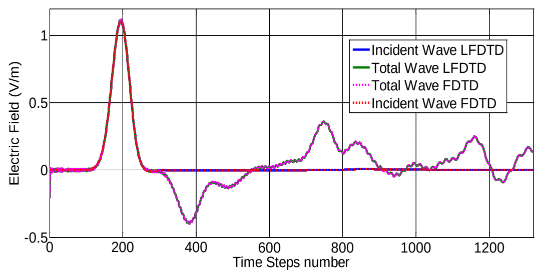

Figure 8: Incident and Total Waves obtained by simulating the Rectangular Patch Antenna fed by UML.

By using linear tapered microstrip line rather than the uniform microstrip line, the Patch Antenna is matched from 6 GHz to 18.5 GHz (Figure 10). In addition, we can achieve good level of matching by increasing the ration R expressed as:

$$R = \frac{L_1}{l - L_1} \quad (49)$$

where L_1 uniform portion's length and l is is the total length of the microstrip line (Figure 5). In the case of sinus tapered microstrip line, the antenna's matching band ranges from 6.5 GHz to 18.5 GHz (Figure 11). So, the more the magnitude increases the higher the matching level is. The matching level depends on the number and the magnitudes of ripples. All calculations in this paper have been performed on an $Intel^R\ Core^{TM} i_3$ 2.30 GHz Machine.

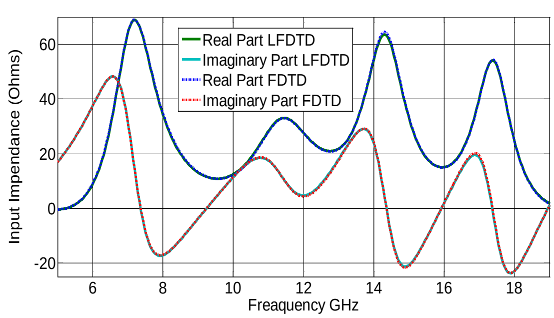

Figure 9: Input Impedance of the rectangular patch antenna fed by UML.

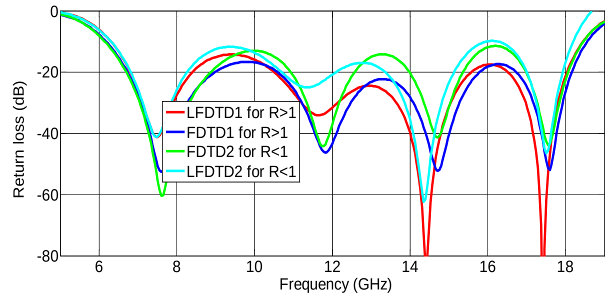

Figure 10: Return Loss S_{11} of the rectangular patch antenna fed by linear tapered microstrip line.

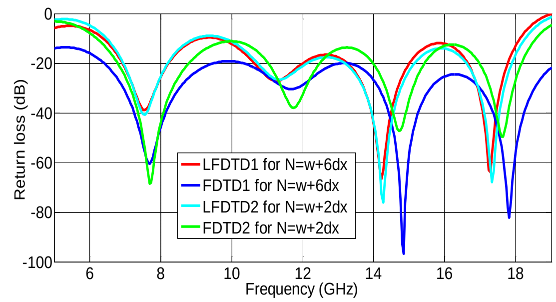

Figure 11: Return Loss $S_{11}$of the rectangular patch antenna fed by sinus tapered microstrip line.

7. Conclusion

Simulation based upon Laguerre Polynomials leads to unconditionally stable $FDTD$ method. A node numbering scheme to achieve tri-diagonal a matrix has been proposed in this paper and the resulting matrices are sparse and suitable for LU Decomposition. Consequently, the huge sparse matrix obtained from the conventional $LFDTD$ formulation is easily handled. The time steps Δt_{FDTD} and Δt_{LFDTD} were taken equal and the numerical examples indicate that the $LFDTD$ method need more memory storage. Therefore, there are several applications (microwaves devices, filters, connectors, couplers, Patch antennas) that potentially can make the use of the new efficient implementation of $Laguerre - FDTD$ formulation. The new programming code has been applied to simulate the Rectangular Patch Antenna fed by microstrip lines. In order to achieve Ultra Wideband Matching, the uniform microstrip is replaced by non uniform microstrip lines. Configurations such as linear and sinus tapered microstrip lines contribute to have good level of matching with Ultra Bandwidth.

References

[1] A. Taflove, “Computational Electrodynamics: THE FINITE−DIFFERENCE TIME−DOMAIN”, Artech House, Boston. London.

[2] DENNIS M. SILLIVAN, “Electromagnetic Simulation Using the FDTD Method. IEEE Press, 445 Hoes Lane, P.O. Box 1331, Piscataway, NJ 08855-1331.

[3] Takefumi Namiki, “A New FDTD Algorithm Based on Alternating Direction Implicit Method”, IEEE TRANSACTIONS ON MICROWAVE THEORY AND TECHNIQUES, VOL. 47, NO. 10, OCTOBER 1999.

[4] Fenghua Zheng, Zhizhang Chen, and Jiazong Zhang, “Toward the Development of a Three-Dimensional unconditionally Stable Finite-Difference Time-Domain Method”, IEEE TRANSACTIONS ON MICROWAVE THEORY AND TECHNIQUES, VOL. 48, NO. 9, SEPTEMBER 2000.

[5] Fenghua Zheng and Zhizhang Chen, “Numerical Dispersion Analysis of the Unconditionally Stable 3-D ADI–FDTD Method”, IEEE TRANSACTIONS ON MICROWAVE THEORY AND TECHNIQUES, VOL. 49, NO. 5, MAY 2001.

[6] Guilin Sun and Christopher W. Trueman, “ACCURACY OF THREE UNCONDITIONALLY-STABLE FDTD SCHEMES FOR SOLVING MAXWELL’S EQUATIONS”, ACES JOURNAL, VOL. 18, NO. 4, NOVEMBER 2003.

[7] Young-Seek Chung , Tapan K. Sarkar , Baek Ho Jung, and Magdalena Salazar-Palma, “An Unconditionally Stable Scheme for the Finite-Difference Time-Domain Method”, IEEE TRANSACTIONS ON MICROWAVE THEORY AND TECHNIQUES, VOL. 51, NO. 3, MARCH 2003.

[8] Young-Seek Chung, Tapan K. Sarkar ,Baek Ho Jung, Magdalena Salazar-Palma , Zhong Ji, Seongman Jang, and Kyungjung Kim, “Solution of Time Domain Electric Field Integral Equation Using the Laguerre Polynomials”, IEEE TRANSACTIONS ON ANTENNAS AND PROPAGATION, VOL. 52, NO. 9, SEPTEMBER 2004.

[9] Guilin Sun and Christopher W. Trueman, “Approximate Crank–Nicolson Schemes for the 2-D Finite-Difference Time-Domain Method for TE_z Waves”, IEEE TRANSACTIONS ON ANTENNAS AND PROPAGATION, VOL. 52, NO. 11, NOVEMBER 2004.

[10] Guilin Sun and Christopher W. Trueman, “Efficient Implementations of the Crank–Nicolson Scheme for the Finite-Difference Time-Domain Method”, IEEE TRANSACTIONS ON MICROWAVE THEORY AND TECHNIQUES, VOL. 54, NO. 5, MAY 2006.

[11] Omar Ramadan, “Unconditionally Stable Crank–Nicolson Nearly PML Algorithm for Truncating Linear Lorentz Dispersive FDTD Domains”, IEEE TRANSACTIONS ON MICROWAVE THEORY AND TECHNIQUES, VOL. 54, NO. 6, JUNE 2006.

[12] Masoud Movahhedi, Abdolali Abdipour , Alexandre Nentchev, Mehdi Dehghan, and Siegfried Selberherr, “Alternating-Direction Implicit Formulation of the FiniteElement Time-Domain Method”, IEEE TRANSACTIONS ON MICROWAVE THEORY AND TECHNIQUES, VOL. 55, NO. 6, JUNE 2007.

[13] B. H. Jung and T. K. Sarkar, “ANALYSIS OF TRANSIENT ELECTROMAGNETIC SCATTERING WITH PLANE WAVE INCIDENCE USING MODFDM”, Progress In Electromagnetic Research, PIER 77, 111–120, 2007.

[14] Krishna Srinivasan, “Multiscale EM and Circuit Simulation Using the Laguerre-FDTD Scheme for Package Aware Integrated-Circuit Design”, Georgia Institute of Technology August 2008

[15] Krishna Srinivasan, Pradeep Ya dav, A. Ege Engin, Madhava n Swa minathan and Myunghyun Ha, “Fast EM/Circuit Transient Simulation Using Laguerre Equivalent Circuit (SLeEC)”, IEEE TRANSAC TIONS ON ELECTROMAGNETIC C OMPATIBILITY, VOL. 51, NO. 3, AUGUST 2009.

[16] Myunghyun Ha, Krishna Srinivasan, and Madhavan Swaminathan, “Transient Chip-Package Cosimulation of Multiscale Structures Using the Laguerre-FDTD Scheme”, IEEE TRANSACTIONS ON ADVANCED PACKAGING, VOL. 32, NO. 4, NOVEMBER 2009.

[17] B. H. Jung and Z. Mei and T. K. Sarkar, “TRANSIENT WAVE PROPAGATION IN A GENERAL DISPERSIVE MEDIA USING THE LAGUERRE FUNCTIONS IN A MARCHING-ON-IN-DEGREE (MOD) METHOD-OLOGY”, Progress In Electromagnetics Research, Vol. 118, 135–149, 2011.

[18] Yan-Tao Duan, Bin Chen, Da-Gang Fang , and Bi-Hua Zhou, “Efficient Implementation for 3-D Laguerre-Based Finite-Difference Time-Domain Method”, IEEE TRANS-ACTIONS ON MICROWAVE THEORY AND TECHNIQUES, VOL. 59, NO. 1, JANUARY 2011.

[19] Myunghyun Ha, “EM SIMULATION USING THE LAGUERRE-FDTD SCHEME FOR MULTISCALE 3-D INTERCONNEC- TIONS”, Georgia Institute of Technology December 2011.

[20] X. L. Travassos, Jr. ,S. L. Avila, D. Prescott , A. Nicolas , and L. Krähenbühl, “Optimal Configurations for Perfectly Matched Layers in FDTD Simulations”, IEEE TRANS-ACTIONS ON MAGNETICS, VOL. 42, NO. 4, APRIL 2006.

[21] Girish Kumar K. P. Ray, “Broadband Microstrip Antennas”, Artech House, Boston London.

Nabih EL OUAZZANI received the Ph.D degree in microwave circuits, especially microwave filters, from the university of Limoges - France at the IRCOM Institute (Institute of research in microwave and optical communications) in 1995. Since 1995 he has been a professor at the faculty of science and techniques – Fès, Morocco (FST –Fès). He has been carrying out many activities with respect to research and education. The disciplines that are relevant to his expertise are high frequency technology and telecommunication technology. He is also involved in the research area of electromagnetic compatibility (EMC) in VLSI and MMIC circuits. The methods of simulation and analysis are based upon modelling and applying numerical codes such as FDTD and MNA. He has authored and co-authored numerous publications in the field of recursive microwave filters, planar circuit interconnexions, skin effect and arbitrary non uniform transmission lines. He coorganized many international conferences related the ICT and telecommunication subjects in Morocco and participated in scientific committees. He is now at the head of the team of EMC research within the LSSC laboratory (Laboratory of signals, systems and components) at the FST-Fès (Morocco).

NGENDAKUMANA Gaspard received Bachelor and Master degrees in science and Technology and in telecommunications and microwave devices from Faculty of Science and Technology - Cadi Ayyad University in Marrakech and National School of Applied Sciences in Fès - University Sidi Mohamed Ben Abdellah, Morocco in 2009 and 2011, respectively. He is currently pursuing his Ph.D. degree in science and engineering techniques at the Faculty of Science and Technology in Fès - University Sidi Mohamed Ben Abdellah. His research interests include numerical method, Electromagnetic Compatibility, Microwave antennas, embedded and RFID systems, RF and microwave applications.