Effectiveness of Orthogonal Benzzoubeir-Laguerre Moments on the Analysis of Two-Dimensional Synthetic Image Color

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Abstract
Information, communication and technology have known a larger innovation in the field of research. This technology has a several directions of studies, among the axis most necessary is the image processing. This field underwent a major change by adopting new procedures of analysis, such as Image Analysis by orthogonal Moments. Among this orthogonal moments family: Zernike Moments, Tchebichef Moments and Legendre Moments... This article presents a new set of orthogonal Benzzoubeir-Laguerre moments. These new moments can be effectively employed as devices of model in the analysis of the two-dimensional images. Most important also, this article contains three innovating aspects: 1. The first invention relates to the creation of a new method for treatment and analysis of the digital images 2D: Image analysis by orthogonal Benzzoubeir-Laguerre moment. This new method supplements the need for any calculation of the various existing applications and the experimental results prove in a conclusive way the effectiveness of this new techni-que, which one can regard it as a new descriptor of analysis. 2. The second invention relates to the creation of new mathematical applications: New definition of orthogonal coefficient in the new finite interval [0.1]. New definition of continuous orthogonal Benzzoubeir-Laguerre moments. New definition of discreet (total discretization) orthogonal Benzzoubeir-Laguerre moments. 3. The third invention relates to the creation of a new automatic procedure for the treatment and analysis of two-dimensional images 2D; who allows to analyze the various types of images (synthetic, real…) , with a minimum time, and a smaller error.

1. Introduction

For several years, image processing has known a great innovation in the field of research and the development of the technical, data-processing tools, and mathematical concept. The mathematical concept of moments has been around for many years and has been utilized in many fields ranging from mechanics and statistics to pattern recognition and image understanding. Describing images with moments instead of other more commonly used image features, means that global properties of the image are used rather than local properties. Historically, the first significant work considering moments for pattern recognition was performed by Hu [1]. From methods of algebraic invariants, he derived a set of seven moment invariants, using non-linear combinations of geometric moments. These invariants remain the same under image translation, rotation and scaling. Since then, moments and functions of moments have been used as shape descriptors in a variety of applications in image analysis, like visual pattern recognition [2], [3], object classification [4], template matching [5], edge detection [6], pose estimation [7], robot vision [8], and data compression [9].In all these application, geometric moments and their extensions in the form of radial and complex moments have played important roles in characterization of image shape in extracting features that are invariant with image plane transformation.

Teague [10] introduced moments with orthogonal basis functions, with the additional property of minimal information redundancy in a moment set. In this class Legendre and Zernike moments have been extensively researched in the recent past, and several new techniques have emerged involving orthogonal moment based feature detectors [11], [12], [13].

R. Mukundan [14] and S.Benzzoubeir [15], introduces a new set of orthogonal moment functions based on the discrete Tchebichef polynomials. The Tchebichef moments can be effectively used as pattern features in the analysis of two-dimensional images. The implementation of moments proposed in this paper does not involve any numerical approximation, since the basis set is orthogonal in the discrete domain of the image coordinate space.

I show that it is possible to reconstruct the image by orthogonal Benzzoubeir-Laguerre moments, these new moments based on the orthogonal Laguerre polynomials define on another new finite interval [0.1] with a new coefficient of orthogonality,that I defined in the same finite interval and know by the name of Orthogonal Benzzoubeir-Laguerre coefficient

This article is organized as follows: In section 2: the general definition of continuous and discreet moments in image (2D). Section 3: definition of continuous and discreet geometric moments in image (2D). Section 4, 5: recall the definition of continuous and discrete orthogonal

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Legendre and Zernike moments. Section 6: Explanation and comment for difficulties of discritization in the conventional Laguerre moments in infinite interval. Section 7: justifications and reasons to define the orthogonal Benzzoubeir-Laguerre moments.Sections 8 Justification to choose a new orthogonal Benzzoubeir-Laguerre coefficient. Variation of this new orthogonal coefficient compared with the conventional Laguerre coefficient. Sections 9, 10: new theorems of Continuous and Discreet Orthogonal Benzzoubeir-Laguerre Moments. To finite by reconstructed image with these new Moments. In Sections 11, present a role of recurrence equation in the analysis of image. Experimental results are given in section 12: presents the test data and results used to validate the theoretical framework presented above and also to establish the feature representation capability of orthogonal Benzzoubeir-Laguerre moments through synthetic image reconstruction .A comparative analysis between orthogonal Benzzoubeir-Laguerre, Legendre and geometric moments is also given in the table (IV) of synthetic images reconstruction. A comparative reconstruction error by Benzzoubeir-Laguerre and Legendre is given in Fig.15 and in the tableV.

2. General Definition of Moments

The general definition of moment functions $\psi_{pq}(x, y)$, of order $(p+q)$, of an image intensity function $f(x, y)$ can be given as follows:

$$\Psi_{pq} = \int_x \int_y \psi_{pq}(x, y) f(x, y) dxdy. \quad p, q = 0.1.2... \quad (1)$$

Where $\psi_{pq}(x, y)$, is the moment weighting kernel (also known as the basis function); the basis functions may have a range of useful properties that may be passed onto the moments, producing descriptions which can be invariant under rotation and scale. To apply the moments in digital images, (1) needs to be expressed in discrete form:

$$\Psi_{pq} = \sum_x \sum_y \psi(x, y) f(x, y). \quad (2)$$

Can distinguish two types of moments, not orthogonal basis functions result in not orthogonal moments, for example: Geometric moments; and orthogonal basis functions result in orthogonal moments, where as Legendre moment, Zernike moments, Tchebichef moments...

3. Geometric Moments

For several years, geometric moments are the most popular types of moments and have been used for a number of image processing tasks. The two-dimensional geometric moment of order $(p+q)$ of a function $f(x, y)$ is defined:

$$m_{pq} = \int\int x^p y^q f(x, y) dxdy, \quad (3)$$

The two-dimensional moments for a discrete image of size $(N \times N)$, is given by:

$$m_{pq} = \sum\sum x_i^{\,p} y_j^{\,q} f(i, j) \Delta x_i \Delta y_j, \quad (4)$$

The monomial product is the basis function for this moment definition. Thus, geometric moments are not orthogonal since this basis function is not orthogonal. The uniqueness theorem states that the moment set is unique for a given image function and the exis-tence theorem states that the moments of all orders exist. These two theorems give rise to the reconstruction property of moments.

4. The Continuous and Discrete Orthogonal Zernike Moments

4.1 Continuous orthogonal Zernike moments

$$Z_{nl} = \frac{(n+1)}{\pi} \iint_{r,\theta} R_{nl}(r) \times e^{-\hat{j}l\theta} f(r, \theta) dr d\theta;$$

$$\hat{j} = \sqrt{-1}, \qquad |l| \leq n, \qquad n - |l| \text{ is even } (5)$$

4.2 Discreet orthogonal Zernike moments

In this equation, R_{nl} are the Zernike radial polynomials of degree n . A discrete approximation of (5) is :

$$Z_{nl} = \frac{2(n+1)}{\pi(N-1)^2} \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} R_{nl}(r_{ij}) \times e^{-j\theta_{ij}} f(i, j);$$

$$0 \leq r_{ij} \leq 1 \quad (6)$$

Where the image coordinate transformation to the interior of the unit circle is given by:

$$r_{ij} = \sqrt{(c_1 i + c_2)^2 + (c_1 j + c_2)^2},$$

$$\theta_{ij} = \tan^{-1}\left(\frac{c_1 j + c_2}{c_1 i + c_2}\right), \quad c_2 = -\frac{1}{\sqrt{2}}, \quad c_1 = \frac{\sqrt{2}}{N-1}$$

$$(i, j) \in \{0,1,2,..., N-1\} \quad (7)$$

Therefore these orthogonal moments of Zernike have a polar structure and a radial distribution, from where all the pixels locating at interior circle are treated; on the other hand the pixels locating outside the circle are not treated (FIG), the watch that the four corners of the image are not treated. So the orthogonal moments of Zernike go strongly increased the error rate; contrary to the moments Cartesian which analyses all and all pixels of the image.

5. The Continuous and Discrete Orthogonal Legendre Moments

5.1 Continuous Orthogonal Legendre Moments

The two most important orthogonal moments that have found several applications in the field of image shape representation are the Legendre moments and the Zernike moments. The Legendre moments of order $(p+q)$ are defined as:

$$\lambda_{pq} = \frac{(2p+1)(2q+1)}{4}\int_{-1}^{1}\int_{-1}^{1} P_p(x)P_q(y)f(x,y)dxdy,$$

$$p, q = 0,1,2,3,... \quad (8)$$

Where $P_n(x)$ is the Legendre polynomial of order n.

5.2 Discreet Orthogonal Legendre Moments

On an image coordinate space $(i, j) \in \{0.1.2....N-1\}$ the above moment integral has the following discrete approximation:

$$\lambda_{pq} = \frac{(2p+1)(2q+1)}{(N-1)^2}\sum_{i=0}^{N-1}\sum_{j=0}^{N-1} P_p\left(\frac{2i-N+1}{N-1}\right)\times P_p\left(\frac{2j-N+1}{N-1}\right) f(i,j) \quad (9)$$

The inverse moment transform which follows from the orthogonal of Legendre polynomials in the continuous domain, can be similarly expressed with:

$$f(i,j) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\lambda_{mn}P_m\left(\frac{2i-N+1}{N-1}\right)P_n\left(\frac{2j-N+1}{N-1}\right),$$

$$i, j = 0,1,2,3,...., N-1. \quad (10)$$

6. Difficulties of Discretization in the Conventional Orthogonal Laguerre Moments in Infinite Interval $[0.+\infty[$

6.1 The Conventional Continuous Orthogonal Laguerre Moments

The Conventional Continuous Orthogonal Laguerre moments in interval $[0.+\infty[$ of order $(m+n)$, are defined:

$$\lambda_{mn} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tilde{L}_m(x)\tilde{L}_n(y)f(x,y)dxdy, \quad (11)$$

$$\tilde{L}_m(x) = e^{-x}L_m(x), \text{ And } \tilde{L}_m(y) = e^{-y}L_m(y),$$

$$m, n = 0,1,2,... \quad (12)$$

6.2 Difficulty the discretization of conventional continuous orthogonal Laguerre Moments

The expression (11) present:

Difficulties of the discretization in the infinite interval $[0.\infty[$.

The infinite word does not have significance on the physical field.

The discretization on an infinite interval is unrealisable.

The coefficient of orthogonality in finite interval $[0.1]$ has great variations compared to that calculate in the interval $[0.\infty[$ (see Fig1 and Fig 2).

With this reason, it is necessary to develop a new method which eliminates all the difficulties from the discretization; and respecting all the logical sequences, by justifying the choice of each new application.

7. Reasons to Define a New Orthogonal Benzzoubeir-Laguerre Moments

7.1 Reasons

First logical reason: why the definition of new method for analysis by orthogonal Benzzoubeir-Laguerre moments?

This new creativity defined a new method for treatment and analysis of the digital images 2D. This new method supplements the need for any calculation of the various existing applications.

Second logical reason: why the discretization?

First of all, the image is together of pixels, each value of pixel presents an intensity of image. Thereafter, the image it is a discrete system, so all the mathematical applications or formulas must be discrete and not continuous. One more, the discretization will be realizable if and only if the interval is finished.

Third logical reason why the complete discretization?

The orthogonal Benzzoubeir-Laguerre moments are completely discrete moments, and its formula (17) does not have any function of uncertainty. Knowing that, the complete discretization is a discretization which respects purely mathematical rules. The objective, to eliminate all the functions from uncertainties $(\Delta x, \Delta y)$. Consequently, these new moments do not have any approximation or an uncertainty on the treatment and the analysis of the image.

Fourth logical reason: why the orthogonality?
The moments of Benzzoubeir-Laguerre are orthogonal moments; then what they are defined on the same basis of the orthogonal moments of Laguerre. Knowing that the advantage of orthogonality, it is that it eliminates data-processing instability when the size of the image is large.
Fifth logical reason why the choice a new finite interval [0.1]?

1) The Laguerre moments calculate in infinite interval $[0.+\infty[$ are completely different to this calculate in another finite interval $[a.b]$ with $[a.b]\subset[0.+\infty[$;

$$\int_0^\infty\int_0^\infty g(x,y)dxdy \neq \int_a^b\int_a^b g(x,y)dxdy, \quad (13)$$

$$g(x,y)=\tilde{L}_m(x)\tilde{L}_n(y)f(x,y), \quad (14)$$

2) Orthogonal Benzzoubeir-Laguerre coefficient defined in new finite interval $[0.1]$ has a great variation compared to that calculates in other finite interval. (See the Table I and Fig1). Then the orthogonal Benzzoubeir-Laguerre coefficient is completely different to this of Laguerre coefficient. With to these last conditions presented in 1 and 2, and according to the difficulties presented in paragraph 6. It is necessary to define the orthogonal Benzzoubeir-Laguerre Moments.

7.2 For what these new moments take a new name: Benzzoubeir-Laguerre moments?

The orthogonal Benzzoubeir Laguerre moments present three innovating aspects:

1. The first invention relates to the creation of a new method for the treatment and the analysis of the digital images 2D. The experimental results prove in a conclusive way the effectiveness of this new technique, which one can regard it as a new descriptor of device.
2. The second invention relates to the creation of new mathematical formulates:
New definition of orthogonal Benzzoubeir-Laguerre coefficient.
New definition of continuous orthogonal Benzzoubeir-Laguerre moments.
New definition of discreet (the complete discretization) orthogonal Benzzoubeir-Laguerre moments. The orthogonal Benzzoubeir-Laguerre moments are completely discreet and its formula (17) does not have any function of uncertainty. Therefore these new moments, do not have any approximation or an uncertainty on the level of the treatment and analysis of image. What makes the treatment faster and the error rate is smaller. 3. The third invention relates to the creation of a new automatic procedure for the treatment and the analysis of two-dimensional digital images 2D. Some works in the image processing are based on the calculate of matrix or on the product of convolution. But the disadvantages of these two applications are:
Increasing of execution time.
Instability of data-processing when the size of image is large.
Respect the dimensional order of each matrix or each vector which describes the mathematical formulas or equations.
To avoid these three problems, and to have an effective solution, third invention presents a new procedure, which treats and analyzes pixels by pixels, the various types of two-dimensional images. According to the flow chart presented in the table(III).

8. Reasons to Calculate a Orthogonal Benzzoubeir-Laguerre Coefficient in Finite Interval

8.1 Justification the variation of orthogonal Benzzoubeir-Laguerre Coefficient

8.1.1 In finite interval $[0.1]$

Some values of orthogonal Benzzoubeir-Laguerre coefficient, calculated in finite interval $[0.1]$; present in the table I:

TABLE I

x	0	0.1	0.2	0.4	0.6	0.8	1
C(x)	0	0.095	0.181	0.329	0.451	0.551	0.632
C	$\prec 1$	$\prec 1$	$\prec 1$	$\prec 1$	$\prec 1$	≤ 1	≤ 1

The variation of orthogonal Benzzoubeir-Laguerre coefficient in finite interval $[0.1]$ is given in the figure1.

8.1.2 In infinite interval $[1.+\infty[$

Some values of orthogonal Benzzoubeir-Laguerre coeffecient, calculated of to values $x \geq 1$, present in the table II.

TABLE II

x	1.2	1.8	2	3	4	5	6
C(x)	0.699	0.835	0.865	0.950	0.982	0.993	0.99
C	≈ 1	≈ 1	≈ 1	≈ 1	≈ 1	≈ 1	≈ 1

Variation of orthogonal Benzzoubeir-Laguerre coefficient in all $x \geq 1$, (see Fig2).

8.2 Explanation and commentary

We remark, the coefficient of orthogonality has a larger variation in interval $[0,1]$, for all $0 \le x \le 1$. On the other hand, for all $x \ge 1$, the coefficient of orthogonality tends towards 1.

Now, neglecting the great variation of x in the interval $[0,1]$, and supposing that the coefficient of orthogonality is equal to 1; then in this case, not respecting the standards of quality for the treatment and the analysis of image. Because if the coefficient of orthogonality is equal to 1in the interval $[0,1]$:

The error rate will be increased.
The logical reasoning will be ignored.
The mathematical rules will not be respected.

8.3 Conclusion:

1) The coefficient of orthogonality defined in finite interval $[0.1]$ is different to this calculate in infinite interval from 0 to ∞. Even thing for a new continuous and discrete orthogonal moments.
2) The variation of orthogonal Benzzoubeir-Laguerre coeffi cient tends towards 1 for any elements superior to 1, which is identical to the orthogonal Laguerre coefficient calculate in the old interval $[0.+\infty[$ see the Table II and Fig 2.

9. New Definition of Continuous Orthogonal Benzzoubeir-Laguerre Moments

Theorem1:
Continuous Orthogonal Benzzoubeir-Laguerre moments as defined by:

$$BL_{pq} = \frac{1}{(c_{(BL)})^2} \iint_D \tilde{L}_p(x)\tilde{L}_q(y) f(x,y)\,dxdy, \qquad (15)$$

$$\tilde{L}_p(x) = e^{-x} L_p(x),\ \ \tilde{L}_q(y) = e^{-y} L_q(y), \qquad (16)$$

$L_p(x),\ L_q(y)$: Laguerre polynomials of order (p,q) respectively.

f, Presents the intensity of the pixel coordinates $(i,j),\ D = [0.1],$ The new field of definition the variable (x,y).

$c_{(BL)}$: Orthogonal Benzzoubeir-Laguerre coefficient.

Since, the continuous orthogonal moments of Benzzoubeir-Laguerre are defined in finite interval $[0.1]$, then the discretization is realizable.

10. New Definition of Discrete Orthogonal Benzzoubeir-Laguerre Moments

10.1 Definition of Continuous Orthogonal Benzzoubeir-Laguerre Moments

Theorem2:
On an image coordinate space $(i,j) \in \{0,1,2,...,N-1\}$; the above moment integral has new following discrete approximation in interval $[0,1]$:

$$BL_{pq} = \frac{1}{(N-1)^2 (c_{(BL)_n})^2} \sum_{i=0}^{N-1}\sum_{j=0}^{N-1} \tilde{L}_p(x_i)\tilde{L}_q(y_j) f(i,j),$$
$$p,q = 0.1.2..., \qquad (17)$$

10.2 Reconstructed Image by orthogonal Benzzoubeir- Laguerre moments

Theorm3:
Moments reconstructed by the orthogonal Benzzoubeir-Laguerre polynomials in the discrete base can be similarly expressed by:

$$f(i,j) = \sum_{m=0}\sum_{n=0} BL_{mn} \tilde{L}_m\left(\frac{i}{N-1}\right)\tilde{L}_n\left(\frac{j}{N-1}\right),$$
$$m,n = 0.1.2..., \qquad (18)$$

11. Role of Recurrence Equation on the Analysis of Image

Formerly, to analysis an image, by the moments. It was obliged to write all the expressions of the polynomials for each order given, (see TABLEIII relation (α)). On the other hand, with the new method, one gives only the equation of recurrence by initializing the first two polynomials, (see relation (β)). Advantageously, with this new algorithm, the experimental results prove in a conclusive way its effectiveness:

The method of image analysis is easier and simplified.
The execution time is minimal.
The rate of calculation is minimal.
The number of order is increased.

TABLE III

1. Give the order n, m such as n>m.
2. Write the expressions of all the polynomials P until n.

$$\begin{cases} P_0(x) \\ P_1(x) \\ . \\ . \\ P_m(x) \\ until \\ P_n(x) \end{cases} (\alpha) \quad \Rightarrow \begin{cases} P_0(x), P_1(x) \\ \text{until } \{n\} \\ \text{write the recurrence equation} \end{cases} \quad (\beta)$$

3. With the order (m,n), calculate moments (17) .
4. With the order (m,n), calculate the image reconstructed (18).

12. Experimental Results

Since, the orthogonal Benzzoubeir-Laguerre moments keep the same properties as those of the Legendre moments (even Cartesian space, even bases orthogonal, even finished interval of the variables, even distribution...).Then, the tests employed to validate the theoretical framework are based on the comparison between Benzzoubeir-Laguerre and Legendre moments. In this section presents the test data and results used to validate the theoretical framework presented above. And also to establish the feature representation capability of orthogonal Benzzoubeir-Laguerre moments through image reconstruction. A comparative analysis between orthogonal Benzzoubeir-Laguerre moments, geometric moments and Legendre moments is also given in the table (IV). A binary image of the letter "L" of pixel (20x20) (see Fig.3). The orthogonal Benzzoubeir-Laguerre moments, Legendre and geometric moments of the first few orders of the test image, computed using (17), (9), and (4), respectively, are given in Table IV, the table also shows the uniform range of orthogonal Benzzoubeir-Laguerre moments for different orders. The last table (V), present the error for each order calculate by the two methods: Benzzoubeir-Laguerre moments and Legendre moments.

TABLE IV Benzzoubeir-Laguerre, Legendre and Geometric Moments of The Teste in Image "L".

Order of Moments		Benzzoubeir-Laguerre	Legendre Moments	Geometric Moments
P	Q	BL_{pq}	$\lambda^{Leg[-1.1]}_{pq}$	m_{pq}
0	0	1.7872	1.7729	640
0	1	1.0053	0.2519	7008
1	0	0.3850	-1.0067	95776
0	2	-0.0620	-0.3715	1451376
1	1	-0.3555	-0.3648	23297152
2	0	1.0816	-0.3779	6288
0	3	0.6151	0.2387	69912
1	2	0.2437	0.7043	969288
2	1	-0.0255	-0.3520	14842872
3	0	0.5056	-1.7761	77648
0	4	0.2962	-0.5626	868632
1	3	0.1280	1.6737	12161608
2	2	0.0703	1.1842	1099368
3	1	0.0548	-0.7479	12285972
4	0	-0.2381	0.8818	17000768

TABLEV

Reconstructed Synthetic Image " L " (N=20)				
	Fig4	Fig 6	Fig 8	Fig12
$Error_{BL}$	0.1872	6. 2766	7.8123	23. 2863
Using Benzzoubeir-Laguerre Moments				
	Fig5	Fig 7	Fig 9	Fig13
$Error_{Leg}$	0.1729	16.6136	758.7052	1.2159e+006
Using Legendre Moments				
Maximum order of Moments	0	30	100	700

It may be noted that the total number of moment terms from order zero up to order n, in the case of both Legendre and Benzzoubeir-Laguerre moments is:

$$\Omega = \frac{(n+1)(n+2)}{2}, \quad (19)$$

Of order 4, we must have the 15 valour of Moments (see a table III). $\underbrace{\lambda_{00}, \lambda_{01}, \lambda_{10}, \lambda_{02}, \lambda_{11}, \lambda_{20}, \ldots, \lambda_{40}}_{15 elements},$.

Generally, the error reconstructed by Benzzoubeir-Laguerre moments varies regularly, gradually and converges towards a fixed value, of all the different orders. On the other hand, the error reconstructed by Legendre moments varies instan-taneous, spontaneous and diverges for any order superior to 100 of image "L" (see table IV). With the initial order, the moments calculated by the two methods are equal, more still, the two reconstructed images are identical, in companies of the same error (see Fig4. Fig5). For all , there is a total absence of all information of image reconstructed by Legendre moments (see Fig7. Fig9. Fig11. Fig13). Advantageously with the Benzzoubeir-Laguerre moments, information of image reconstructed still exists for any (seeFig6.Fig8.Fig10.Fig12).Finally figure 16 present the graphic interpretation of the two methods:

The reconstructed error calculated by Legendre moments varies overall instantaneous and spontaneous, according to three remarkable classes: For all the orders lower than 50, the error values converge towards a finished value. For all the orders vary between 60 and 100, the error varies frequent and remarkable. For all the higher orders strictly than 100, all the error values strongly diverge at 10^3 to 10^6 .

According to the new invention of Benzzoubeir-Laguerre moments, reconstruction error varies completely gradually, uniformly and convergent (see Fig16).

12. Conlusion

A new set of discrete orthogonal moment features based on Benzzoubeir-Laguerre moments has been proposed in this paper. And I added to the domain of research, a new technique based on calculation improved of moment by the new method

More, this article presents three new inventions:

1) Creation of a new method for the treatment and analysis of the digital images 2D: The experimental results prove in a conclusive way the effectiveness of this new invention, which results in the convergence of error for all the values with higher order. Contrary, with the method of Legendre, the error varies overall way, instantaneous and spontaneous, in company of a remarkable divergence.

2) Creation of new tools mathematical, and the new definitions and property of moments.

3) Creation of new automatic procedure for the treatment and analysis of two-dimensional digital images 2D:

The two methods follow the same new algorithmic stages: giving new data-processing programming; who minimizes the time of execution, improves calculation and facilitates the method of analysis of the various images.

Then, the experimental results conclusively prove the effectiveness of Benzzoubeir-Laguerre moments as feature descriptors of analysis, compared with geometric and Legendre moments. The result show, that the orthogonal Benzzoubeir-Laguerre moments are superior to the conventional orthogonal moments, such as Legendre moments Zernike moments and geometric moments.

Future applications: Creation and innovation of the new methods for the treatment and the analysis of the two-dimensional images, comparison with other moments, compression, rotation….

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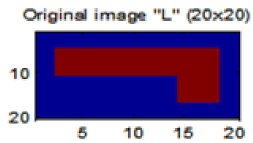


FIG 3

Image Reconstructed by different orders:

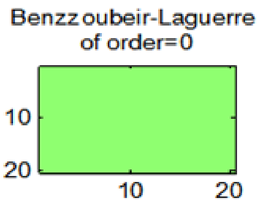


FIG 4

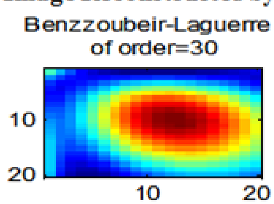


FIG 6

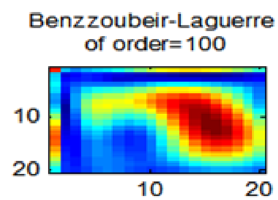


FIG 8

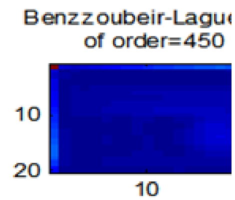


FIG 10

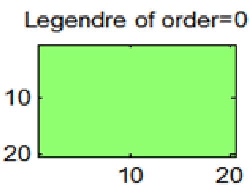


FIG 5

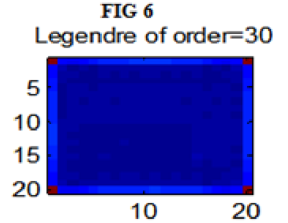


FIG 7

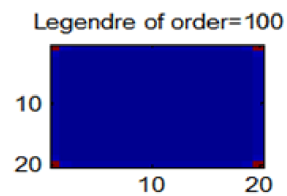


FIG 9

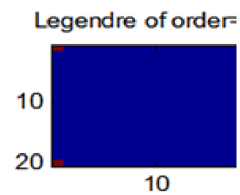


FIG 11

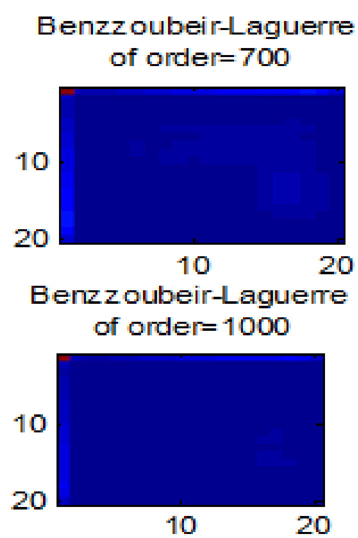


FIG 12
FIG 14

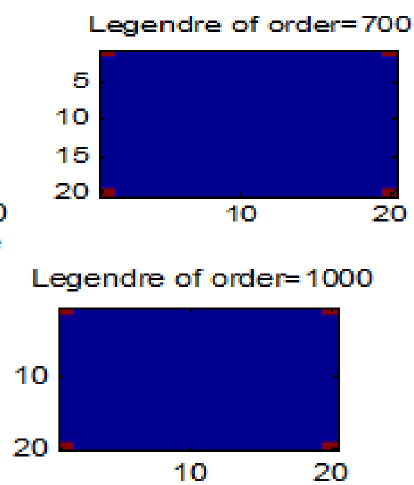


FIG 13
FIG 15

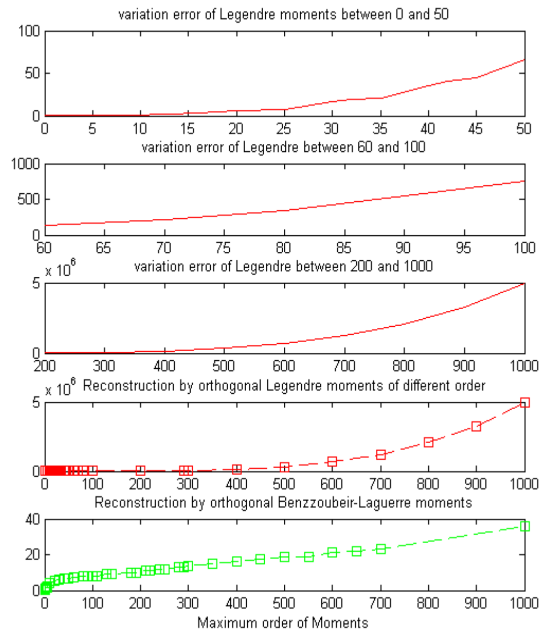
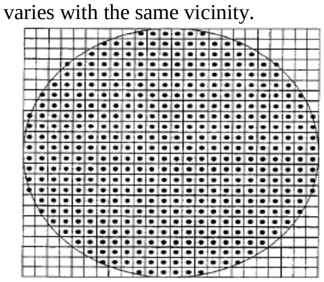


Fig16. Comparative reconstruction of errors with synthetic image (L=20x20) by Benzzoubeir-Laguerre and Legendre moments. The obstruction of the rectangles shows that the error varies with the same vicinity.



(FIG)

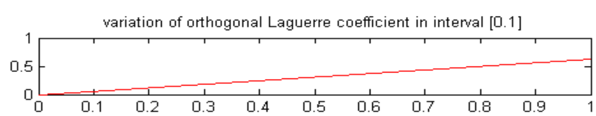


Fig.1

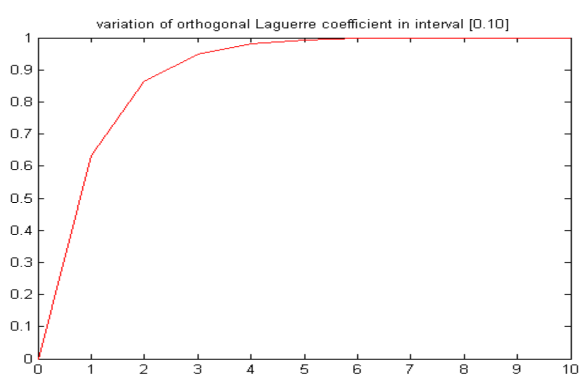


Fig.2

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