Image and Inverse Image of Refinement of Fuzzy Topogenous Order

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Abstract:
In this paper, the concept of fuzzy (semi-) topogenous order in the framework of fuzzy topologies, fuzzy proximities and fuzzy uniformities have been introduced. The refinement of fuzzy (semi-) topogenous order has been researched. On this basis, the image and inverse image of refinement of fuzzy (semi-) topogenous order have been defined by general order homomorphism (GOH). Some important properties of them have been obtained.

Keywords:
Fuzzy （semi-）topogenous order, refinement, GOH，image and inverse image.

1. Introduction

In his classic paper [1] of 1965, Zadeh introduced the fundamental concept of a fuzzy set. Subsequently, Chang [2] and others extended some basic concepts from general topology to fuzzy sets and developed a theory of fuzzy topological spaces. Katsaras [3] combined order structure with fuzzy topological structure and made an initial research. Recently, katsaras and petalas [4-6] introduced the fuzzy syntopogenous structure and studied the unified theory of fuzzy topology, fuzzy proximity and fuzzy uniformity. Wang [7,8] studied refinement of semi-topogenous order on completely distributive lattice and refinement of fuzzy (semi-) topogenous order. In this paper, we define image and inverse image of refinement of fuzzy (semi-) topogenous order by GOH, and study some important properties.

2. Preliminaries

In this paper, we use notation, which is standard for the "fuzzy mathematics", usually with-out explanation. I stands for the unit interval $[0,1]$ and let $I_1=[0,1)$. I^X denotes the family of all fuzzy subsets of a given set X. We will denote fuzzy sets by lower case Greek letters such as μ,λ,υ. For $\underline{\alpha}$ denotes fuzzy set which assumes the value α at each $x\in X$.

Definition2.1 [3] A binary relation η on I^X is a katsaras fuzzy topogenous order on X, if it satisfies the following axioms: (1) $(\underline{1},\underline{1}),(\underline{0},\underline{0})\in\eta$, (2)if $(\mu,\lambda)\in\eta$,

Then $\mu\le\lambda$, (3) if $\mu\le\mu_1,\lambda_1\le\lambda$ and $(\mu_1,\lambda_1)\in\eta$, then

$(\mu,\lambda)\in\eta$, (4) $(\mu_1\vee\mu_2,\lambda)\in\eta$ iff

$(\mu_1,\lambda)\in\eta,(\mu_2,\lambda)\in\eta$

and $(\mu,\lambda_1\wedge\lambda_2)\in\eta$ iff $(\mu,\lambda_1)\in\eta,(\mu,\lambda_2)\in\eta$.

Definition2.2 [4] A function $\tau:I^X\to I$ is called a fuzzy topology on X if it satisfies the following conditions:

(1) $\tau(\underline{0})=\tau(\underline{1})=1$, (2) $\tau(\mu_1\wedge\mu_2)\ge\tau(\mu_1)\wedge\tau(\mu_2)$ for each

$\mu_1,\mu_2\in I^X$, (3) $\tau(\vee_{i\in\Gamma}\mu_i)\ge\wedge_{i\in\Gamma}\tau(\mu_i)$ for

any $\{\mu_i\}_{i\in\Gamma}\subset I^X$. The pair (X,τ) is called a fuzzy topological space.

Let τ_1 *and* τ_2 be fuzzy topologies on X. We say τ_1

Manuscript received January 5, 2015
Manuscript revised January 20, 2015

is finer than τ_2 (or τ_2 is coarser than τ_1) iff $\tau_2(\lambda) \le \tau_1(\lambda)$ for all $\lambda \in I^X$. Let (X, τ_1) and (Y, τ_2) be fuzzy topological space. A function $f:(X,\tau_1) \to (Y,\tau_2)$ is called a fuzzy continuous map if $\tau_2(\lambda) \le \tau_1(f^{-1}(\lambda))$ for all $\lambda \in I^X$.

Definition2.3 [5] A function $P: I^X \times I^X \to \{0,1\}$ is called a fuzzy proximity on X, if it satisfies the following axioms: (1) $P(\mu,\rho) = P(\rho,\mu)$, (2) $P(\underline{1},\underline{0}) = 0$, (3) if $P(\mu,\lambda) = 0$, then $\mu \le \underline{1} - \lambda$, (4) $P(\mu, \rho \vee \lambda) = P(\mu,\rho) \vee P(\mu,\lambda)$, (5) if $P(\mu,\lambda) = 0$, there exists $\rho \in I^X$ such that $P(\mu,\rho) = 0 = P(\underline{1} - \rho, \lambda)$. The pair (X, P) is a Artico fuzzy proximity space.

Notation2.1 [6] Let X be a set and Ω_X be the set of all mappings $\alpha: I^X \to I^X$ such that: (1) $\alpha(\underline{0}) = \underline{0}$, (2) $\alpha(\mu) \ge \mu$, (3) $\alpha(\vee_{i\in\Gamma} \mu_i) = \vee_{i\in\Gamma} \mu_i$.

Remark2.1 (1) If $\alpha_{1,}\alpha_2 \in \Omega_X$, then $\alpha_1 \wedge \alpha_2 \in \Omega_X$ where $(\alpha_1 \wedge \alpha_2)(\mu) = \wedge\{\alpha_1(\mu_1) \wedge \alpha_2(\mu_2) | \mu = \mu_1 \vee \mu_2\}$,

(2) If $\alpha \in \Omega_X$, then $\alpha^{-1} \in \Omega_X$, where

$\alpha^{-1}(\mu) = \wedge\{\lambda \in I^X | \alpha(\underline{1} - \lambda) \le \underline{1} - \mu\}$.

Definition2.4 A subset U of Ω_X is called a Hutton fuzzy uniformity on X satisfying for $\alpha, \beta \in \Omega_X$, the following condition: (1) $\alpha \wedge \beta \in U$ iff $\alpha \in U$ and $\beta \in U$, (2)there exists $\alpha \in U$, (3)If $\alpha \in U$, there exists $\beta \in U$ such that $\beta \circ \beta \le \alpha$, (4) If $\alpha \in U$, then $\alpha^{-1} \in U$. The pair (X, U) is said to be a Hutton fuzzy uniform space.

3. The Fuzzy (Semi-)Topogenous Order

Definition3.1 [3] A function $\eta: I^X \times I^X \to I$ is called a fuzzy semi-topogenous order on X, if it satisfies the following axioms: (FT1) $\eta(\underline{1},\underline{1}) = \eta(\underline{0},\underline{0}) = 1$, (FT2) if $\eta(\mu,\lambda) \ne 0$, then $\mu \le \lambda$, (FT3) if $\mu \le \mu_1, \lambda_1 \le \lambda$, then $\eta(\mu_1,\lambda_1) \le \eta(\mu,\lambda)$.

Proposition3.1 Let η be a fuzzy semi-topogenous order on X and let the mapping $\eta^s: I^X \times I^X \to I$ defined by $\eta^s(\lambda,\mu) = \eta(\underline{1} - \mu, \underline{1} - \lambda), \forall \lambda, \mu \in I^X$.

Then η^s be a fuzzy semi-topogenous order on X.

Definition3.2. A fuzzy semi-topogenous order η is called symmetric if $\eta = \eta^s$, that is (FT4)

$\eta(\lambda,\mu) = \eta(\underline{1} - \mu, \underline{1} - \lambda), \forall \lambda, \mu \in I^X$.

Definition3.3. A fuzzy semi-topogenous order η is called fuzzy topogenous if for any $\lambda, \lambda_1, \lambda_2, \mu, \mu_1, \mu_2 \in I^X$

(FT5) $\eta(\lambda_1 \vee \lambda_2, \mu) = \eta(\lambda_1, \mu) \wedge \eta(\lambda_2,\mu)$, (FT6)

$\eta(\lambda, \mu_1 \wedge \mu_2) = \eta(\lambda,\mu_1) \wedge \eta(\lambda,\mu_2)$.

Definition3.4. A fuzzy semi-topogenous order η is called perfect if (FT7) $\eta(\vee_{i\in\Gamma}\lambda_i, \mu) = \wedge_{i\in\Gamma}\eta(\lambda_i,\mu),$.

for any $\{\mu, \lambda_i | i \in \Gamma\} \subset I^X$. A perfect fuzzy semi-topogenous order η is called biperfect if: (FT8)

$\eta(\lambda, \wedge_{i\in\Gamma}\mu_i) = \wedge_{i\in\Gamma}\eta(\lambda,\mu_i)$, for any $\{\lambda, \mu_i | i \in \Gamma\} \subset I^X$

Theorem3.1 Let $\eta_1, \eta_2 : I^X \times I^X \to I$ be perfect (resp. fuzzy topogenous, biperfect) fuzzy semi-topogenous order on X. Define the composition $\eta_1 \circ \eta_2$ of η_1 and η_2 on X by $\eta_1 \circ \eta_2(\lambda, \mu) = \sup_{\upsilon \in I^X}(\eta_1(\lambda, \upsilon) \wedge \eta_2(\upsilon, \mu))$. Then $\eta_1 \circ \eta_2$ is a perfect (resp. fuzzy topogenous, biperfect) fuzzy semi-topogenous order on X.

4. The Refinement of Fuzzy Topogenous Order

Definition4.1[7]. Let η be a fuzzy semi-topogenous order on X and $\xi \in I^X$,we consider a binary relation $\eta * \xi$ on I^X as follows: $\eta * \xi(\mu, \lambda)$ iff there exists $\delta \in I^X$ such that $\eta * \xi(\mu, \lambda) = \eta(\mu, \delta)$ and $\mu \vee (\delta \wedge \xi) \le \lambda$.

Theorem4.1 If η be a semi-topogenous order on X, then $\eta * \xi$ is a semi-topogenous order on X..

proof. (1) Let $\delta = \underline{0}$, then $\underline{0} \vee (\underline{0} \wedge \xi) \le \underline{0}$ and $\eta * \xi(\underline{0}, \underline{0}) = \eta(\underline{0}, \underline{0}) = 1$. Let $\delta = \underline{1}$, then $\underline{1} \vee (\underline{1} \wedge \xi) \le \underline{1}$ and $\eta * \xi(\underline{1}, \underline{1}) = \eta(\underline{1}, \underline{1}) = 1$. (2) if $\eta * \xi(\mu, \lambda) \ne 0$, there exists $\delta \in I^X$ such that $\eta * \xi(\mu, \lambda) = \eta(\mu, \delta) \ne 0$ then $\mu \le \delta$ and $\mu \vee (\delta \wedge \xi) \le \lambda$, so $\mu \le \lambda$. (3) if $\mu \le \mu_1, \lambda_1 \le \lambda$, there exists $\delta \in I^X$ such that $\mu \vee (\delta \wedge \xi) \le \mu_1 \vee (\delta \wedge \xi) \le \lambda_1 \le \lambda$ and $\eta * \xi(\mu_1, \lambda_1) = \eta(\mu_1, \delta) \le \eta(\mu, \delta) = \eta * \xi(\mu, \lambda)$. So $\eta * \xi$ is a semi-topogenous order on X..

Theorem4.2 If η be a semi-topogenous order on X , then (1) $\eta * \underline{0} = \le$; (2) $\eta * \underline{1} = \eta$.

proof. (1) if $\eta * \underline{0}(\mu, \lambda)$, then there exists $\delta \in I^X$ such that $\eta * \underline{0}(\mu, \lambda) = \eta(\mu, \delta)$ and $\mu \vee (\delta \wedge \underline{0}) \le \lambda$ such that $\mu \le \lambda$. Conversely, if $\mu \le \lambda$, let $\delta = \underline{1}$, then $\eta(\mu, \underline{1})$ and $\mu \vee (\underline{1} \wedge \underline{0}) \le \lambda$, so $\eta * \underline{0}(\mu, \lambda)$.

(2) if $\eta * \underline{1}(\mu, \lambda)$, then there exists $\delta \in I^X$ such that $\eta * \underline{1}(\mu, \lambda) = \eta(\mu, \delta)$ and $\mu \vee (\delta \wedge \underline{1}) \le \lambda$ i.e. $\delta \le \lambda$ so $\eta(\mu, \lambda)$. Conversely, if $\eta(\mu, \lambda)$, let $\delta = \lambda$, have $\eta(\mu, \delta)$ and $\mu \vee (\delta \wedge \underline{1}) \le \lambda$, so $\eta * \underline{1}(\mu, \lambda)$.

Theorem4.3 If η, η_1 be a semi-topogenous order on X and $\eta \le \eta_1$, $\xi, \xi_1 \in I^X$ and $\xi_1 \le \xi$, then $\eta * \xi \le \eta_1 * \xi_1$.

proof. if $\eta * \xi(\mu, \lambda)$, then there exists $\delta \in I^X$ such that $\eta * \xi(\mu, \lambda) = \eta(\mu, \delta)$ and $\mu \vee (\delta \wedge \xi) \le \lambda$. Since $\eta \le \eta_1$ and $\xi_1 \le \xi$, then $\eta_1(\mu, \delta)$ and $\mu \vee (\delta \wedge \xi_1) \le \mu \vee (\delta \wedge \xi) \le \lambda$ then $\eta_1 * \xi_1(\mu, \lambda)$.so $\eta * \xi \le \eta_1 * \xi_1$.

Remark4.1 If η be a semi-topogenous order on X and $\xi, \xi_1 \in I^X$, then (1) if $\xi_1 \le \xi$, then $\eta * \xi \le \eta * \xi_1$, (2) $\eta \le \eta * \xi \le "\le"$, $\forall \xi \in I^X$

Theorem4.4 If a fuzzy semi-topogenous order η is symmetric, then $\eta * \xi$ is symmetric too.

Proof: if $\eta = \eta^s$, has $\eta(\mu, \lambda) = \eta(\underline{1} - \lambda, \underline{1} - \mu)$, $\forall \lambda, \mu \in I^X$, Since $(\eta * \xi)^s(\mu, \lambda) = \eta * \xi(\underline{1} - \lambda, \underline{1} - \mu)$, then there exists $\delta \in I^X$ such that $\eta * \xi(\underline{1} - \lambda, \underline{1} - \mu) = \eta(\underline{1} - \lambda, \delta)$ and $\underline{1} - \lambda \vee (\delta \wedge \xi) \le \underline{1} - \mu$, Then $\eta(\mu, \delta)$ and $\mu \vee (\delta \wedge \xi) \le \lambda$ has $\eta * \xi(\mu, \lambda)$, so $(\eta * \xi)^s(\mu, \lambda) = \eta * \xi(\mu, \lambda)$.

5. Image and Inverse Image of Refinement of Fuzzy Topogenous Order

Definition5.1 Let (X, η_1), (Y, η_2) be fuzzy topogenous space, mapping $f : I^X \to I^Y$ is called a GOH, if it satisfies the following axioms: (1) $f(\alpha) = \underline{0}$ iff $\alpha = \underline{0}$, (2) For any $\alpha_i \in I^X$ $(i \in I)$, $f(\vee_{i \in I} \alpha_i) = \vee_{i \in I} f(\alpha_i)$, (3) For any $\beta_j \in I^Y$ $(j \in J)$, $f^{-1}(\vee_{j \in J} \beta_j) = \vee_{j \in J} f^{-1}(\beta_j)$ there for any $\beta \in I^Y$ have $f^{-1}(\beta) = \vee\{\alpha \in I^X | f(\alpha) \le \beta\}$.

Definition5.2 Let mapping $f : I^X \to I^Y$ be a GOH, and $\eta * \xi$ be a semi-topogenous order on X, define a binary relation $f(\eta * \xi)$ on Y as follows: $f(\eta * \xi)$ (α, β) iff there exists $\mu, \lambda \in I^X$ such that $\eta * \xi$ (μ, λ) and

$\alpha \le f(\mu)$, $f(\lambda) \le \beta$.

Theorem5.1 Let $\eta * \xi$ be a semi-topogenous order on X, then $f(\eta * \xi)$ be a semi-topogenous order on Y, and we call $f(\eta * \xi)$ is image of $\eta * \xi$ by f.
proof. (1) Since $\eta * \xi\ (\underline{0},\underline{0}) = 1$ and $f(\underline{0}) = \underline{0}$, then $f(\eta * \xi)\ (\underline{0},\underline{0}) = 1$. As $\eta * \xi\ (\underline{1},\underline{1}) = 1$ and $f(\underline{1}) = \underline{1}$, then $f(\eta * \xi)\ (\underline{1},\underline{1}) = 1$. (2) If $f(\eta * \xi)\ (\alpha,\beta) \ne 0$ iff there exists $\mu,\lambda \in I^X$ such that $\eta * \xi\ (\mu,\lambda) \ne 0$ and $\alpha \le f(\mu)$, $f(\lambda) \le \beta$, then $\alpha \le f(\mu) \le f(\lambda) \le \beta$ so $\alpha \le \beta$. (3) Let $\alpha \le \alpha_1$, $\beta_1 \le \beta$ and $f(\eta * \xi)\ (\alpha_1,\beta_1)$ iff there exists $\mu,\lambda \in I^X$ such that $\eta * \xi\ (\mu,\lambda)$ and $\alpha \le \alpha_1 \le f(\mu)$, $f(\lambda) \le \beta_1 \le \beta$, then $f(\eta * \xi)\ (\alpha,\beta)$, namely $f(\eta * \xi)\ (\alpha_1,\beta_1) \le f(\eta * \xi)\ (\alpha,\beta)$.

Theorem5.2 Let $f : I^X \to I^Y$ be a GOH, then $f(\eta * \xi) \le f(\eta) * f(\xi)$
proof. If $f(\eta * \xi)\ (\alpha,\beta)$ iff there exists $\mu,\lambda \in I^X$ such that $\eta * \xi\ (\mu,\lambda)$ and $\alpha \le f(\mu)$, $f(\lambda) \le \beta$, iff there exists $\delta \in I^X$ such that $\eta(\mu,\delta)$ and $\mu \vee (\delta \wedge \xi) \le \lambda$, and $\alpha \le f(\mu)$, $f(\lambda) \le \beta$. Then there exist $f(\delta) \in I^Y$, such that $f(\eta)\,(f(\mu), f(\delta))$ and $\alpha \le f(\mu)$, then $f(\eta)\,(\alpha, f(\delta))$ and $\alpha \vee (f(\delta) \wedge f(\xi)) \le f(\mu \vee (\delta \wedge \xi)) \le f(\lambda) \le \beta$, i.e. there exists $f(\delta) \in I^Y$ such that $f(\eta)\,(\alpha, f(\delta))$ and $\alpha \vee (f(\delta) \wedge f(\xi)) \le \beta$, so $f(\eta) * f(\xi)\ (\alpha,\beta)$.

Proposition5.1 Let $f : I^X \to I^Y$ be a GOH, η,η_1 be a semi-topogenous order on X, and $\xi,\xi_1 \in I^X$, then (1) If $\eta \le \eta_1$ implies $f(\eta * \xi) \le f(\eta_1 * \xi)$. (2) If $\xi \le \xi_1$ Implies $f(\eta * \xi_1) \le f(\eta * \xi)$. (3) $f(\eta) \le f(\eta * \xi) \le f(\le)$, (for any $\xi \in I^X$).

Proposition5.2 Let $f : I^X \to I^Y$ and $g : I^Y \to I^z$ be GOH, η be semi-topogenous order on X, and $\xi \in I^X$, then $(g \circ f)\ (\eta * \xi) = g(f(\eta * \xi)) \le g(f(\eta)) * g(f(\xi))$.

Definition5.3 Let mapping $f : I^X \to I^Y$ be a GOH, and $\eta * \xi$ be a semi-topogenous order on Y, define a binary relation $f^{-1}(\eta * \xi)$ on X as follows: $f^{-1}(\eta * \xi)\ (\mu,\lambda)$ iff $\eta * \xi(f(\mu), \underline{1} - f(\underline{1} - \lambda))$.

Theorem5.3 Let mapping $f : I^X \to I^Y$ be a GOH, and $\eta * \xi$ be a semi-topogenous order on Y, then for any $\mu,\lambda \in I^X$ have $f^{-1}(\eta * \xi)\ (\mu,\lambda)$ iff there exists $(\alpha,\beta) \in I^Y$, such that $\eta * \xi\ (\alpha,\beta)$ and $\mu \le f^{-1}(\alpha)$, $f^{-1}(\beta) \le \lambda$.
proof. If $f^{-1}(\eta * \xi)\ (\mu,\lambda)$, then $\eta * \xi(f(\mu), \underline{1} - f(\underline{1} - \lambda))$, let $\alpha = f(\mu)$, $\beta = \underline{1} - f(\underline{1} - \lambda)$, so have $\eta * \xi\ (\alpha,\beta)$ and $\mu \le f^{-1}(f(\mu)) = f^{-1}(\alpha)$, $f^{-1}(\beta) = f^{-1}(\underline{1} - f(\underline{1} - \lambda)) \le \lambda$. Conversely, if there exists $\alpha,\beta \in I^Y$ such that $\eta * \xi\ (\alpha,\beta)$ and $\mu \le f^{-1}(\alpha)$, $f^{-1}(\beta) \le \lambda$, then $f(\mu) \le f(f^{-1}(\alpha)) \le \alpha$, $\beta \le \underline{1} - [ff^{-1}(\underline{1} - \beta)] \le \underline{1} - (\underline{1} - \lambda))$, so $\eta * \xi(f(\mu), \underline{1} - f(\underline{1} - \lambda))$, i.e. $f^{-1}(\eta * \xi)\ (\mu,\lambda)$.

Theorem5.4 Let $\eta * \xi$ be a semi-topogenous order on Y, then $f^{-1}(\eta * \xi)$ be a semi-topogenous order on X , and we call $f^{-1}(\eta * \xi)$ is inverse image of $\eta * \xi$ by f .
proof. (1) Since $\eta * \xi\ (\underline{0},\underline{0}) = 1$ and $f^{-1}(\underline{0}) = \underline{0}$, then $f^{-1}(\eta * \xi)\ (\underline{0},\underline{0}) = 1$. As $\eta * \xi\ (\underline{1},\underline{1}) = 1$ and $f^{-1}(\underline{1}) = \underline{1}$, then $f^{-1}(\eta * \xi)\ (\underline{1},\underline{1}) = 1$. (2) If $f^{-1}(\eta * \xi)\ (\mu,\lambda) \ne 0$ iff there exists $\alpha,\beta \in I^Y$ such that $\eta * \xi\ (\alpha,\beta) \ne 0$ and $\mu \le f^{-1}(\alpha)$, $f^{-1}(\beta) \le \lambda$, then $\mu \le f^{-1}(\alpha) \le f^{-1}(\beta) \le \lambda$ so $\mu \le \lambda$. (3) Let $\mu \le \mu_1$, $\lambda_1 \le \lambda$ and $f^{-1}(\eta * \xi)\ (\mu_1,\lambda_1)$ iff there exists $\alpha,\beta \in I^Y$ such that $\eta * \xi\ (\alpha,\beta)$ and $\mu \le \mu_1 \le f^{-1}(\alpha)$, $f^{-1}(\beta) \le \lambda_1 \le \lambda$, then $f^{-1}(\eta * \xi)\ (\mu,\lambda)$, namely $f^{-1}(\eta * \xi)\ (\mu_1,\lambda_1) \le f^{-1}(\eta * \xi)\ (\mu,\lambda)$.

Theorem5.5 Let $f : I^X \to I^Y$ be a GOH, $f^{-1}(\eta * \xi) \le f^{-1}(\eta) * f^{-1}(\xi)$.
proof. If $f^{-1}(\eta * \xi)\ (\mu,\lambda)$ iff there exists $\alpha,\beta \in I^Y$, such that $\eta * \xi\ (\alpha,\beta)$ and $\mu \le f^{-1}(\alpha)$, $f^{-1}(\beta) \le \lambda$. iff there exists $\delta \in I^Y$, such that $\eta(\alpha,\delta)$ and $\alpha \vee (\delta \wedge \xi) \le \beta$, and $\mu \le f^{-1}(\alpha)$, $f^{-1}(\beta) \le \lambda$ .Then there exists $f^{-1}(\delta) \in I^X$, such that $f^{-1}(\eta)\,(f^{-1}(\alpha), f^{-1}(\delta))$ and

$\mu \leq f^{-1}(\alpha)$, then $f^{-1}(\eta)\,(\mu, f^{-1}(\delta))$ and $\mu \vee (f^{-1}(\delta) \wedge f^{-1}(\xi)) \leq f^{-1}(\alpha \vee (\delta \wedge \xi)) \leq f^{-1}(\beta) \leq \lambda$, i.e. there exists $f^{-1}(\delta) \in I^X$, such then $f^{-1}(\eta)\,(\mu, f^{-1}(\delta))$ and $\mu \vee (f^{-1}(\delta) \wedge f^{-1}(\xi)) \leq \lambda$, so $f^{-1}(\eta) * f^{-1}(\xi)\,(\mu, \lambda)$.

Proposition5.3 Let $f : I^X \to I^Y$ be a GOH, η, η_1 be a semi-topogenous order on Y, and $\xi, \xi_1 \in I^Y$, then (1) If $\eta \leq \eta_1$ implies $f^{-1}(\eta * \xi) \leq f^{-1}(\eta_1 * \xi)$. (2) If $\xi \leq \xi_1$ Implie $f^{-1}(\eta * \xi_1) \leq f^{-1}(\eta * \xi)$. (3) $f^{-1}(\eta) \leq f^{-1}(\eta * \xi)$ $\leq f^{-1}(\leq)$, (for any $\xi \in I^Y$).

Proposition5.4 Let $f : I^X \to I^Y$ and $g : I^Y \to I^z$ be GOH, η be semi-topogenous order on Z, and $\xi \in I^Z$, then $(g \circ f)^{-1}\,(\eta * \xi) = f^{-1}(g^{-1}(\eta * \xi))$.

Acknowledgment

The project is supported by the soft science plan from Science and Technology Department of Sichuan Province under Grant No.2012ZR0125.

References

[1] L. A. Zadeh, Fuzzy Set, Inform and Control, 8 (1965), 338-353
[2] O. L. Chang, Fuzzy Topological Space, J.Math.Anal.Appl., 24 (1968), 182-190
[3] A. K. Katsaras, Fuzzy syntopogenous structures compatible with Lowen fuzzy uniformities and Artico-Moresco fuzzy proximities, Fuzzy Sets and Systems, 36 (1990), 375-393
[4] A.P. Sostak, On a fuzzy topological structures. Suppl. Rend. Cire. Mat Palermo Ser. II, 11 (1985), 89-103.
[5] G. Artico and R. Moresco, Fuzzy proximities, and totally bounded fuzzy uniformities, J. Math. Anal. Appl., 99 (1984), 320-337
[6] B. Hutton, Uniformities on fuzzy topological space, J. Math. Anal. Appl. 58 (1977), 559-571.
[7] H. Wang, The Refinement of Fuzzy Topogenous structure and It’s Properties. Fuzzy Systems and Math. 25(4) (2011), 83-86
[8] H. Wang, On Weakly Compact Sets in L-Fuzzy Topological Spaces, International Journal of Computer Science and Network Security, 10(6) (2010), 333-336